Written re-exam for the M. Sc. in Economics, Winter 2015-16

Advanced Macroeconomics

Master's course

February 15, 2016

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 5 pages in total.

The weighting of the problems is:

Problem 1: 30 %, Problem 2: 50 %, Problem 3: 10 %. Problem 4: 10 %.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 Consider a Blanchard OLG model for a closed economy with a public sector, public debt, and lump-sum taxation. The dynamics of the economy are described by the differential equations

$$\dot{C}_t = (F_K(K_t, L) - \delta - \rho)C_t - m(\rho + m)(K_t + B_t), \tag{1}$$

$$\dot{K}_t = F(K_t, L) - \delta K_t - C_t - G, \qquad K_0 > 0 \text{ given}, \tag{2}$$

$$\dot{B}_t = [F_K(K_t, L) - \delta]\, B_t + G - T_t, \qquad B_0 > 0 \text{ given}, \tag{3}$$

the condition

$$\lim_{t\to\infty} B_t e^{-\int_0^t [F_K(K_s,L)-\delta]ds} \leq 0, \tag{4}$$

and a requirement that households satisfy their transversality conditions. Here, C_t is aggregate private consumption, K_t is physical capital, L is population = labor supply, B_t is public debt, G is government spending on goods and services, T_t is net tax revenue (= gross tax revenue−transfer payments), and F is an aggregate neoclassical production function with constant returns to scale and satisfying the Inada conditions. The other symbols stand for parameters and all these are positive; L and G are positive constants. A dot over a variable denotes the derivative w.r.t. time t.

a) Briefly interpret the equations (1) - (3), the weak inequality (4), and the parameters.

b) Assuming $B_0 > 0$, and a balanced budget for all $t \geq 0$, construct a phase diagram for the resulting two-dimensional dynamics. Make sure you explain how the curves and arrows you draw are implied by the model. Illustrate the path the economy follows for an arbitrary $K_0 > 0$. It is understood that G and B_0 are "modest" relative to the production possibilities of the economy, given this K_0. Make sure you explain how the illustrated path is implied by the model.

We will first do comparative dynamics.

c) Suppose that two countries, Country I and Country II, are both well described by the model. The countries are similar at time $t = 0$, except that Country 1 has less debt than Country 2, that is, $B_0^I < B_0^{II}$ (and as a consequence the countries possibly also differ w.r.t. T_0). The countries have the same K_0. Comment on the implied long-run differences between the countries.

Now we shift to dynamic analysis of just Country 1. Suppose Country I has been in its steady state until time $t_0 > 0$. Then, suddenly the government changes fiscal policy so that $T_t = \bar{T}$, where $\bar{T}$ is a constant which is smaller than the tax revenue in the old steady state.

d) Define what is meant by fiscal policy being sustainable. Is the fiscal policy $(G, \bar{T})$ sustainable? Why or why not? *Hint:* there are different approaches; one approach uses that if a is a positive constant, then $\int_{t_0}^{\infty} e^{-a(t-t_0)}dt = 1/a$.

Suppose that at time $t_1 > t_0$, taxation in country I again changes such that for $t \geq t_1$ the government budget is balanced and people rightly expect it to remain so. Between t_0 and t_1 we can not precisely pin down the evolution of the economy because the market participants did not know in advance *when* a fiscal tightening - or debt default - would occur. Yet, qualitatively something *can* be said about the position of the economy in the (K, C) plane immediately after time t_1.

e) Construct a phase diagram in the (K, C) plane to illustrate a possible position of the economy immediately after time t_1 as well as the movement of the economy over time afterwards.

f) Illustrate by graphical time profiles the corresponding motion over time of T_t, B_t, C_t, and K_t for $t \geq t_1$. Comment.

Problem 2 Consider a single firm that is a price taker in all markets. The firm chooses a plan $(L_t, I_t)_{t=0}^{\infty}$ to maximize

$$V_0 = \int_0^{\infty} \left(F(K_t, L_t) - G(I_t, K_t) - w_t L_t - pI_t \right) e^{-rt} dt \quad \text{s.t.} \tag{1}$$

$$L_t \geq 0, I_t \text{ free (i.e., no restriction on } I_t), \tag{2}$$

$$\dot{K}_t = I_t - \delta K_t, \qquad K_0 > 0 \text{ given}, \tag{3}$$

$$K_t \geq 0 \text{ for all } t \geq 0. \tag{4}$$

Here F is a neoclassical production function with constant returns to scale and satisfying the Inada conditions. The stock of capital and the input of labor are denoted K_t and L_t, respectively; G is a function representing the capital installation costs, I_t is gross investment, w_t is the real wage, $p > 0$ is a constant real price of investment goods, $r > 0$ a constant real interest rate, and $\delta > 0$ a constant capital depreciation rate. The installation cost function satisfies

$$G(0, K) = 0, \;\; G_I(0, K) = 0, \;\; G_{II}(I, K) > 0, \quad \text{and} \quad G_K(I, K) \leq 0,$$

for all (I, K).

a) Set up the current-value Hamiltonian. Let the adjoint variable be denoted q_t. Derive the first-order conditions and state the necessary transversality condition (TVC). *Hint:* in this problem the necessary TVC has the standard form for an infinite horizon optimal control problem with discounting.

b) Interpret q_t. Show that the optimal level of investment at time t is a function of q_t, p, and K_t.

From now on we consider the special case $G(I, K) = \frac{I^2}{2K}$.

c) Express the optimal I_t/K_t as a function of q_t and p.

Assume that the firm is a representative firm in a small open economy (SOE) with perfect competition in all markets and free mobility of financial capital, but no mobility of labor across borders. The real interest rate and the price on investment goods that the firm faces are given from the world market and identical to r and p, respectively. The labor force in the SOE is a constant, $\bar{L}$.

d) Determine the equilibrium real wage in the SOE at time t.

e) Show that the firm's first-order conditions result in two coupled differential equations in K_t and q_t.

f) Construct the corresponding phase diagram. Comment! *Hint:* it can be shown that the curve representing $\dot{q} = 0$ has negative slope (at least in a neighborhood of the steady state).

g) Determine the steady-state value of q.

h) Illustrate in the phase diagram the path followed by (K_t, q_t) for an arbitrary $K_0 > 0$. Comment!

i) Assume that until time $t_0 > 0$, the system has been in steady state with $(K, q) = (K^*, q^*)$. Then an unexpected shift in p to the level $p' > p$ occurs. Assume that the new price is rightly expected to stay at the new level forever. Illustrate in the phase diagram (possibly a new one) the path followed by (K_t, q_t) for $t > t_0$. *Hint:* it can be shown that the new saddle path necessarily will be positioned *above* the old (a fact which to some people is surprising).

j) Illustrate in a new diagram the time profile for q_t and K_t for $t \geq 0$. Give an intuitive explanation of the long-run effect on K_t and q_t, respectively, of the rise in p.

Problem 3 Consider the market for some financial asset. Suppose the market participants have rational expectations and focus on the expected rate of return vis-a-vis the risk-free interest rate, r, which we assume is a positive constant. Thus the no-arbitrage condition is

$$\frac{d_t + E_t p_{t+1} - p_t}{p_t} = r > 0, \qquad t = 0, 1, 2, \dots. \tag{*}$$

where d_t is the dividend, p_t is the market price of the asset at time t, and E_t is the expectation operator conditional on the information available up to and including period t. Consider the stochastic process

$$b_t = \beta^{-t} x_t, \qquad t = 0, 1, 2, \dots \qquad 0 < \beta < 1, \qquad b_0 > 0, \tag{**}$$

where $x_{t+1} = x_t + \varepsilon_{t+1}$, with $E_t \varepsilon_{t+1} = 0$, $t = 0, 1, 2, \dots$.

Suppose somebody claims that the asset price p_t contains a rational bubble component following the process (**). Can we rule out that this claim might hold? Why or why not? *Hint:* for a process b_t to be a rational bubble, it is required that b_t has the property $b_t = a E_t b_{t+1}$, $t = 0, 1, 2, \dots$, where $a = (1 + r)^{-1}$.

Problem 4 Short questions

a) What is precisely meant by Ricardian equivalence?

b) Two economists – one from MIT and one from Chicago – are walking down the street. The MIT economist sees a 100 dollar note lying on the sidewalk and says: "Oh, look, what a fluke!". "Don't be silly, obviously it is false", laughs the Chicago economist, "if it wasn't, someone would have picked it up". Briefly discuss in relation to the theoretical concepts of arbitrage and equilibrium.

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