Written exam for the M. Sc. in Economics, Winter 2015-16

Advanced Macroeconomics

Master's course

January 11, 2016

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 5 pages in total.

The weighting of the problems is:

Problem 1: 25 %, Problem 2: 45 %, Problem 3: 20 %. Problem 4: 10 %.[1]

[1]The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 Consider a small open economy (SOE) in a monetary union which is not also a fiscal union. There is a constant real interest rate r on financial capital in the world market. There is free mobility of financial capital. Time is discrete, $t = 0, 1, 2, \ldots$. Notation is:

$G_t \equiv$ real government spending on goods and services in period t,
$T_t \equiv$ real net tax revenue (= gross tax revenue − transfer payments) in period t,
$B_t \equiv$ real public debt (one-period bonds) at the start of period t, $B_0 > 0$ given,
$r_t \equiv$ real interest rate on public debt,
$Y_t \equiv$ real GDP in period t,
$b_t \equiv B_t/Y_t$.

Until further notice world market conditions are tranquil and the following circumstances hold for SOE:

- its government bonds are perfect substitutes vis-a-vis other countries' bonds so that $r_t = r$;
- its Y_t grows at a constant exogenous rate g_Y satisfying $0 < g_Y < r$;
- its government budget deficit is exclusively financed by issuing debt (and any budget surplus by redeeming debt).
- $G_t = \gamma Y_t$, and $T_t = \tau Y_t$, where γ and τ are given constants satisfying $0 < \gamma < 1$ and $0 < \tau < 1$. We refer to γ, τ, and b_t as the spending-income ratio, the tax-income ratio, and the debt-income ratio, respectively.

a) Set up an equation giving B_{t+1} as determined by B_t and the real budget deficit in period t. Derive from this an equation showing how b_{t+1} emerges from b_t.

b) Find the minimum value, $\hat{\tau}$, of the tax-income ratio needed for fiscal sustainability. *Hint:* a difference equation $x_{t+1} = ax_t + c$, where a and c are constants, $a \neq 1$, has the solution $x_t = (x_0 - x^*)a^t + x^*$, where $x^* = c/(1-a)$.

c) What will happen if $\tau < \hat{\tau}$?

From now on we assume that $\tau = \hat{\tau}$. We also assume that the debt-income ratio of SOE is at the same time "high".

Suppose that a global financial and economic crisis breaks out and also hits SOE. As an implication, $\hat{\tau}$ is only slightly below the tax-income ratio, $\bar{\tau}$, that under the current circumstances maximizes (net) tax revenue (because on top of supply effects of taxes come contractive demand effects). Suppose also that the actual spending-income ratio, γ, is only slightly above the level $\bar{\gamma}$ which is the minimum politically tolerable spending-income ratio of SOE. The world market interest rate is still r.

d) Are "fundamentals" of SOE in accordance with fiscal sustainability under these circumstances? Why or why not? *Hint:* compare $\hat{\tau} - \gamma$ and $\bar{\tau} - \bar{\gamma}$.

e) May the "high" debt-income ratio of SOE under the given circumstances trigger a government debt default through the mechanism of self-fulfilling rational expectations? Why or why not?

Problem 2 Consider a single firm with production function

$$Y_t = F(K_t, L_t),$$

where Y_t, K_t, and L_t are output, capital input, and labor input per time unit at time t, respectively, while F is a neoclassical production function with CRS and satisfying the Inada conditions. Time is continuous. The increase per time unit in the firm's capital stock is given by

$$\dot{K}_t = I_t - \delta K_t, \qquad \delta > 0,$$

where I_t is gross investment per time unit at time t and δ is the capital depreciation rate. There is perfect competition in all markets and no uncertainty. The real interest rate faced by the firm is a positive constant r. Cash flow (in real terms) at time t is

$$R_t = F(K_t, L_t) - w_t L_t - I_t - G(I_t),$$

where w_t is the real wage and $G(I_t)$ represents capital installation costs and is a function satisfying

$$G(0) = G'(0) = 0, \; G''(I) > 0.$$

a) Set up the firm's intertemporal production and investment problem as a standard optimal control problem, given that the firm wants to maximize its market value.

b) Let the adjoint variable be denoted q_t. Derive the first-order conditions and state the necessary transversality condition, TVC. *Hint:* the TVC has the standard form for an infinite horizon optimal control problem with discounting.

c) What is the economic interpretation of q_t?

d) On the basis of one of the first-order conditions, show that the firm's chosen labor input is such that the capital-labor ratio at time t is a function of w_t. *Hint:* it is convenient to express the marginal productivity of labor by the production function in intensive form.

Suppose from now on that $w_t = w$ for all $t \geq 0$, where w is a positive constant. Let the corresponding optimal capital-labor ratio be denoted $\bar{k}$.

e) Show that the optimal investment level, I_t, can be written as an implicit function of q and that $\dot{q}$ can be expressed in terms of q and $\bar{k}$.

f) Construct a phase diagram for the (K, q) dynamics, assuming that a steady state with $K > 0$ exists. Let the steady state value of K be denoted K^*. For an arbitrary $K_0 > 0$, indicate in the diagram the movement of the pair (K_t, q_t) along the optimal path. *Hint:* since the setup differs from the examples in syllabus, the phase diagram may look somewhat different from what you are used to; moreover, as you know from one of the policy regimes in Blanchard's dynamic IS-LM model, the saddle path in a phase diagram *may* coincide with the line representing no change in the asset price.

g) In another diagram draw the time profiles of q_t, I_t, and K_t. Comment on why, in spite of the marginal productivity of capital in the steady state exceeding $r + \delta$, there is no incentive to increase K above K^*.

h) It is common to call K^* the "desired capital stock". Express the desired capital stock as an implicit function of r, δ, and w. How does the desired capital stock depend on r and w, respectively? Indicating the sign is enough. *Hint:* a simple approach can be based on curve shifting.

i) Show that optimal net investment, $I_t^n \equiv I_t - \delta K_t$, equals $\delta(K^* - K_t)$. This rule is called the capital adjustment principle. What is intuitively the contents of the rule?

Problem 3 In this problem we consider the dynamic IS-MP model with a Phillips curve. Time is continuous. The economy is closed. Notation is as follows:

$$\begin{aligned}
\pi_t &\equiv \dot{P}_t/P_t \equiv \text{ the inflation rate,} \\
\pi_t^e &\equiv \text{expected inflation rate,} \\
Y_t &\equiv GNP \text{ at time } t, \\
y_t &\equiv \log Y_t, \\
r_t^e &\equiv \text{ expected real interest rate faced by borrowing households and firms,} \\
i_t &\equiv \text{ the central bank's policy rate.}
\end{aligned}$$

The equations of the model are

$$\begin{aligned}
y_t &= \mu - \beta r_t^e, \qquad \mu > 0, \beta > 0, && (1) \\
\dot{\pi}_t &= \delta(y_t - y^*), \qquad \delta > 0,\ y^* > 0, \qquad \pi_0 \text{ given}, && (2) \\
r_t^e &\equiv i_t + \omega(\mu) - \pi_t^e, \qquad \omega(\mu) \geq 0, \qquad \omega'(\mu) < 0, && (3)
\end{aligned}$$

and

$$\begin{aligned}
i_t &= \max\left[0, \hat{\imath} + \alpha_1(y_t - y^*) + \alpha_2(\pi_t^e - \hat{\pi})\right], && (4) \\
\text{where } \hat{\imath} &\equiv \hat{r} - \omega(\bar{\mu}) + \hat{\pi}, \quad \hat{\pi} > 0, \quad 0 < \omega(\bar{\mu}) < \hat{r} + \hat{\pi}, \quad \alpha_1 \geq 0, \quad \alpha_2 > 1.
\end{aligned}$$

The shift parameter μ reflects the "state of confidence". In "normal" times it takes the value $\bar{\mu}$. Our dynamic model has five endogenous variables: $y_t, \pi_t, \pi_t^e, r_t^e$, and i_t. Remaining symbols are parameters. A subset of these are linked through the definition

$$\hat{r} \equiv \frac{\bar{\mu} - y^*}{\beta}.$$

The model as presented consists of only four equations, (1), (2), (3), and (4), while there are five endogenous variables. One may "close" the model by adding adaptive expectations or rational expectations or some other expectations formation hypothesis. We leave the model "open" in this regard.

a) Consider the short run, that is, a fixed t. Taking expected and actual inflation as given, construct an IS-MP diagram in the (y, i) plane.

b) Indicate in the diagram whether and how the IS curve and the MP curve, respectively, shift if expected inflation shifts upward. Indicate also in the diagram whether and how the IS curve and the MP curve, respectively, shift if actual inflation shifts upward. Comment.

c) Presupposing that the zero lower bound is not binding, show that the short-run equilibrium value of y_t can be written as a linear (affine) function of the expected inflation rate. Draw the graph of this function, i.e., the "AD curve", in a diagram in the (y, π^e) plane.

d) Give a brief account of conclusions concerning the dynamics and dynamic responses to shocks implied by the model.

Problem 4

In the wake of the full-blown financial and economic crisis in late 2008 and 2009 a large fall in employment occurred in many countries, not least in the U.S. Two different stories could in principle explain this sharp fall in employment. One is a "Schumpeterian story" emphasizing technological and structural change. The other is a "Keynesian story".

a) A believer of the Schumpeterian story would expect "total separations", "quits", and "hiring" to rise during the recession. In contrast, a believer of the Keynesian story would expect "layoffs and discharges" to rise and "hiring" and "quits" to fall (the terms in quotation marks are the terms used by the Bureau of Labor Statistics in the U.S.). Briefly explain why each of the two types of believers would expect so.

b) What does the data on labor market flows in the U.S. published by the Bureau of Labor Statistics tell us in relation to these two types of explanations?

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