

Written exam for the M. Sc. in Economics, Winter 2014-15

**Advanced Macroeconomics**

Master's course

January 15, 2015

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

**This exam question consists of 5 pages in total.**

The weighting of the problems is:

Problem 1: 50 %, Problem 2: 35 %, Problem 3: 15 %.<sup>1</sup>

---

<sup>1</sup>The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

**Problem 1** The Blanchard OLG model for a closed economy is described by the two differential equations

$$\dot{\tilde{k}}_t = f(\tilde{k}_t) - \frac{\lambda + b}{b}\tilde{c}_t - (\delta + g + b - m)\tilde{k}_t, \quad \tilde{k}_0 > 0 \text{ given}, \quad (1)$$

$$\dot{\tilde{c}}_t = \left[ f'(\tilde{k}_t) - \delta - \rho + \lambda - g \right] \tilde{c}_t - b(\rho + m)\tilde{k}_t, \quad (2)$$

and the condition that for any fixed pair  $(t_0, v)$ , where  $t_0 \geq 0$  and  $v \leq t_0$ ,

$$\lim_{t \rightarrow \infty} a_{t,v} e^{-\int_{t_0}^t (f'(\tilde{k}(s)) - \delta + m) ds} = 0. \quad (3)$$

Notation:  $\tilde{k}_t \equiv K_t/(T_t L_t)$  and  $\tilde{c}_t \equiv C_t/(T_t N_t) \equiv c_t/T_t$ , where  $K_t$  and  $C_t$  are aggregate capital and aggregate consumption, respectively;  $N_t$  is population,  $L_t$  is labor supply, and  $T_t$  is the technology level, all at time  $t$ ;  $f$  is a production function on intensive form, satisfying  $f(0) = 0$ ,  $f' > 0$ ,  $f'' < 0$ , and the Inada conditions. Finally,  $a_{t,v}$  is financial wealth at time  $t$  of an individual born at time  $v$ . The remaining symbols stand for parameters and we assume all these are strictly positive. Furthermore,  $\rho \geq b - m \geq 0$  and  $\lambda < \delta + \rho + g$ .

- a) Briefly interpret (1), (2), and (3), including the parameters.
- b) Draw a phase diagram and illustrate the path the economy will follow, given some arbitrary positive initial value of  $\tilde{k}$ . Can the divergent paths be ruled out? Why or why not?

The model entails a simple theory of the rate of return in the long run,  $r^*$ . For example, the model implies that  $r^*$  must be above a certain value defined by the parameters  $\rho$ ,  $\lambda$ , and  $g$ .

- c) By appealing to the phase diagram you have already drawn, indicate what this value must be.

Since Laurence Summers' speech at the IMF's 2013 Annual Research Conference there has been a debate and a concern about the possibility of "secular stagnation" (a better name might be "lasting stagnation", in Danish "varig stagnation"). At least three simultaneous empirical circumstances have been in focus: 1) the downward trend in the demographic parameters  $b$ ,  $m$ , and  $n \equiv b - m$  (whereby  $b$  has gone even more down than  $m$ ); 2) the downward trend in real interest rates; and 3) the event of a binding zero lower bound, ZLB, on the nominal policy rate coming increasingly into sight, first in Japan, then in the US, and subsequently also in the Eurozone, and the alarming duration of this state of affairs.

Let us first check whether the present model envisages any connection between 1) and 2).

- d) How will a lower  $b$  affect  $r^*$  in the present model? You are only supposed to make a comparative analysis, considering  $b$  as a shift parameter. *Hint:* standard curve shifting in the phase diagram does not work here, but another graphical argument is possible based on the fact that in steady state the equations for the  $\dot{\tilde{k}}_t = 0$  and  $\dot{\tilde{c}}_t = 0$  loci are simultaneously satisfied; elimination of  $\tilde{c}$  gives an equation in  $\tilde{k}$  which can be ordered so that terms involving  $\tilde{k}$  appear on the left-hand side while the constant  $(\lambda + b)(\rho + m)$  is isolated on the right-hand side; a graph of the left-hand side as a function of  $\tilde{k}$  will be helpful.
- e) Consider a simultaneous downward shift in  $b$  and  $m$  so that also  $n$  is lowered. How will such a parameter shift affect  $r^*$  in the present model?

As technology growth is the result of new ideas and ideas come from human beings, there is a concern that reduced  $n$  tends to reduce  $g$ .

- f) How will a lower  $g$  affect  $r^*$  in the present model?

We will now interpret the present model as describing only the long-run trend of the economy. Around this trend business cycle fluctuations occur, primarily due to shifts in aggregate demand. We interpret our  $r^*$  as the “natural” rate of interest consistent with continuing “full employment” (NAIRU employment) in the absence of shocks.

- g) Briefly give a few examples of the kind of shocks that may trigger shifts in aggregate demand.
- h) After a severe adverse demand shock recovery requires for some time a real interest rate considerably below the natural rate. When trying to foster this, conventional monetary policy reaches a barrier when the ZLB becomes binding. Explain.
- i) How does a reduced  $r^*$  affect the likelihood of the ZLB to become binding? Why?
- j) Briefly discuss alternative conceivable policies that may lessen a possible tendency to “secular stagnation”.

**Problem 2** Consider the following attempt at a “microfoundation” of Keynesian short-run theory. (i) There is a given “large” number,  $m$ , of firms and equally many horizontally differentiated products. (ii) Each firm supplies its own differentiated product on which it has a monopoly and which is an imperfect substitute for the other products. (iii) A price change by one firm has only a negligible effect on the demand faced by any other firm.

Firm  $i$  has the production function

$$y_i = n_i^\alpha, \quad 0 < \alpha < 1,$$

where  $y_i$  is output and  $n_i$  is labor input,  $i = 1, 2, \dots, m$ .

Suppose aggregate output demand in the period considered is

$$Y^d = \frac{M}{\beta P},$$

where  $M$  is base money (there is no private banking sector),  $P$  is the “general price level”, and  $\beta \in (0, 1)$  is a parameter reflecting consumers’ patience.

Let the nominal wage rate be denoted  $W$  and let  $\eta$  be a parameter,  $\eta > 1$ . Given  $W$ ,  $P$ , and  $M$ , firm  $i$  sets a price  $P_i$  so as to maximize its profit

$$\begin{aligned} \Pi_i &= P_i y_i - W y_i^{1/\alpha} = P_i \left( \frac{P_i}{P} \right)^{-\eta} \frac{M}{m\beta P} - W \left( \left( \frac{P_i}{P} \right)^{-\eta} \frac{M}{m\beta P} \right)^{1/\alpha} \\ &\equiv \Pi(P_i, P, W, M). \end{aligned} \quad (*)$$

a) Briefly interpret (\*).

Let firm  $i$ ’s profit maximizing price be denoted  $\bar{P}_i$  and suppose this price is set in advance, given the expectation  $M^e = M$ , the same for all firms. Suppose the actual money supply turns out to be  $M' > M$ . Now, the firm contemplates whether to change its price or keep it unchanged, assuming the general price and wage levels remain unchanged. There is a given “menu cost”  $c > 0$ .

- b) Assuming that the difference  $\Delta M \equiv M' - M$  is “infinitesimal”, does the firm have an incentive to change price? Why or why not? Give an algebraic argument for your answer as well as a graphical illustration.
- c) Is the logic different for a finite  $\Delta M$ ? Why or why not?
- d) Relate your answers at b) and c) to the *rule of the minimum*.
- e) The above “story” has something to say in relation to at least one of the three stylized short-run facts often emphasized by Keynesians and listed in Chapter 19 of Lecture Notes. What are these stylized facts and which one of them is in particular related to the above “story”?

We now change the scenario. We assume that firm  $i$  has chosen its preset price  $\bar{P}_i$  on the basis of an *expected* nominal wage  $W^e = W$ , in addition to the expected money supply  $M^e = M$ . This time it is the actual nominal wage,  $W'$ , that turns out higher than expected. Again, the firm contemplates whether to change its price or keep it unchanged, assuming the general price level remains unchanged. There is still a given “menu cost”  $c > 0$ .

- f) Replacing  $M$  by  $W$  in questions b) and c), answer these questions in the new scenario.

- g) On the basis of your answers to f), discuss whether there is one of the three stylized facts from e) that the menu cost theory is not helpful in explaining.

**Problem 3**     *Short questions*

- a) What is meant by the No-Ponzi-Game condition of the government?
- b) The No-Ponzi-Game condition of the government and the intertemporal budget constraint of the government are closely related. In what sense?
- c) “A given fiscal policy is sustainable if and only if it maintains compliance with the intertemporal budget constraint of the government.” True or false? Briefly discuss.

—