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A suggested solution to the problem set at the exam in **Advanced Macroeconomics 2** February 19, 2009

(4-hours closed book exam)[1]

As formulated in the course description, a score of 12 is given if the student's performance demonstrates (a) accurate and thorough understanding of the concepts, methods and models in the course, (b) knowledge of the major empirical regularities for aggregate economic variables, and (c) ability to use these theoretical tools and this empirical knowledge to answer macroeconomic questions.

1. Solution to Problem 1

a) The parent cares about the descendants' utility:

$$U_t = u(c_{1t}) + (1+\rho)^{-1}u(c_{2t+1}) + (1+R)^{-1}(1+n)U_{t+1}.$$

By forward substitution of U_{t+1}, U_{t+2}, etc., we arrive at (*), since $\lim_{j\to\infty}(\frac{1+n}{1+R})^{j+1}U_{t+j+1} = 0$, in view of $R > n$ and no technical progress.

b) We insert into U_t the effective intergenerational discount rate $\bar{R}$, defined by $1+\bar{R} \equiv (1+R)/(1+n)$, and the two period budget constraints in order to consider the objective of the parent as a function, $\tilde{U}_t$, of the decision variables, s_t and b_{t+1}. Then, wrt. s_t we get

$$\begin{aligned}\frac{\partial \tilde{U}_t}{\partial s_t} &= -u'(c_{1t}) + (1+\rho)^{-1}u'(c_{2t+1})(1+r) = 0, \text{ i.e.,} \\ u'(c_{1t}) &= (1+\rho)^{-1}u'(c_{2t+1})(1+r). \qquad \text{(FOC1)}\end{aligned}$$

Wrt. b_{t+1} we get, when the constraint $b_{t+1} \geq 0$ is not binding (Case a),

$$\begin{aligned}\frac{\partial \tilde{U}_t}{\partial b_{t+1}} &= (1+\rho)^{-1}u'(c_{2t+1})\left[-(1+n)\right] + (1+\bar{R})^{-1}u'(c_{1t+1})\cdot 1 = 0, \text{ i.e.,} \\ (1+\rho)^{-1}u'(c_{2t+1}) &= (1+\bar{R})^{-1}u'(c_{1t+1})\frac{1}{1+n}. \qquad \text{(FOC2a)}\end{aligned}$$

[1] The solution below contains more details and more precision than can be expected at a four hours exam.

When the constraint $b_{t+1} \geq 0$ is binding (Case b), we get

$$\begin{aligned}\frac{\partial \tilde{U}_t}{\partial b_{t+1}} &= (1+\rho)^{-1}u'(c_{2t+1})\left[-(1+n)\right] + (1+\bar{R})^{-1}u'(c_{1t+1}) \cdot 1 \leq 0, \text{ i.e.,} \\ (1+\rho)^{-1}u'(c_{2t+1}) &\geq (1+\bar{R})^{-1}u'(c_{1t+1})\frac{1}{1+n}. \qquad \text{(FOC2b)}\end{aligned}$$

Comment: in Case a the optimal b_{t+1} satisfies (FOC2a) which says that the parent's utility cost of increasing the bequest by one unit in an interior optimum must equal the discounted utility benefit derived from the next generation having $1/(1+n)$ more units per member for consumption in the same period. In Case b, however, the optimal $b_{t+1} = 0$, i.e., we have a corner solution, which has the property that the parent's utility cost of increasing the bequest by one unit either exceeds or equals the discounted utility benefit derived from the next generation having $1/(1+n)$ more units per member for consumption in the same period.

c) Since the agents are at an interior solution and $r = R$ (the modified golden rule property), the economy must be in a steady state. A detailed proof goes as follows. Inserting (FOC2a) on the right-hand side of (FOC1) gives

$$u'(c_{1t}) = (1+\bar{R})^{-1}u'(c_{1t+1})\frac{1+r}{1+n}. \qquad (**)$$

Given $r = R$, we have $1 + \bar{R} \equiv (1+r)/(1+n)$ so that (**) gives

$$u'(c_{1t}) = u'(c_{1t+1}), \qquad (1.1)$$

which implies $c_{1t} = c_{1t+1}$, since $u'' < 0$. By (FOC1), forwarded one period, we have

$$u'(c_{2t+2}) = (1+\rho)(1+r)^{-1}u'(c_{1t+1}) = (1+\rho)(1+r)^{-1}u'(c_{1t}) = u'(c_{2t+1}),$$

by (1.1) and (FOC1), respectively. It follows that $c_{2t+1} = c_{2t+2}$. Thus, the economy is in a steady state.

d) (i) is the IBC of the government saying that the PDV of spending equals initial total wealth. (ii) represents a re-ordering of IBC and says that the PDV of taxes equals PDV of spending plus initial debt. (iii) is a further re-ordering of IBC saying that the PDV of primary surpluses equals initial debt. The three equations are just different ways of saying that the intertemporal government budget constraint is satisfied (but not "over-satisfied").

e) The first equation reflects that the PDV of the consumption plan of the representative dynasty cannot exceed its initial total wealth; and the optimal consumption plan will imply equality. The second equation defines the human wealth of the dynasty as the PDV of the stream of after-tax income to the young plus the pension to the old.

f) The consumption path $(c_{2t+i}, c_{1t+i})_{i=0}^{\infty}$ is not affected because the dynasty will not feel reacher. Although the pensions are now higher, sooner or later taxes will have to be increased in order that the IBC of the government can be satisfied. The PDV of these extra taxes will exactly match the PDV of the extra pensions. As the question is framed, this answer is sufficient. But a more detailed argument goes like this: Adding B and H gives

$$\begin{aligned} B_t + H_t &= L_t \sum_{i=0}^{\infty} \frac{(1+n)^i}{(1+r)^{i+1}} \left[\sigma_{t+i} - \frac{\pi}{1+n} + w_{t+i} - \sigma_{t+i} + \frac{\pi}{1+n} \right] \\ &= L_t \sum_{i=0}^{\infty} \frac{(1+n)^i}{(1+r)^{i+1}} w_{t+i}. \end{aligned}$$

We see that the time profiles of π and σ have vanished and cannot affect $B_t + H_t$, hence, cannot affect the total wealth of the dynasty, $A_t + H_t = K_t + A_t^f + B_t + H_t$, where A_t^f is net foreign assets. Thus, *Ricardian equivalence* holds.

g) The consumption path $(c_{2t+i}, c_{1t+i})_{i=0}^{\infty}$ is not affected because the dynasty will not feel reacher. Although taxes on the young are now lower, sooner or later taxes will have to be increased in order that the IBC of the government can be satisfied. The PDV of the tax cuts will exactly match the PDV of the extra taxes later.

h) No, since the economy is, at least initially, in steady state, we have $c_{1t} = c_{1t+1}$ so that $u'(c_{1t}) = u'(c_{1t+1})$, implying that (**) no longer holds. Indeed, in view of $R > r$, we have $1 + R = (1 + \bar{R})(1 + n) > 1 + r$, so that from $u'(c_{1t}) = u'(c_{1t+1})$ follows

$$u'(c_{1t}) > (1+\bar{R})^{-1} u'(c_{1t+1}) \frac{1+r}{1+n}.$$

Substituting (FOC1) into this gives

$$(1+\rho)^{-1} u'(c_{2t+1}) > (1+\bar{R})^{-1} u'(c_{1t+1}) \frac{1}{1+n}. \tag{1.2}$$

This shows that the bequest motive cannot be operative (we are in Case b). Indeed, the parent would prefer to leave negative bequests, but that is forbidden. Hence, $b_{t+1} = 0$. Thus, Ricardian equivalence no longer holds. The reason is that when the non-negativity constraint on bequests is binding, the links between generations is broken and the economy no longer has an infinitely-lived representative agent.

i) Yes, resource allocation is affected because the old parent now has the opportunity to partly realize the preference for more own consumption, still leaving no bequests.

j) Yes, resource allocation is affected because the young parent now has the opportunity to partly realize the preference for more own consumption, both in the first and the second period of life, still leaving no bequests in the second period.

k) In contrast to the Barro model, the Ramsey model leads to Ricardian Equivalence *unconditionally.* Essentially, this is because the Ramsey model ignores the non-negativity constraint on bequests and is therefore unconditionally a representative agent model. In the basic OLG models, like Diamond's or Blanchard's, there is no bequest motive, and so these models are not representative agent models. Hence, Ricardian equivalence does not hold in these models.

ℓ) Positive features of the Barro model include: the model takes inheritance into account, constitutes a theoretically interesting benchmark case, and provides a nice interpretation of the Ramsey model. More problematic features include: the model does not properly take into account that a large fraction of a population typically does not leave bequests and that an even smaller fraction does it in the altruistic form assumed by Barro. The data does not give much support for the Barro model.

2. Solution to Problem 2

The decision problem, as seen from period 0, is:

$$\max E_0(U_0) = E_0[\sum_{t=0}^{T-1}(\log c_t - \gamma\frac{\sigma}{1+\sigma}\ell_t^{(1+\sigma)/\sigma})(1+\rho)^{-t}] \qquad \text{st.} \tag{2.1}$$

$$c_t > 0, 0 \leq \ell_t \leq 1, \tag{2.2}$$

$$a_{t+1} = (1+r_t)a_t + w_t\ell_t - c_t, \qquad a_0 \text{ given}, \tag{2.3}$$

$$a_T \geq 0. \tag{2.4}$$

a) Defining $\tilde{U}_t \equiv (1+\rho)^t U_t$, the remainder of the problem as seen from period t ($t = 0, 1, ..., T-1$) is:

$$\max \ E_t\tilde{U}_t = (1+\rho)^t E_t U_t$$

$$= \log c_t - \gamma\frac{\sigma}{1+\sigma}\ell_t^{(1+\sigma)/\sigma} + (1+\rho)^{-1}E_t[\log c_{t+1} - \gamma\frac{\sigma}{1+\sigma}\ell_{t+1}^{(1+\sigma)/\sigma} + ...] \tag{2.5}$$

$st.$ (2.2) - (2.4), $\quad a_t$ given.

To solve the problem we will use the substitution method. First, from (2.3) we have

$$c_t = (1+r_t)a_t + w_t\ell_t - a_{t+1}, \qquad \text{and} \tag{2.6}$$
$$c_{t+1} = (1+r_{t+1})a_{t+1} + w_{t+1}\ell_{t+1} - a_{t+2}.$$

Substituting these expressions into (2.5), the problem is reduced to one of maximizing the function $E_t\tilde{U}_t$ wrt. $(\ell_t, a_{t+1}), (\ell_{t+1}, a_{t+2}), ..., (\ell_{T-1}, a_T)$. We get

$$\frac{\partial E_t\tilde{U}_t}{\partial \ell_t} = \frac{1}{c_t}w_t - \gamma\ell_t^{1/\sigma} = 0,$$

that is,

$$\gamma\ell_t^{1/\sigma} = \frac{1}{c_t}w_t \qquad t = 0, 1, 2, ..., T-1, \tag{*}$$

and

$$\frac{\partial E_t\tilde{U}_t}{\partial a_{t+1}} = \frac{1}{c_t}\cdot(-1) + (1+\rho)^{-1}E_t[\frac{1}{c_{t+1}}(1+r_{t+1})] = 0,$$

that is,

$$\frac{1}{c_t} = (1+\rho)^{-1}E_t[\frac{1}{c_{t+1}}(1+r_{t+1})], \qquad t = 0, 1, 2, ..., T-2. \tag{**}$$

In view of the solvency condition (2.4), in the last period, period $T-1$, consumption must be

$$c_{T-1} = (1+r_{T-1})a_{T-1} + w_{T-1}\ell_{T-1},$$

since it is not optimal to end up with $a_T > 0$ (indeed, the transversality condition is $a_T = 0$).

b) The first-order condition (*) describes the trade-off between leisure in period t and consumption in the same period. The condition says that in the optimal plan, the cost (in terms of current utility) of increasing labor supply by one unit is equal to the benefit of obtaining an increased labor income and using this increase for extra consumption (i.e., marginal cost = marginal benefit).

The other first-order condition, (**), describes the trade-off between consumption in period t and consumption in period $t+1$, as seen from period t. The optimal plan must satisfy that the current utility loss by decreasing consumption c_t by one unit is equal to the discounted expected utility gain next period by having $1+r_t$ extra units available for consumption, where $1+r_t$ is the gross return on saving one more unit (i.e., marginal cost = marginal benefit).

c) We rewrite (*) as

$$\gamma\ell_t^{1/\sigma}c_t = w_t. \tag{*'}$$

Apart from the finite horizon (which is not important in this context), the intertemporal utility function above could easily be a specification of the preferences of a representative household in a RBC model. Further, the RBC theory maintains that factor prices are always such that there is no unemployment. Hence, the prediction from the RBC theory is the same as that from condition (*), namely that, since employment is procyclical and fluctuates almost as much as GDP, and consumption and employment are positively correlated, real wages will also be procyclical and fluctuate almost as much as output. But according to the stylized fact (iii), real wages are only weakly procyclical and do not fluctuate much. This is one of the often mentioned difficulties faced by RBC theory.

d) By replacing t by $t+1$ in (*) we get

$$\gamma \ell_{t+1}^{1/\sigma} = \frac{1}{c_{t+1}} w_{t+1}$$

so that

$$\left(\frac{\ell_t}{\ell_{t+1}}\right)^{1/\sigma} = \frac{c_{t+1}}{c_t} \frac{w_t}{w_{t+1}}. \tag{2.7}$$

Ignoring uncertainty, (**) gives

$$\frac{c_{t+1}}{c_t} = (1+\rho)^{-1}(1+r_{t+1}).$$

Substituting this into (2.7) and solving gives

$$\frac{\ell_t}{\ell_{t+1}} = (1+\rho)^{-\sigma}\left(\frac{w_t}{w_{t+1}/(1+r_{t+1})}\right)^{\sigma}. \tag{2.8}$$

We see from this expression that σ is the elasticity of ℓ_t/ℓ_{t+1} wrt. the relative factor price, $w_t/\left[w_{t+1}/(1+r_{t+1})\right]$. Hence, σ measures what is called the *elasticity of intertemporal substitution* (in labor supply). From microeconometric studies we have estimates of this parameter. These estimates indicate the parameter is quite small, at least for men (in the range 0 to 1.5, in many studies considerably below 1). And since fluctuations in w_t/w_{t+1} in the data are also small, it is difficult to reconcile the theory with the stylized fact (i) saying that employment fluctuates almost as much as GDP.

e) If fluctuations in the real wage are almost negligible, is it then likely that fluctuations in r_{t+1} could be a driving force behind fluctuations in employment? According to equation (2.8) one might be tempted to answer "yes". At least (2.8) indicates a positive relationship between ℓ_t/ℓ_{t+1} and r_{t+1}. The interpretation of this relation is that a high interest rate has a negative substitution effect on leisure in the current period, hence positive substitution effect on current labor supply.

But when the real wage doesn't fluctuate much, and an attempt is made to explain fluctuations in employment by fluctuations in the real interest rate, then, by (*), one would expect a *negative* correlation between employment and consumption. But the stylized fact (ii) tells the opposite.

f) We now reintroduce uncertainty. Indeed, there is now also uncertainty as to the prospect of employment in the future. The decision problem, as seen from period 0, can now be written:

$$\begin{aligned} \max E_0(U_0) &= E_0[\sum_{t=0}^{T-1}(\log c_t - \gamma\frac{\sigma}{1+\sigma}\ell_t^{(1+\sigma)/\sigma})(1+\rho)^{-t}] \qquad \text{st.} \\ c_t &> 0, 0 \leq \ell_t \leq \min(z_t, 1), \\ a_{t+1} &= (1+r_t)a_t + w_t\ell_t - c_t, \qquad a_0 \text{ given}, \\ a_T &\geq 0. \end{aligned}$$

where $z_t \geq 0$ is the exogenous maximum employment offered the household in period t (this constraint comes from the demand side in the labor market).

When the employment constraint $\ell_t \leq z_t$ is binding, (*) is replaced by

$$\gamma\ell_t^{1/\sigma} \leq \frac{1}{c_t}w_t. \tag{2.9}$$

The interpretation of the possibility that $<$ obtains is: although in the optimal plan, the cost (in terms of current utility) of increasing labor by one unit is *less* than the benefit of obtaining an increased labor income and using this increase for extra consumption, this desired increase in employment cannot be realized, due to the exogenous employment constraint (involuntary unemployment).

g) Yes, within this extended framework it is possible to reconcile theory with the stylized facts. Indeed, rewriting (2.9) as

$$\gamma\ell_t^{1/\sigma}c_t \leq w_t,$$

we see that when $<$ is in force, there is scope for employment to be procyclical and fluctuate almost as much as GDP (fact (i)) and for consumption and employment to be positively correlated (fact (ii)), whereas real wages do not fluctuate much (fact (iii)).

3. Solution to Problem 3

a) In new-Keynesian theory, "nominal rigidities" refers to the tenet that nominal prices (and wages) are (in the short run) far from exhibiting the flexibility assumed in neoclassical theory, where nominal prices immediately adjust so that Walrasian market clearing obtains. The new-Keynesian tenet is that shifts in aggregate demand has little influence on nominal prices. Instead, produced quantities adjust.

By "real rigidities" (or "real price rigidities") new-Keynesians mean that relative prices, for example the real wage, exhibit low sensitivity to changes in the corresponding quantities, for example employment.

b) In industrialized countries the individual firms typically have market power and face downward-sloping demand curves. Hence, they are price setters and set prices above marginal costs so that there is scope for quantity adjustment when for instance the demand curve shifts outward. An increase in the money supply tends, via the monetary transmission mechanisms, to increase aggregate demand (and so push the firms' demand curves outward). If adjusting price is associated with "menu costs", it is possible that these exceed the potential profit gain from adjusting price (which is generally small anyway, due to the envelope theorem). In such a case the individual firm prefers to adjust the produced quantity instead. In this way changes in the money supply may have real effects. This explains the importance of *nominal rigidities*, caused by menu costs.

But this is only half the story in the sense that nominal rigidities are only a necessary, not a sufficient condition for noticeable real effects of changes in the money supply. Expanding output implies an increase in the demand for labor. And if the economy were always *on* an upward-sloping aggregate labor supply curve, a substantial and persistent increase in employment would require a large increase in wages (because microeconometric studies indicate that labor supply is not very elastic). A large increase in wages would imply substantial upward shifts in the firms' marginal cost curves, and then it would be too costly for the firms not to adjust their prices − unless one assumes implausibly high menu costs. Therefore, many new-Keynesians emphasize that the industrialized economies are typically *not* on the aggregate labor supply curve. Instead, usually there is more or less *involuntary unemployment* (a pool of persons without job, but willing to take a job at the going or even a lower wage). In the short term it therefore takes only little, if any, rise in the real wage to permit a large expansion of employment. This explains the importance or *real rigidities* for the real effects of money supply changes.

c) Theories of the functioning of labor markets (efficiency wages, bargaining, social norms) lead to the hypothesis that the *level* of the expected real wage is negatively related to the rate of unemployment. Theory thus predicts a *wage curve*, for example of this form:

$$w_t - p_t^e = \beta v_t + (1-\beta)\alpha_t - bu_t + \varepsilon_t, \tag{3.1}$$

where w is the log of the nominal wage, p^e is the log of the expected price level, β is a constant $\in [0, 1]$, v_t is the reservation wage (the minimum real wage at which the worker is willing to supply labor), α_t a measure of labor productivity, b a positive parameter, u the unemployment rate, and ε white noise.

In contrast, a *Phillips curve* is an empirical relationship where the *change* in wages or prices is negatively related to unemployment. For example a wage Phillips curve can have this form:

$$w_t - w_{t-1} = a + (p_{t-1} - p_{t-2}) - bu_t + \varepsilon_t, \tag{3.2}$$

where a is a positive constant.

By reasonable hypotheses about how the reservation wage depends on the actual real wage (in the previous period) and on productivity, a *level* formulation as in (3.1) *may* be consistent with a *change* formulation as in (3.2). Blanchard and Katz (1999) find such hypotheses, together with (3.2), consistent with US data, but less so with European data. They interpret this as reflecting the difference between the US labor market and the typical European labor markets characterized by more influential labor unions, more stringent hiring and firing regulations, and perhaps also a greater role of the underground economy. An interesting implication of this theory is that in Europe the "natural" rate of unemployment should be sensitive to permanent shifts in factors such as the level of energy prices, payroll taxes, or real interest rates, whereas in the US it should not.

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