Written exam for the M. Sc. in Economics 2009-I

Advanced Macroeconomics 2

Master's Course

Re-exam February, 2009

(4-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The weighting of the problems is:

Problem 1: 55 %, Problem 2: 35 %, Problem 3: 10 %.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 Consider a small open economy, SOE, with perfect mobility of goods and financial capital across borders, but no mobility of labour. Domestic and foreign financial claims are perfect substitutes. The real rate of interest at the world financial market is a constant, r. Time is discrete. People live for two periods, as young and as old. As young they supply one unit of labour inelastically. As old they do not work. As in the Barro dynasty model we consider single-parent families with a bequest motive. Each parent belonging to generation t has $1+n$ descendants, $n<r$ and n constant. There is perfect competition on all markets, no uncertainty, and no technical progress. Notation is

$$\begin{aligned} L_t &= \text{ number of young in period } t, \\ \tilde{T}_t &= \text{ real gross tax revenue in period } t, \\ \sigma_t &= \tilde{T}_t/L_t = \text{ a lump-sum tax levied on the young in period } t, \\ B_t &= \text{ real government debt as inherited from the end of period } t-1. \end{aligned}$$

In every period each old receives the same pension payment, π, from the government. From time to time the government runs a budget deficit (surplus) and in such cases the deficit is financed by bond issue (withdrawal). That is,

$$B_{t+1} - B_t = rB_t + \pi L_{t-1} - \tilde{T}_t,$$

where B_0, L_0, and π are given (until further notice, π is constant). Thus, the pension payments are, along with interest payments on government debt, the only government expenses. The government always preserves solvency in the sense that sooner or later tax revenue is adjusted to satisfy the intertemporal government budget constraint (more about this below).

An individual belonging to generation t chooses saving, s_t, and bequest, b_{t+1}, to each of the descendants so as to maximize

$$U_t = \sum_{i=0}^{\infty}(1+\bar{R})^{-i}\left[u(c_{1t+i}) + \frac{1}{1+\rho}u(c_{2t+i+1})\right] \qquad (*)$$

s.t.

$$\begin{aligned} c_{1t} + s_t &= w_t - \sigma_t + b_t, \\ c_{2t+1} + (1+n)b_{t+1} &= (1+r)s_t + \pi, \qquad b_{t+1} \geq 0, \end{aligned}$$

and taking into account the optimal responses of the descendants. Here $1+\bar{R} \equiv (1+R)/(1+n)$, where $R > n \geq 0$ (both R and n constant). Also $\rho > -1$ is constant. The period utility function u satisfies the "no fast" assumption and $u' > 0, u'' < 0$. Negative bequests are forbidden by law.

a) How comes that the preferences of the single parent can be expressed as in (*)?

b) Derive the first-order conditions for the decision problem, taking into account that two cases are possible, namely that the constraint $b_{t+1} \geq 0$ is binding and that it is not binding. Comment.

Suppose it so happens that $R = r$ and that, at least for a while, circumstances are such that the agents are at an interior solution (i.e., $b_{t+1} > 0$). We define a steady state of this economy as a path along which c_{1t} and c_{2t} do not change over time.

c) Is the economy in a steady state? Why or why not? *Hint:* combine the first-order conditions and use that $R = r$.

As seen from the beginning of period t the intertemporal government budget constraint is:

$$\sum_{i=0}^{\infty} \pi L_{t+i-1}(1+r)^{-i-1} = \sum_{i=0}^{\infty} \tilde{T}_{t+i}(1+r)^{-i-1} - B_t \Rightarrow \quad \text{(i)}$$

$$L_t \sum_{i=0}^{\infty} \sigma_{t+i} \frac{(1+n)^i}{(1+r)^{i+1}} = L_t \sum_{i=0}^{\infty} \frac{\pi}{1+n} \frac{(1+n)^i}{(1+r)^{i+1}} + B_t \Rightarrow \quad \text{(ii)}$$

$$L_t \sum_{i=0}^{\infty} \frac{(1+n)^i}{(1+r)^{i+1}} \left[\sigma_{t+i} - \frac{\pi}{1+n}\right] = B_t. \quad \text{(iii)}$$

d) Briefly explain in economic terms what each row here expresses.

e) The intertemporal budget constraint of the representative dynasty is

$$L_{t-1} \sum_{i=0}^{\infty} \frac{(1+n)^i}{(1+r)^{i+1}} [c_{2t+i} + (1+n)c_{1t+i}] = A_t + H_t,$$

where A_t is aggregate financial wealth in the economy and H_t is aggregate human wealth (after taxes):

$$H_t = L_t \sum_{i=0}^{\infty} \frac{(1+n)^i}{(1+r)^{i+1}} (w_{t+i} - \sigma_{t+i} + \frac{\pi}{1+n}).$$

Briefly explain in economic terms these two equations.

f) Suppose that in period $t+1$, π is increased (a little) to a higher constant level, before the bequest b_{t+1} is decided. Is the consumption path $(c_{2t+i}, c_{1t+i})_{i=0}^{\infty}$ affected? Why or why not?

g) Given π, suppose that for some periods there is a (small) tax cut so that $\tilde{T}_{t+i} < \pi L_{t+i-1} + rB_{t+i}$, that is, a budget deficit is run. Is the consumption path $(c_{2t+i}, c_{1t+i})_{i=0}^{\infty}$ affected? Why or why not?

Now suppose instead that $R > r$ (but still $r > n$) and that the economy is, at least initially, in steady state.

h) Will the bequest motive be operative? Why or why not?

i) Suppose π is increased (a little) to a higher level without σ_t being immediately adjusted correspondingly. Is resource allocation affected? Why or why not?

j) Given π, suppose a tax cut occurs so that for some periods a budget deficit is run. Is resource allocation affected? Why or why not?

k) In a few words relate the results of your analysis to the conclusions from other dynamic general equilibrium models you know of.

ℓ) In a few words assess the Barro model of infinitely-lived families linked through bequests.

Problem 2 Consider a decision problem in discrete time for a given household facing uncertainty. To begin with we assume, in accordance with new classical theory, that the household never expects having to face the problem of getting less employment than desired at the going wage. As seen from period 0, the decision problem is:

$$\max E_0(U_0) = E_0[\sum_{t=0}^{T-1}(\log c_t - \gamma\frac{\sigma}{1+\sigma}\ell_t^{(1+\sigma)/\sigma})(1+\rho)^{-t}] \qquad \text{s.t.} \tag{1}$$

$$c_t > 0, 0 \leq \ell_t \leq 1, \tag{2}$$

$$a_{t+1} = (1+r_t)a_t + w_t\ell_t - c_t, \qquad a_0 \text{ given}, \tag{3}$$

$$a_T \geq 0, \tag{4}$$

where c = consumption, ℓ = labor supply, a = financial wealth, r = real rate of return on financial wealth, and w = real wage. The parameters γ, σ, and ρ are all positive. We assume the upper boundary, 1, to labor supply is large enough so as to be never binding, given the environment in which the household acts. The symbol E_0 (generally E_t) denotes the mathematical expectation conditional on the information available in period 0 (generally t). This information includes knowledge of all realizations of the variables up to period 0, including that period. There is uncertainty about future values of r_t and w_t, but the household knows the stochastic processes which these variables follow.

a) Derive two first-order conditions, the first of which (call it (*)) describes the trade-off between consumption and labor supply in, say, period t, and the second of which (call it (**)) describes the trade-off between consumption in period t and consumption in period $t+1$, both conditions as seen from period t ($t = 0, 1, ...$). *Hint:* consider maximization of $E_t\tilde{U}_t$ for $t = 0, 1, 2, ...$, where $\tilde{U}_t \equiv (1+\rho)^t U_t$.

b) Interpret the two first-order conditions.

Among the "stylized facts" of business cycle fluctuations (based on time series data after detrending) are the following:

(i) Employment (aggregate labor hours) is procyclical and fluctuates almost as much as GDP.

(ii) Aggregate consumption and employment are positively correlated.

(iii) Real wages are weakly procyclical and do not fluctuate much.

c) Are these facts supportive or the opposite for the RBC theory in the light of the condition (*)? Discuss. *Hint:* it will prove convenient to rewrite (*) such that w_t is isolated on one side of the equation; ignoring the finite horizon, the decision problem above can be seen as that of the representative agent in an RBC model.

d) In order to simplify the discussion, suppose for a moment there is no uncertainty. Then find ℓ_t/ℓ_{t+1} as a function of w_t/w_{t+1}. From this expression, give an interpretation of the parameter σ. Relate this to the discussion under c), taking into account the empirical evidence concerning the elasticity of intertemporal substitution in labor supply.

e) Within the market-clearing framework of the RBC approach, if fluctuations in the real wage are almost negligible, is it then likely that fluctuations in r_{t+1} could be a driving force behind fluctuations in employment? Relate your answer to your result under d), the condition (*), and the stylized facts above. *Hint:* given that fluctuations in the real wage are almost negligible, we can on the basis of (*) sign the expected correlation between consumption and employment and compare with fact (ii).

We now reintroduce the existence of uncertainty and reconsider the household's decision problem under the Keynesian hypothesis that the household may have to face rationing in the labor market and uncertainty concerning the prospect of employment in the future. That is, for $t = 0, 1, 2, ...$, we replace (2) by the constraint $c_t > 0$, $0 \leq \ell_t \leq \min(z_t, 1)$, where $z_t \geq 0$ is the exogenous maximum employment offered to the household in period t. The current z_t is known by the household, but not the future values.

f) Show that when z_t is binding, the equality sign in (*) is replaced by a weak inequality sign. Write down the new (*) and interpret.

g) Is it possible within this framework to reconcile theory with the stylized facts? Why or why not?

Problem 3 *Short questions*

a) In new-Keynesian theory, what does "nominal rigidities" mean and what does "real rigidities" mean?

b) Both nominal and real rigidities are important for persistence of real effects of changes in the money supply. Give a brief intuitive explanation.

c) Briefly describe the difference between the concepts "wage curve" and "Phillips curve". Is it possible to unify them theoretically? Empirically? Comment.

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