Advanced macroeconomics 2 Short Note 5.

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The IS-LM-CC model

We briefly here present the IS-LM-CC model of Bernanke and Blinder (1988). It is a stylized model of the monetary transmission mechanism when bonds and bank loans are not perfect substitutes, but there is no credit rationing. The model is static (only one period is considered).

A closed economy is considered. There is a public sector with two agents, a government and a central bank (from now CB). And there is a private sector also with two agents, the consolidated commercial banks and the "public" (households and non-bank firms). Until further notice, the CB is assumed to use the monetary base, M_0, as its instrument. It can change M_0 by an open market operation. All bonds are short-term and are traded at a centralized bonds market. The commercial banks act as financial intermediaries and supply unsecured bank loans at a customer credit market. The model can be considered as an extension of the standard Keynesian IS-LM model. The addition of CC in the title of the model can be interpreted as an abbreviation of "customer credit" or "commodity and credit".

1 The model

Notation is:[1]

$$
\begin{aligned}
i_B &= \text{nominal interest rate on bonds,} \\
i_L &= \text{nominal interest rate on loans (bank credit),} \\
D &= \text{demand deposits (earn no interest),} \\
\sigma &= \text{required reserve-deposit ratio, } \sigma \in [0,1)\,, \\
M_0 &= \text{monetary base,} \\
E &\equiv M_0 - \sigma D = \text{ excess reserves (earn no interest),} \\
L^s &= \text{supply of bank loans (credit),} \\
\rho &= \text{a shift parameter measuring perceived riskiness of offering bank loans,} \\
B &= \text{nominal stock of government bonds held by the private sector,} \\
\bar{B} - B &= \text{nominal stock of government bonds held by CB,} \\
P &= \text{price level,} \\
Y &= \text{real aggregate output,} \\
G &= \text{real government spending on goods and services.}
\end{aligned}
$$

Note that L does not here refer to liquidity, but to loans, and that ρ has a completely different meaning than in Bernanke and Blinder (1988). The superscripts s and d signify "supply" and "demand", respectively. We ignore currency so that the monetary base is identical to bank reserves. The CB can change M_0 through an open market operation whereby $dM_0 = -dB$.

The consolidated balance sheet of the commercial banks reads:

Commercial banks

Assets	Liabilities
M_0 = reserves	D = demand deposits
L^s = loans	
B_b = bonds held by banks	$\bar{N}$ = net worth = 0
total	total

Exogenous variables in the analysis are: $\bar{B}, G, P, M_0, \sigma, \rho, \bar{N}$, and expected inflation. We imagine a shift in G is fully tax-financed and hence does not change $\bar{B}$. For simplicity, $\bar{N} = 0$ and $P = 1$.

[1] Our notation is somewhat different from that in Bernanke and Blinder (1988).

1.1 The supply of money and bank loans

From the balance sheet of the consolidated commercial banks follows the adding-up constraint

$$M_0 + L^s + B_b = D + \bar{N} = D, \tag{1}$$

or, by subtraction of required reserves on both sides,

$$E + L^s + B_b = (1-\sigma)D.$$

For simplicity, the fraction of disposable deposits held as excess reserves is assumed to depend only on the interest rate on bonds (which is the more liquid substitute for reserves). Thus

$$E = e(i_B)(1-\sigma)D, \qquad e(i_B) \in [0,1]\,, \quad e_{i_B} < 0. \tag{2}$$

The fraction of disposable deposits used for supplying bank loans depends positively on the interest rate obtainable on these and negatively on the opportunity cost, the interest rate on bonds. That is,

$$L^s = \ell(i_B, i_L, \rho)(1-\sigma)D, \qquad \ell(i_B, i_L) \in [0,1]\,, \quad \ell_{i_B} < 0, \ell_{i_L} > 0, \ell_\rho < 0.$$

Since currency is ignored, money supply, M_1^s, equals deposits, D. The actual reserve-deposit ratio is

$$\frac{M_0}{D} = \frac{\sigma D + E}{D} = \sigma + e(i_B)(1-\sigma) \equiv mm(i_B)^{-1},$$

where the second equality follows from (2), and $mm(i_B)$ is the money multiplier, $mm_{i_B} > 0$. Thus, money supply is

$$M_1^s = D = mm(i_B)M_0. \tag{3}$$

1.2 The demand for money and bank loans

With the price level P equal to 1, we can ignore P. The amount of money (deposits) and loans, respectively, demanded by the public is described by:

$$M_1^d = M(i_B, Y), \qquad M_{i_B} < 0, M_Y > 0,$$

and

$$L^d = C(i_B, i_L, Y), \qquad C_{i_B} > 0, C_{i_L} < 0, C_Y > 0.$$

Money demand depends on Y due to the transactions motive and on the opportunity cost of holding money as measured by the interest rate on bonds. The role of the interest rate on bank loans is taken into account through the demand function for bank loans, here denoted $C(\cdot)$ (C for "credit"). The positive dependence of the demand for credit on Y captures the transactions demand for credit.

The demanded assets and liabilities must satisfy the balance sheet constraint

$$M_1^d + B_p = L^d + W, \tag{4}$$

where B_p is the stock of bonds held by the public and W is nominal net financial wealth of the private sector, that is,

$$W = M_0 + B + \bar{N} = M_0 + B, \tag{5}$$

since we have assumed that banks' net worth, $\bar{N}$, is vanishing.

1.3 Equilibrium

There are three asset markets to consider, the money market, the credit market, and the bonds market. Interest rates are assumed to adjust to create equilibrium on all three. Thus, considering the first two markets, we have

$$M_1^s = M_1^d \quad \text{i.e.} \quad mm(i_B)M_0 = M(i_B, Y), \tag{(MM)}$$

and

$$L^s = L^d \quad \text{i.e.} \quad \ell(i_B, i_L, \rho)(1-\sigma)mm(i_B)M_0 = C(i_B, i_L, Y). \tag{CC}$$

Equilibrium in the third market, the bonds market, requires

$$B_b + B_p = B.$$

This equilibrium condition *must* hold, given equilibrium in the two other asset markets. This follows from the balance sheet constraints. Indeed,

$$\begin{aligned}
B &= W - M_0 = B_p + M_1^d - L^d - M_0 \qquad \text{(by (5) and (4))} \\
&= B_p - L^s + M_1^s - M_0 = B_p - L^s + D - M_0 \qquad \text{(by (MM), (CC), and (3))} \\
&= B_p - L^s + B_b + L^s = B_p + B_b \qquad \text{(by (1))}.
\end{aligned}$$

Finally, the asset market equilibrium conditions are combined with equilibrium in the output market. Since the expected inflation rate is regarded as an exogenous constant, the real interest rate on bonds and bank loans varies one-to-one with the nominal rates, i_B and i_L, respectively. Thus, equilibrium in the output market can be expressed as

$$Y = Y^d(Y, i_B, i_L) + G, \qquad 0 < Y^d < 1, Y^d_{i_B} < 0, Y^d_{i_L} < 0. \tag{YY}$$

We thus have three equations, (MM), (CC), and (YY), and three endogenous variables, Y, i_B, and i_L.

2 Analysis

It is convenient to derive a graphical representation of the model in (Y, i_B) space, similar to that of the standard IS-LM model. For given M_0, equilibrium in the money market immediately provides an "LM curve", which we here, to avoid confusion about what *our* L refers to, will call the *MM curve.* Indeed, the equation (MM) gives i_B as an implicit function of Y and M_0 :

$$i_B = i_{MM}(Y, M_0).$$

The partial derivatives can be found by taking the total differential on both sides of (MM):

$$mm(i_B)dM_0 + M_0 mm_{i_B} di_B = M_{i_B} di_B + M_Y dY. \tag{6}$$

We find $\partial i_B / \partial Y_{|MM}$ by setting $dM_0 = 0$ and reordering:

$$\frac{\partial i_B}{\partial Y}_{|MM} = \frac{M_Y}{M_0 mm_{i_B} - M_{i_B}} > 0. \tag{7}$$

This gives the positive slope of the MM curve in Fig. 1.

The description of equilibrium in the output market is a little more cumbersome. First, note that equilibrium in the loans market gives the interest rate on loans as an implicit function of Y, i_B, ρ, and M_0 :

$$i_L = f(Y, i_B, \rho, M_0). \tag{8}$$

The partial derivatives can be found by taking the total differential on both sides of (CC):

$$\begin{aligned} &(1-\sigma)\left[\ell(i_B, i_L, \rho)\left(mm(i_B)dM_0 + M_0 mm_{i_B} di_B\right) + mm(i_B)M_0(\ell_{i_B} di_B + \ell_{i_L} di_L + \ell_\rho d\rho)\right] \\ = \; &C_{i_B} di_B + C_{i_L} di_L + C_Y dY. \end{aligned} \tag{9}$$

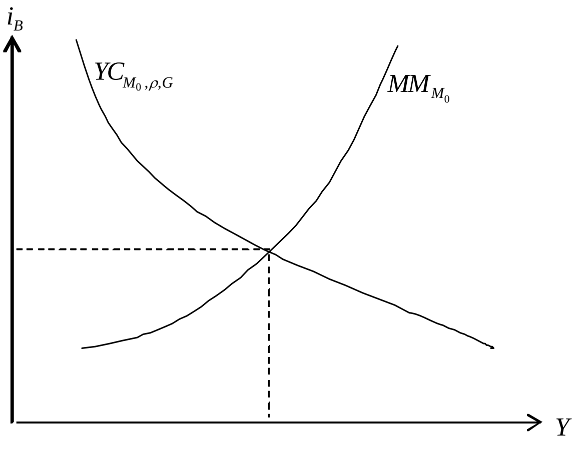

Figure 1: The YC-MM cross (for fixed M_0, ρ, and G).

We find the partial derivative of f wrt. Y by setting $di_B = d\rho = dM_0 = 0$ and reordering:

$$f_Y = \frac{C_Y}{(1-\sigma)mm(i_B)M_0\ell_{i_L} - C_{i_L}} > 0. \tag{10}$$

Similarly, we find the partial derivative of f wrt. i_B by setting $dY = d\rho = dM_0 = 0$ in (9) and reordering:

$$f_{i_B} = \frac{C_{i_B} - (1-\sigma)\left[\ell(i_B, i_L, \rho)M_0 mm_{i_B} + mm(i_B)M_0\ell_{i_B}\right]}{(1-\sigma)mm(i_B)M_0\ell_{i_L} - C_{i_L}} > 0, \tag{11}$$

assuming, as Bernanke and Blinder do, that mm_{i_B} is not "too large". And we find the partial derivative of f wrt. ρ by setting $dY = di_B = dM_0 = 0$ in (9) and reordering:

$$f_\rho = \frac{-(1-\sigma)mm(i_B)M_0\ell_\rho}{(1-\sigma)mm(i_B)M_0\ell_{i_L} - C_{i_L}} > 0. \tag{12}$$

Finally, the partial derivative of f wrt. M_0 is found by setting $dY = di_B = d\rho = 0$ in (9) and reordering:

$$f_{M_0} = \frac{-(1-\sigma)\ell(i_B, i_L, \rho)mm(i_B)}{(1-\sigma)mm(i_B)M_0\ell_{i_L} - C_{i_L}} < 0. \tag{13}$$

Substituting (8) into (YY) gives

$$Y = Y^d(Y, i_B, f(Y, i_B, \rho, M_0)) + G. \tag{YC}$$

Instead of the standard IS equation we thus arrive at an YC equation (YC for "output and credit"), which depends on the supply of base money, M_0, and the perceived riskiness of offering bank loans. The YC equation gives i_B as an implicit function of Y, ρ, and M_0 :

$$i_B = i_{YC}(Y, \rho, M_0).$$

We find the partial derivatives of this function by taking the total differential on both sides of (YC):

$$dY = Y_Y^d dY + Y_{i_B}^d di_B + Y_{i_L}^d (f_Y dY + f_{i_B} di_B + f_\rho d\rho + f_{M_0} dM_0) + dG. \tag{14}$$

We find $\partial i_B / \partial Y_{|YC}$ by setting $d\rho = dM_0 = dG = 0$ and reordering:

$$\frac{\partial i_B}{\partial Y}_{|YC} = \frac{1 - Y_Y^d - Y_{i_L}^d f_Y}{f_{i_B}} < 0, \tag{15}$$

where f_Y from (10) can be inserted. For given ρ and M_0, the YC equation thus defines a negative relationship between i_B and Y, cf. Fig. 1 depicted as the downward-sloping *YC curve*. In contrast to a standard IS curve, the position of the YC curve depends not only on G, but also on the supply of base money and the perceived riskiness of offering bank loans.

Since an upward-sloping MM curve and a downward-sloping YC curve can only cross once, a solution (Y, i_B) to the model is unique. We can thus write Y and i_B as implicit functions of the exogenous variables we are interested in:

$$Y = g(\rho, M_0, G), \tag{16}$$

$$i_B = h(\rho, M_0, G). \tag{17}$$

An alternative version of the model would consider i_B as the monetary policy instrument and then let M_0 adjust endogenously.

In Exercise Problem XI.2 you are asked to use this model for a series of economic questions.

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