Advanced Macroeconomics. Exercises

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Solutions to problems VII.2, VII.3, and VII.4 about the housing market and issues in monetary economics

VI.2 *A partial equilibrium model of the housing service market and the house market.*

a) $D(R_t, A, PDV(wl)) = S_t$. By taking the total differential, we get $D_1 dR_t + D_2 dA + D_3 dPDV(wl) = dS_t$, so that the implicit function $R_t = \tilde{R}(S_t,\, A,\, PDV(wL))$ has the partial derivatives $\tilde{R}_1 = 1/D_1 < 0$, $\tilde{R}_2 = -D_2/D_1 > 0$ and $\tilde{R}_3 = -D_3/D_1 > 0$.

b) Since $S_t = \alpha H_t$, we have $R_t = \tilde{R}(\alpha H_t,\, A,\, PDV(wL)) \equiv R(H_t)$, where $R' = \tilde{R}_1' \alpha < 0$.

c) In the absence of uncertainty differences across the two alternative assets they must have the same expected after-tax rate of return.

d) $p_t = \frac{(1-\tau_R)\alpha R(H_t)+\dot{p}_t^e}{(1-\tau_r)r+(1-\tau_R)\delta+\sigma}$. Let $N \equiv (1-\tau_r)r + (1-\tau_R)\delta + \sigma$. Then we have

$\frac{\partial p_t}{\partial r} = -\frac{[(1-\tau_R)\alpha R(H_t)+\dot{p}_t^e](1-\tau_r)}{N^2} < 0$, since $\dot{p}_t^e$ is not "too negative",

$\frac{\partial p_t}{\partial \tau_r} = \frac{[(1-\tau_R)\alpha R(H_t)+\dot{p}_t^e]r}{N^2} > 0,$

$\frac{\partial p_t}{\partial \sigma} = -\frac{(1-\tau_R)\alpha R(H_t)+\dot{p}_t^e}{N^2} < 0,$

$\frac{\partial p_t}{\partial H_t} = \frac{(1-\tau_R)\alpha R'(H_t)}{N} < 0,$

$\frac{\partial p_t}{\partial \dot{p}_t^e} = \frac{1}{N} > 0.$

e) The graph is shown in Fig. 1.

f) $pT_I(I,H) - 1 = 0$. This gives $I = M(p,H)$ with $\partial I/\partial p = -1/(p^2 T_{II}) > 0$.

g) $pT_I(I,H) = 1$ can now be written $T_I(\frac{I}{H}, 1) = \frac{1}{p}$, from which we get I/H as an implicit function $\frac{I}{H} = \tilde{m}(\frac{1}{p})$, where $\tilde{m}(1) = 0$ since $T_I(0,H) = 1$, and $\tilde{m}' = 1/T_{II}(\frac{I}{H}, 1) < 0$; defining $m(p) \equiv \tilde{m}(\frac{1}{p})$, we see that $m(1) = 0$ and $m' > 0$.

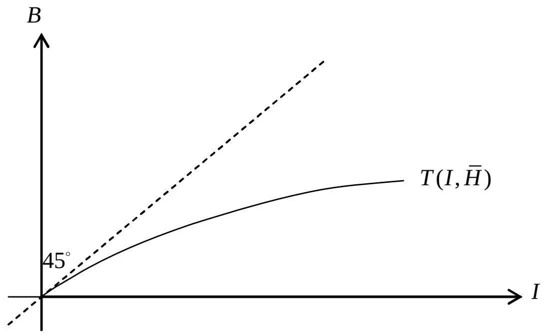

Figure 1:

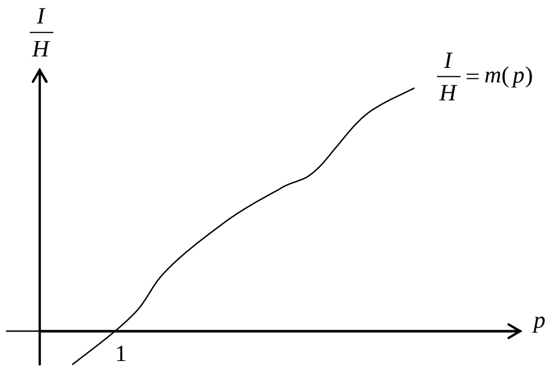

Figure 2:

Like Tobin's q the real price p is a market value of an asset, houses, whose supply changes only slowly. This is because of strictly convex adjustment costs. Instead of the stock of houses adjusting instantaneously, we get residential investment as an increasing function of p in a similar way as investment in business fixed capital tends to be an increasing function of Tobin's q.

h) $\dot{H}_t = [T(m(p_t), 1) - \delta]\, H_t = (b(p_t) - \delta)H_t$, where $b(p_t) \equiv T(m(p_t), 1)$ and $b(1) = T(m(1), 1) = T(0, 1) = 0$, $b' = T_I m' > 0$.

i) $\dot{p}_t = [(1 - \tau_r)r + (1 - \tau_R)\delta + \sigma]\, p_t - (1 - \tau_R)\alpha R(H_t)$, where $R' < 0$. The phase diagram is analogue to those in Lecture Notes, Chapter 10.

j) When $H > 0$, $\dot{H} = 0$ implies $b(p) = \delta > 0$; the unique p satisfying this equation is p^*. Since $b(1) = 0$ and $b' > 0$, we have $p^* > 1$. We have $\dot{p} = 0$ for $p = \frac{(1-\tau_R)\alpha R(H)}{(1-\tau_r)r+(1-\tau_R)\delta+\sigma}$; the unique H satisfying this equation for $p = p^*$ is H^*.

k) The new long-run level of H is $H^{*\prime} > H^*$, because $R' < 0$. On impact p jumps up to the point where the vertical line $H = H^*$ crosses the new (downward-sloping) saddle path.[1] The intuition is that the after-tax return on owning a house has been increased. Hence, by arbitrage the market price p rises to a level such that the after-tax *rate* of return is as before, namely equal to $(1-\tau_r)r$. After t_1, due to the high p relative to the unchanged building cost schedule, H increases gradually and p falls gradually (due to R falling with the rising H). This continues until the new steady state "is reached" with unchanged p^*, but higher H.

ℓ) 1. In the *short* run, H, hence also S, is given. The rental rate R, hence also p, adjusts in the same direction as the demand curve for housing services moves.

2. In the *longer* run (i.e., without new disturbances), it is H, hence also S, that adjusts. Their adjustment is in a direction indicated by the sign of $p - p^*$. On the other hand the house price, p, remains in a neighborhood of the constant cost-determined level, p^*.

This long-run price level equals the marginal building costs when building activity exactly matches the physical wearing down of houses so that the stock of houses is stationary. Due to the positive relationship between building productivity and H, the marginal building costs are unchanged in the long run. The long-run level of H is at the level required for the rental rate $R(H)$ to yield an after-tax rate of return on owning a house equal to $(1-\tau_r)r$. This level of H is H^*. The corresponding level of R is $R^* = R(H^*)$, which is that level at which the demand for housing services equals the long-run supply, i.e., $D(R^*, A, PDV(wl)) = S^* = \alpha H^*$.

VII.3

a) By definition, $\mathcal{MC} = \partial I/\partial B = \partial \mathcal{TC}(B,H)/\partial B$. Taking the total differential on both sides of $T(I,H) = B$ gives $T_I dI + T_H dH = dB$ so that $\mathcal{MC} = \partial I/\partial B = 1/T_I(I,H) >$

[1] In the text of the problem divergent paths (rational bubbles) are ruled out. Otherwise a rational bubble could arise, that is, p could jump to a value *above* the new saddle path and follow the corresponding diverging trajectory in the phase diagram. But such an ever-expanding, deterministic rational bubble does not seem plausible. We are not acquainted with such ever-expanding incidents in real world situations. In a stochastic model of the housing market, however, we could allow for *stochastic* rational bubbles that sooner or later burst. This kind of bubbles is less implausible. Yet, although it takes time to build new houses, the fact that houses have clearly defined reproduction costs, is likely to imply a ceiling on the ultimate level of p and then, by backward induction even a stochastic rational bubble can not get started in the first place. If we want to include bubble theory in our housing market model, we should rather appeal to the behavioral finance literature, where market psychology (herding, fads, etc.) is emphasized.

0, from which follows $\partial\mathcal{MC}/\partial B = \partial^2 I/\partial B^2 = -T_{II}(I,H)/T_I(I,H)^3 > 0$, since $T_{II} < 0$.

b) $\mathcal{MC} = p$ gives $1/T_I(I,H) = p$, which is the same as the first order condition found under f). The marginal cost of building "the first" new house is $\mathcal{MC} = \partial I/\partial B_{|B=0}$ $= 1/T_I(0,H) = 1$.

VI.4 a) Money is an asset (a store of value) which functions as a generally accepted medium of exchange.

b) Usually three functions are associated with money:

1. Its function as a generally accepted medium of exchange.
2. Its function as a store of value.
3. Its function as a unit of account.

Yet, only the two first functions are essential (indispensable) aspects of money. Historically, during hyperinflations, foreign currency has been used as a unit of account, while the national money continued to be the medium of exchange.

c) The statement is problematic. The Sidrauski model uses a "short-cut". It introduces real money balances directly in the utility function instead of letting money just facilitate exchange and thereby indirectly add to utility. A micro-based theory of money demand would show *how* certain pieces of paper by facilitating trade become generally accepted as a medium of exchange and thus acquires purchasing power and becomes demanded.

d) Yes, at least partly. A connection between "money in the utility function" and the "cash-in-advance" constraint (which, by the way, is also a short-cut) is most easily established in *discrete time*. In our standard notation the "cash-in-advance" constraint implies (for the household) that period utility is $u(\min(c_t,\, m_t))$. But on the basis of this we can define a new utility function, $v(c_t,\, m_t) \equiv u(\min(c_t,\, m_t))$. Now, money enters the utility function directly.

e) Money is said to be *neutral* if the level and evolution of real variables like consumption and capital are independent of the level of the money supply. Money is said to be *superneutral* if the level of real variables like consumption and capital *in steady state* are independent of the growth rate of the money supply.

"Neoclassical macroeconomics" refers to macroeconomic theory based on: 1) optimizing agents, 2) markets clear through perfect price flexibility.

There are two statements. As to the first:

It is correct that neoclassical macroeconomics suggests that money is *neutral*. Usually neoclassical macroeconomics also predicts that money is *superneutral*, but not always. For example superneutrality holds for the Sidrauski model, which is a Ramsey-style model extended by money in the utility function. But there exist respectable neoclassical models where superneutrality does not hold (or at best holds only approximately):

1. the Sidrauski model extended with endogenous labour supply;
2. the Sidrauski model extended with "money in the production function";
3. the Sidrauski model extended with a tax system based on nominal income;
4. neoclassical OLG models with money; these models do not generally predict that money is superneutral (this is due to the Tobin effect and the absence, at the aggregate level, of the Keynes-Ramsey rule).

The second statement is outright incorrect. This is because inflation, at least in the long run, is closely related to the growth rate of the money supply. And the inflation rate affects, through the nominal interest rate, the opportunity costs of holding money and thereby affects welfare. According to Milton Friedman's zero interest rate rule, the growth rate of the money supply should be *negative* so as to generate *deflation* and thereby a nominal interest rate, $i = \bar{r} + \pi$, as close to zero as possible (same notation as in chapters 13 and 14). This recommendation is, however, heavily disputed.

f) The statement is incorrect. The Sidrauski model shows that hyperinflation driven by expectations *can* theoretically arise, if the absolute interest elasticity of money demand is above one. However, under "normal circumstances" this elasticity is estimated to be less than one.

g) See the cases mentioned under e).

h) The statement is incorrect. By emphasizing nominal rigidities in the short run New Keynesian Economics predicts non-neutrality of money. Superneutrality of money can, however, be consistent with New Keynesian Economics because superneutrality of money refers to a steady-state or long-run property.

i) The "Friedman zero interest rule" is a policy proposal by Milton Friedman (1969). The proposal is to have a *negative* growth rate in money supply such that the nominal interest rate (the opportunity cost of holding money) is close to zero. Indeed, with nominal interest rate equal to $\bar{r} + \pi$ and $\pi \approx g_M - g_Y$ (notation as in Chapter 14), to obtain $0 = i = \bar{r} + g_M - g_Y$, the central bank should set $g_M \approx g_Y - \bar{r}$, usually a non-positive number.

j) The Sidrauski model and similar neoclassical models may be useful for understanding phenomena where nominal rigidities are unimportant. For example, the Sidrauski model and similar neoclassical models seem well-suited for explaining inflation in *the long run*. But concerning the monetary transmission mechanism in the short run the neoclassical models give a misleading picture, because they ignore nominal rigidities. Thus, when the problem is to conduct countercyclical monetary policy (stabilization policy) the neoclassical models do not give much guidance. In this context, according to many macroeconomists, the Keynesian-oriented models provide a more suitable framework.

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