Christian Groth

A suggested solution to the problem set at the exam in **Advanced Macroeconomics 2** January 8, 2009

(4-hours closed book exam)[1]

As formulated in the course description, a score of 12 is given if the student's performance demonstrates (a) accurate and thorough understanding of the concepts, methods and models in the course, (b) knowledge of the major empirical regularities for aggregate economic variables, and (c) ability to use these theoretical tools and this empirical knowledge to answer macroeconomic questions.

1. Solution to Problem 1

For convenience we repeat the basic differential equations:

$$\dot{\tilde{k}}_t = f(\tilde{k}_t) - \frac{\omega + n + p}{n + p}\tilde{c}_t - (\delta + g + n)\tilde{k}_t, \qquad \tilde{k}_0 > 0 \text{ given}, \tag{1.1}$$

$$\dot{\tilde{c}}_t = \left[f'(\tilde{k}_t) - \delta - \rho + \omega - g\right]\tilde{c}_t - (n + p)(\rho + p)\tilde{k}_t. \tag{1.2}$$

a) In the Blanchard OLG model individuals have finite, but uncertain remaining lifetime. The parameter p is the death rate, i.e., p is the expected number of deaths per time unit, say per year, relatively to the size of population. The model relies on the simplifying assumption that for a given individual the probability of having a remaining lifetime, X, longer than some arbitrary number x is $P(X > x) = e^{-px}$, the same for all (i.e., independent of age). It follows that for any person the probability of dying within one year from now is approximately equal to p. The birth rate, b, is also assumed constant so that $n = b - p$ is the population growth rate. The model appeals to the law of large numbers and considers the actual number of deaths (and births) per year to be indistinguishable from the expected number.

Individuals can buy life annuity contracts from life insurance companies. These companies have negligible administrative costs so that in equilibrium with free entry (zero

[1] The solution below contains more details and more precision than can be expected at a four hours exam.

profits) the rate of return on these contracts is $r_t + p$ until death, where r_t is the risk-free real interest rate. The actuarial bonus p is financed through the wealth transfer to the insurance sector that occurs when depositors die (due to cancelling of the liability of the insurance company).

The equation (1.1) is essentially just national income accounting. Isolating $f(\tilde{k}_t)$ on one side we have aggregate gross income per unit of effective labor on this side and consumption plus gross investment per unit of effective labor on the other side.

Additional parameters: δ is the capital depreciation rate and g is the rate of (Harrod-neutral) technical progress. Finally, ω is the "retirement rate", reflecting that individual labor supply is assumed to decline exponentially at rate ω with increasing age. For fixed ω and b the fraction of the population in the labor force is a constant $b/(\omega + b)$. Hence, the growth rate of the labor force is the same as that of population, n.

As to the first term on the right-hand side of (1.2), notice that instantaneous utility in the Blanchard OLG model is logarithmic, so that the individual Keynes-Ramsey rule is simply $\dot{c}_t = [r_t + p - (\rho + p)]\, c_t = (r_t - \rho)c_t$, where ρ is the pure rate of time preference (impatience) and $\rho + p$ is the effective rate of discount of future utility (the addition of p to this discount rate reflects the probability of not being alive at the date in question). In general equilibrium with perfect competition $r_t = f'(\tilde{k}_t) - \delta$, where δ is the capital depreciation rate. The corresponding growth-corrected Keynes-Ramsey rule would then be $\dot{\tilde{c}}_t = \left[f'(\tilde{k}_t) - \delta - \rho - g\right] \tilde{c}_t$ if labor supply were age-independent. But due to the gradual replacement of dying elder individuals with little labor supply by younger individuals supplying more labor, this becomes modified by $+\omega$ entering the first term on the right-hand side of (1.2).

The second term on the right-hand side of (1.2) represents another aspect of the generation replacement. The arrival of newborns is Nb per time unit. The fact that they have *more human* wealth than those who they displace has already been taken into account. But the newborns enter the economy with *less financial* wealth than the "average citizen". This lowers aggregate consumption by $b(\rho + p)A_t$ per time unit, where A_t is aggregate financial wealth. Indeed, the average financial wealth in the population is A_t/N_t and the consumption effect of this is $(\rho + p)A_t/N_t$, since the propensity to consume out of wealth is $\rho + p$. Thus, *ceteris paribus*, aggregate consumption is reduced by $N_t b(\rho + p)A_t/N_t = b(\rho + p)A_t = (n + p)(\rho + p)A_t$ per time unit. In general equilibrium in the closed economy (without government debt), $A_t = K_t$. Correcting for population and technology growth, we end up with a lowering of $\dot{\tilde{c}}_t$ equal to $(n + p)(\rho + p)\tilde{k}_t$. This explains the second term in (1.2).

b) The equation describing the $\dot{\tilde{k}} = 0$ locus is

$$\tilde{c} = \frac{n+p}{\omega+n+p}\left[f(\tilde{k}) - (\delta+g+n)\tilde{k}\right]. \tag{1.3}$$

The equation describing the $\dot{\tilde{c}} = 0$ locus is

$$\tilde{c} = \frac{(n+p)\,(\rho+p)\,\tilde{k}}{f'(\tilde{k}) - \delta - \rho + \omega - g}. \tag{1.4}$$

Let $\bar{\tilde{k}}$ be defined by

$$f'(\bar{\tilde{k}}) - \delta = \rho - \omega + g. \tag{1.5}$$

That is, $\bar{\tilde{k}}$ is defined as the value of $\tilde{k}$ such that the denominator of (1.4) vanishes. Such a value exists since, in addition to the Inada conditions, the inequality

$$\omega < \delta + \rho + g$$

is assumed to hold. Another key value of $\tilde{k}$ is the golden-rule value, $\tilde{k}_{GR}$, determined by the requirement

$$f'\left(\tilde{k}_{GR}\right) - \delta = n + g.$$

The $\dot{\tilde{k}} = 0$ and $\dot{\tilde{c}} = 0$ loci are illustrated in Fig. 1.1. The $\dot{\tilde{c}} = 0$ locus is everywhere to the left of the line $\tilde{k} = \bar{\tilde{k}}$ and is asymptotic to this line for $\tilde{k} \to \bar{\tilde{k}}$. The phase diagram also shows the steady-state point E where the $\dot{\tilde{c}} = 0$ locus crosses the $\dot{\tilde{k}} = 0$ locus. The corresponding capital intensity is $\tilde{k}^*$, to which corresponds the (growth-corrected) consumption level $\tilde{c}^*$. Fig. 1.1 depicts a case where $\bar{\tilde{k}} \leq \tilde{k}_{GR}$ so that $\tilde{k}^* < \tilde{k}_{GR}$, that is, the economy is dynamically efficient. Yet, since $\rho - \omega < n$ is possible and this inequality implies $\bar{\tilde{k}} > \tilde{k}_{GR}$, dynamic inefficiency cannot be ruled out theoretically (a typical feature of an OLG model).

The directions of movement in the different regions of the phase diagram is determined by the differential equations (1.1) and (1.2), and are shown by arrows. The arrows taken together show that the steady state E is a saddle point. Since we have one predetermined variable, $\tilde{k}$, and one jump variable, $\tilde{c}$, the steady state is saddle-point stable. The saddle path is the only path that satisfies *all* the conditions of general equilibrium (individual utility maximization for given expectations, profit maximization by firms, continuous market clearing, and perfect foresight). The other paths in the diagram either violate the transversality conditions of the individuals (paths that in the long run point South-East) or their NPG condition[2] (paths that in the long run point North-West). Hence,

[2] And therefore also the transversality condition.

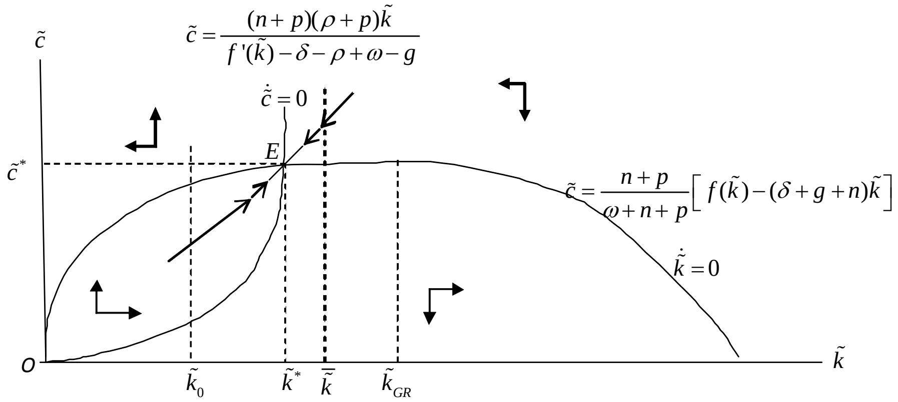

Figure 1.1: Phase diagram of the Blanchard model with retirement.

equilibrium initial consumption is determined as the ordinate, $\tilde{c}_0$, to the point where the vertical line $\tilde{k} = \tilde{k}_0$ crosses the saddle path. Over time the economy moves from this point, along the saddle path, towards the steady state. Thus, for $t \to \infty$,

$$r(t) = f'(\tilde{k}(t)) - \delta \to f'(\tilde{k}^*) - \delta \equiv r^*, \tag{1.6}$$

where r^* is the long-run rate of return.

c) Since $f'' < 0$, r^* is a decreasing function of $\tilde{k}^*$. We therefore focus on how $\tilde{k}^*$ depends on n and g. Given p, a lower n moves the $\dot{\tilde{c}} = 0$ locus clockwise; but in which direction the $\dot{\tilde{k}} = 0$ locus shifts is ambiguous. Hence, instead of just considering simple curve shifting in the phase diagram, we look for another approach. This is provided by the link which refers to the fact that in steady state the right-hand sides of (1.3) and (1.4) are equal. Thus, after ordering

$$\left(\frac{f(\tilde{k})}{\tilde{k}} - (\delta+g+n)\right)\left[f'(\tilde{k}) - \delta - \rho + \omega - g\right] = (\omega+n+p)(\rho+p). \tag{1.7}$$

Let the left-hand side of this equation, considered as a function of $\tilde{k}$, be denoted $h(\tilde{k})$. The downward-sloping curve in Fig. 1.2 illustrates the graph of this function (both the average and marginal products of capital, $f(\tilde{k})/\tilde{k}$ and $f'(\tilde{k})$ are decreasing in $\tilde{k}$). Further, let $RHS(1.7)$ denote the right-hand side of (1.7).

Everything else equal we see that:

$$n \downarrow \quad \Rightarrow \left\{ \begin{array}{c} \text{the } h(\tilde{k}) \text{ curve } \uparrow \\ RHS(1.7) \downarrow \end{array} \right\} \Rightarrow \tilde{k}^* \uparrow \ \Rightarrow r^* \downarrow$$

$$g \downarrow \quad \Rightarrow \left\{ \begin{array}{c} \text{the } h(\tilde{k}) \text{ curve } \uparrow \\ RHS(1.7) \text{ unchanged} \end{array} \right\} \Rightarrow \tilde{k}^* \uparrow \ \Rightarrow r^* \downarrow$$

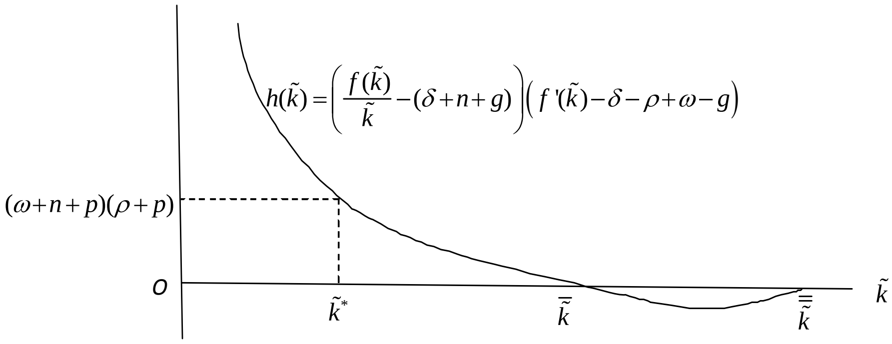

Figure 1.2: Determination of $\tilde{k}^*$.

Thus, unambiguously country B has lower long-run rate of return than country A. Intuitively, lower labor force growth implies less capital "dilution", hence a tendency towards higher capital intensity and lower marginal product of capital. And with lower growth rate in technology, hence in per capita consumption in the long run, a lower excess of the long-run rate of return, r^*, over the pure rate of time preference, ρ, is needed to sustain that lower long-run growth in per capita consumption.

In relation to the debate mentioned, the analysis supports the view that along with lower growth rates in both labor and technology in the future one should expect also a lower long-run rate of return. Admittedly, this support is only indicative, because it is based only on comparative dynamics. The model is not suitable for studying effects of gradual changes in n and g in historical time. The equations of the model depict a situation where the demographic parameters, p, n, and ω have stayed at their current values for a long time and are not changing.

d) The death rate p is likely to *fall*. The inverse of the death rate measures life expectancy which is increasing due to better health conditions. Everything else equal we see that:

$$p \downarrow \;\Rightarrow \left\{ \begin{array}{c} \text{the } h(\tilde{k}) \text{ curve unchanged} \\ RHS(1.7) \downarrow \end{array} \right\} \Rightarrow \tilde{k}^* \uparrow \;\; \Rightarrow r^* \downarrow$$

We see that the effect on r^* of a lower p is unambiguously negative. By itself, higher life expectancy lowers the effective utility discount rate and increases saving and capital accumulation, leading to a higher capital intensity and lower marginal product of capital. The lower p is thus an additional reason for country B to have a lower long-run rate of return.

e) Better health and increased life expectancy are also likely to affect retirement from the labor market, namely to postpone retirement. In the model this corresponds to a *decrease* in ω. The increase in educational attainment, which is often associated with more attractive work conditions (perhaps even enjoyable jobs), is also likely to contribute to a postponement of retirement.

On the other hand, treating retirement as an endogenous economic decision (outside the model), it is not obvious in what direction retirement will respond to higher after-tax real wages in the future (due to expected technical progress). The opportunity cost of early retirement becomes higher. But the higher after-tax real wages during working life imply that one can afford to retire earlier. Thus, in the Slutsky terminology, higher after-tax real wages have negative substitution and income effects on (early) retirement, but a positive wealth effect.

Whatever the direction of the change in ω, the effect on r^* cannot be signed unambiguously. Indeed,

$$\omega \downarrow \ \Rightarrow \left\{ \begin{array}{c} \text{the } h(\tilde{k}) \text{ curve } \uparrow \\ RHS(1.7) \ \uparrow \end{array} \right\} \Rightarrow \tilde{k}^* \ \uparrow\downarrow ?$$

The intuitive interpretation of the ambiguity is the following. On the one hand a lower ω, everything else equal, implies a higher labor force participation rate. This entails more production and capital accumulation in the economy. On the other hand, a lower ω, everything else equal, implies a shorter period as retired and therefore less need to save for old age. This implies *less* saving and capital accumulation in the economy. The net result of these two effects is ambiguous. If the last effect is the stronger one, then a lower ω is a force which, in isolation, increases r^*. Generally we cannot rule out that this force offsets or more than offsets the effect on r^* of the other three forces. Hence, in the end we cannot on this qualitative basis unambiguously conclude whether country A or B has the higher long-run rate of return. A numerical calculation based on a specified production function and calibrated parameter values would be necessary.

f) In our curriculum we have other models with quite specific implications for the long-run rate of return, namely Barro's bequest model, the Sidrauski model (which is a monetary Ramsey model in continuous time), and the RBC model (which is a non-monetary Ramsey model in discrete time). These models are representative agent models that via an aggregate Keynes-Ramsey rule lead to essentially the same "modified golden rule". In Barro's model this takes the form

$$1 + r^* = (1 + R)(1 + g)^{\theta},$$

where r^* is the long-run rate of return, R is the pure intergenerational rate of discount, g is the rate of (Harrod-neutral) technical progress, and θ is the elasticity of marginal utility of consumption. In these models (at least in the versions of our curriculum) a lower growth rate in the labor force will in itself not affect the long-run rate of return. But a lower rate of technical progress has qualitatively the same effect as above.

These models do not consider demographic variables such as life expectancy and retirement rate. Implicitly, the models assume these variables are not important.

2. Solution to Problem 2

For convenience, we repeat the basic definitions:

$$\begin{aligned}
Y_t &= \text{GDP at time } t,\\
T_t &= \text{net tax revenue } (= \text{ gross tax revenue} - \text{transfer payments}) \text{ at time } t,\\
G_t &= \text{government spending on goods and services at time } t,\\
B_t &= \text{public debt at time } t,\\
b_t &\equiv B_t/Y_t = \text{debt-income ratio at time } t,\\
S_t &= \text{primary budget surplus at time } t.
\end{aligned}$$

Time is continuous.

a) In view of the budget deficit being bond financed, we have

$$\dot{B}_t \equiv \frac{dB_t}{dt} = rB_t + G_t - T_t,$$

where the right-hand side is the budget deficit. The primary budget surplus is defined as

$$S_t = T_t - G_t.$$

We assume that $Y_t = Y_0 e^{(g+n)t}$, $G_t = \gamma Y_t$, and $T_t = \tau Y_t$, where g, n, γ, and τ are constants, $g + n > 0$, $0 < \gamma < 1$, and $0 < \tau < 1$. Furthermore, $r > g + n$ and $B_0 > 0$.

b) *Approach 1:* We have

$$\frac{\dot{b}_t}{b_t} = \frac{\dot{B}_t}{B_t} - \frac{\dot{Y}_t}{Y_t} = r - \frac{(\tau - \gamma)Y_t}{B_t} - (g + n),$$

or

$$\dot{b}_t = (r - g - n)b_t - (\tau - \gamma). \tag{2.1}$$

The solution is

$$b_t = (b_0 - b^*)e^{(r-g-n)t} + b^*, \quad \text{where} \quad b^* = \frac{\tau - \gamma}{r - g - n},$$

and b_0 is historically given.

By definition, a given fiscal policy is sustainable if under this policy the government stays solvent (capable of meeting the financial commitments as they fall due).

Since $r-g-n>0$, if fiscal policy is such that $b^* < b_0$, the debt-income ratio explodes. Then the government will in the long run be unable to find buyers for all the debt. The government will become insolvent. Thus, fiscal sustainability requires the policy is such that $b^* \geq b_0$, i.e.,

$$\frac{\tau-\gamma}{r-g-n} \geq b_0 \quad \text{or} \quad \tau-\gamma \geq (r-g-n)b_0.$$

The initial primary surplus is $S_0 = T_0 - G_0 = (\tau-\gamma)Y_0$. Thus, the minimum initial primary surplus is

$$\bar{S}_0 = (r-g-n)b_0 Y_0 = (r-g-n)B_0. \tag{2.2}$$

The corresponding tax rate is

$$\tau = \gamma + (r-g-n)b_0.$$

Approach 2: Here we consider the intertemporal government budget constraint. Since the real interest rate is above the growth rate, fiscal sustainability requires that the government satisfies the intertemporal budget constraint

$$\int_0^\infty G_t e^{-rt} dt \leq \int_0^\infty T_t e^{-rt} dt - B_0,$$

or

$$\int_0^\infty (T_t - G_t) e^{-rt} dt \equiv \int_0^\infty S_t e^{-rt} dt \geq B_0, \tag{2.3}$$

In the present case this requirement amounts to

$$\begin{aligned}\int_0^\infty (\tau-\gamma) Y_t e^{-rt} &= \int_0^\infty (\tau-\gamma) Y_0 e^{(g+n)t} e^{-rt} dt \\ &= (\tau-\gamma) Y_0 \int_0^\infty e^{-(r-g-n)t} dt = (\tau-\gamma) Y_0 \frac{1}{r-g-n} \geq B_0.\end{aligned}$$

From this we again find the minimum initial primary surplus to be as in (2.2).

Now we consider the alternative setup where the expected demographic development is such that with unchanged transfer and taxation rules the net tax rate, T_t/Y_t, will be

$$\tau_t = \tau_0 - (1-e^{-\lambda t})\alpha. \tag{*}$$

Here λ and α are positive parameters and $\alpha > \bar{S}_0/Y_0$, where $\bar{S}_0$ refers to the result in (2.2).

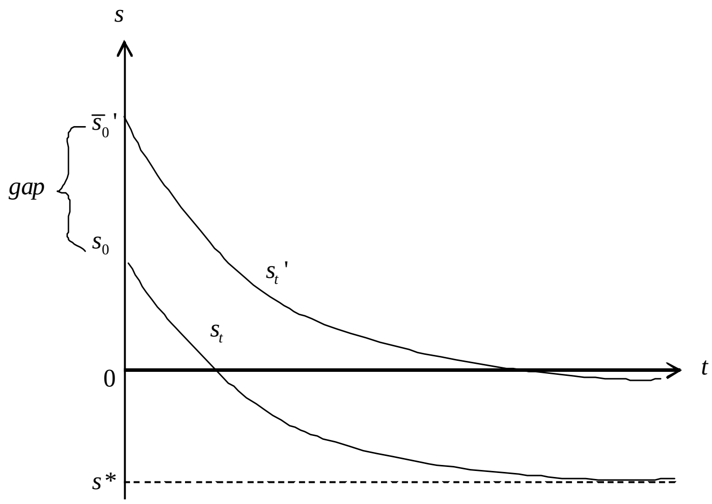

Figure 2.1:

c) A possible interpretation is that an increasing elderly dependency ratio, due to longer life expectancy and lower fertility, leads to more government spending on transfers, say pay-as-you-go pensions, and less gross tax revenue because of a relatively smaller labor force. As a consequence *net* tax revenue as a fraction of GDP tends to fall. The parameter α is then the long-run fall in this fraction and λ is the adjustment speed.

We find

$$\begin{aligned} s_t &\equiv \frac{S_t}{Y_t} = \tau_t - \gamma = \tau_0 - (1 - e^{-\lambda t})\alpha - \gamma = \gamma + \frac{\bar{S}_0}{Y_0} - (1 - e^{-\lambda t})\alpha - \gamma \\ &= \frac{\bar{S}_0}{Y_0} - (1 - e^{-\lambda t})\alpha \rightarrow \frac{\bar{S}_0}{Y_0} - \alpha \equiv s^* < 0 \end{aligned}$$

for $t \rightarrow \infty$. The time profile of s_t is shown in Fig. 2.1.

d) Current fiscal policy $\mathcal{P}$ is not sustainable since $\int_0^\infty S_t e^{-rt} dt = 0 < B_0$.

e) Defining $s_t' \equiv S_t/Y_t$ we have under the new policy, $\mathcal{P}'$,

$$s_t' = \tau_t' - \gamma = \tau_0' - (1 - e^{-\lambda t})\alpha - \gamma \rightarrow \tau_0' - \alpha - \gamma$$

for $t \rightarrow \infty$.

f) The general requirement, (2.3), can be written

$$\begin{aligned} \int_0^\infty s_t Y_t e^{-rt} &= \int_0^\infty s_t Y_0 e^{(g+n)t} e^{-rt} dt \geq B_0, \quad \text{or} \\ \int_0^\infty s_t e^{-(r-g-n)t} dt &\geq \frac{B_0}{Y_0} \equiv b_0. \end{aligned}$$

With the suggested new policy $s_t = s'_t$ and $\tilde{r} \equiv r - g - n$ this gives

$$\int_0^\infty s'_t e^{-\tilde{r}t} dt = \int_0^\infty \left[\tau'_0 - (1 - e^{-\lambda t})\alpha - \gamma\right] e^{-\tilde{r}t} dt = \frac{\tau'_0 - \gamma}{\tilde{r}} - (\frac{1}{\tilde{r}} - \frac{1}{\lambda + \tilde{r}})\alpha \geq b_0. \quad (2.4)$$

The initial primary surplus-income ratio is $s'_0 = \tau'_0 - \gamma$ so that the requirement (2.4) amounts to

$$s'_0 \geq \tilde{r}b_0 + \frac{\lambda}{\lambda + \tilde{r}}\alpha.$$

The *minimum* initial primary surplus-income ratio is thus

$$\bar{s}'_0 = \tilde{r}b_0 + \frac{\lambda}{\lambda + \tilde{r}}\alpha.$$

g) The sustainability gap indicator for policy $\mathcal{P}$ is

$$gap \equiv \bar{s}'_0 - s_0 = \tilde{r}b_0 + \frac{\lambda}{\lambda + \tilde{r}}\alpha - (\tau_0 - \gamma).$$

The gap is indicated in Fig. 2.1. We have

$$\frac{\partial gap}{\partial \tilde{r}} = b_0 - \frac{\lambda}{(\lambda + \tilde{r})^2}\alpha \gtreqless 0 \text{ for } \alpha \lesseqgtr \frac{(\lambda + \tilde{r})^2}{\lambda} b_0,$$

respectively. Two opposite forces are in play. On the one hand $\tilde{r} \uparrow \Rightarrow \tilde{r}b_0 \uparrow$, which implies that the gap tends, ceteris paribus, to be greater with higher growth-corrected interest rate, because the snowball effect of compound interest becomes stronger. On the other hand this effect on the gap may be offset by the fact that $\tilde{r} \uparrow \Rightarrow \frac{\lambda}{\lambda + \tilde{r}}\alpha \downarrow$, which reflects that the present value of the long-run shortfall of the primary budget surplus under policy $\mathcal{P}$ is lower when $\tilde{r}$ is higher. Thus the sign of the net effect on *gap* is generally ambiguous. If the long-run fall in the primary surplus is high relative to the current debt, we get the somewhat surprising result that *gap* decreases with higher growth-corrected interest rate.

3. Solution to Problem 3

For convenience, the equations of the model are repeated here:

$$\dot{Y}_t = \lambda(D(Y_t, r_t, x_t, \alpha) + G - Y_t), \ \lambda > 0, 0 < D_Y < 1, D_r < 0, D_x > 0, D_\alpha > 0, \quad (3.1)$$

$$\frac{M}{P} = L(Y_t, i_t), \qquad L_Y > 0, \ L_i < 0, \quad (3.2)$$

$$i_t = i^* + \frac{\dot{X}_t^e}{X_t}, \quad (3.3)$$

$$r_t \equiv i_t - \pi_t^e, \quad (3.4)$$

$$x_t \equiv \frac{X_t P^*}{P}. \quad (3.5)$$

The variables α, M, P, P^*, and i^* are exogenous and constant. The initial level, Y_0, of output is given. Expectations are rational. Speculative bubbles are assumed away.

a) Evidently, the model is the Blanchard-Fischer version of Dornbusch's overshooting model, i.e., a dynamic IS-LM model for a SOE with a floating exchange rate. It is a model of short-run mechanisms. The price level P is assumed predetermined and constant. The function $D(Y_t, r_t, x_t, \alpha)$ in equation (3.1) represents private aggregate demand. The signs of the partial derivatives of D have the following interpretation: $D_Y > 0$ because private consumption and perhaps also investment depends positively on aggregate production and income (which affect perceived aggregate wealth); a further possible source to the positive dependence of demand on current income is that liquidity constraints may be operative; $D_Y < 1$ because the marginal propensity to spend can, realistically, be assumed less than one (that net exports depend negatively on Y helps in this direction); $D_r < 0$ because consumption (primarily through the wealth effect) and investment depend negatively on the real interest rate; $D_x > 0$ because the Marshall-Lerner condition is implicitly assumed satisfied so that net exports depend positively on the real exchange rate $x \equiv XP^*/P$, cf. (3.5); indeed, x is an indicator of competitiveness. Finally, $D_\alpha > 0$ reflects the role of α as a demand shift parameter.

Equation (3.1) reflects that the adjustment of output to demand takes time; the parameter λ is the speed of adjustment. Equation (3.2) expresses equilibrium in the money market for all t (unlike the output market, asset markets are immediately brought into equilibrium by arbitrage). Naturally, real money demand depends positively on Y through the "transaction motive" and negatively on the (short-term) nominal interest rate, the opportunity cost of holding money. Equation (3.3) is the UIP assumption, saying that domestic and foreign financial assets pay the same expected rate of return when measured in the same currency. Indeed, on the left-hand side appears the interest rate on a bond denominated in domestic currency (henceforth a "domestic bond"). On the right-hand side appears the expected rate of return on investing in a bond denominated in foreign currency (henceforth a "foreign bond") plus the expected rate of depreciation of the domestic currency. This sum equals the expected rate of return on the foreign asset when measured in the domestic currency. The equations (3.4) and (3.5) define the real interest rate, r_t, and the real exchange rate, x_t, respectively.

b) We interpret the assumption of rational expectations (here perfect foresight since there are no stochastic elements in the model) as implying $\pi_t^e = \pi_t = 0$ for all t. Then equation (3.4) reduces to $r_t = i_t$. In view of (3.2) we can write i_t as a function of Y_t and

M/P, that is, $i_t = i(Y_t, M/P)$, where $i_Y = -L_Y/L_i > 0$ and $i_{M/P} = 1/L_i < 0$. Inserting into (3.1) gives

$$\dot{Y}_t = \lambda(D(Y_t, i(Y_t, M/P), X_t P^*/P, \alpha) + G - Y_t). \tag{3.6}$$

Similarly, $\dot{X}_t^e = \dot{X}_t$, so that (3.4) can be written

$$\dot{X}_t = (i(Y_t, M/P) - i^*)X_t. \tag{3.7}$$

In this way the model has been reduced to two coupled differential equations in Y_t and X_t, where Y_t is predetermined and X_t is a jump variable (forward-looking variable).

To draw the corresponding phase diagram, note that (3.6) implies that

$$\dot{Y} = 0 \text{ for } D(Y, i(Y, M/P), XP^*/P, \alpha) + G = Y. \tag{3.8}$$

Take the total differential on both sides wrt. Y, X, M, α, and G (for later use):

$$(D_Y + D_r i_Y)dY + D_r i_{M/P}\frac{1}{P}dM + D_x\frac{P^*}{P}dX + D_\alpha d\alpha + dG = dY.$$

Ordering yields

$$(1 - D_Y - D_r i_Y)dY = D_r i_{M/P}\frac{1}{P}dM + D_x\frac{P^*}{P}dX + D_\alpha d\alpha + dG = dY. \tag{3.9}$$

Setting $dM = d\alpha = dG = 0$, we find

$$\frac{dX}{dY}\Big|_{\dot{Y}=0} = \frac{1 - D_Y - D_r i_Y}{D_x P^*/P} > 0.$$

It follows that the $\dot{Y} = 0$ locus (the "IS curve") is upward-sloping as shown in Fig. 3.1.

Equation (3.7) implies that

$$\dot{X} = 0 \;\text{ for }\; i(Y, M/P) = i^*. \tag{3.10}$$

The value of Y satisfying this equation is unique (because $i_Y \neq 0$) and is called $\bar{Y}$. That is, $\dot{X} = 0$ for $Y = \bar{Y}$, which says that the $\dot{X} = 0$ locus (the "LM curve") is vertical, cf. Fig. 3.1. The figure also indicates the direction of movement in the different regions, as determined by (3.6) and (3.7). We see that the steady-state point, E, is a saddle point. This implies that two and only two solution paths – one from each side – converge towards E.

Given Y_0, at time $t = 0$ the economy must be somewhere on the vertical line $Y = Y_0$. In view of the absence of speculative bubbles, the explosive or implosive paths of X in Fig. 3.1 cannot arise. Hence, we are left with the saddle path, or rather the segment AE of the saddle path, as the unique solution to the model. Gradually over time the economy moves

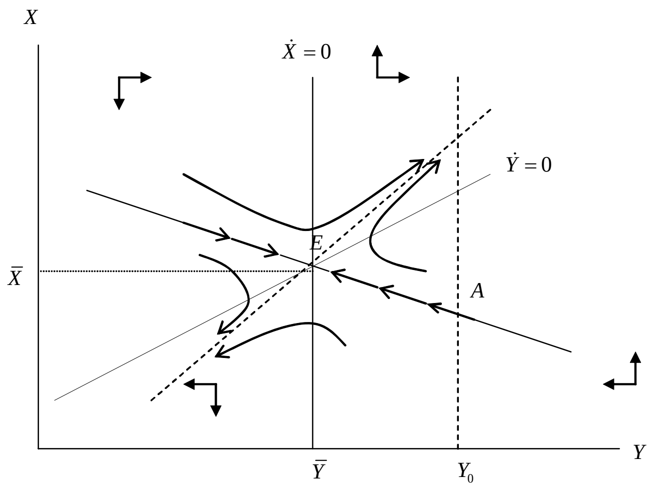

Figure 3.1:

along the saddle path towards the steady state E. If $Y_0 > \bar{Y}$ (as in Fig. 3.1), output is decreasing and the exchange rate increasing (depreciation) during the adjustment process. If instead $Y_0 < \bar{Y}$, the opposite movements occur.

c) The recession in the leading economies in the world implies less exports from our SOE to these countries. The change in the demand parameter to $\alpha' < \alpha$ can be seen as reflecting this. The whole setup might be interpreted as a crude description of, for example, the Swiss or the Swedish economy in the present situation of an almost global recession.

d) The interpretation of the situation is that the SOE, being initially in steady state, is unexpectedly disturbed at time t_0 by two permanent shocks. These shocks take the form that the demand parameter shifts to $\alpha' < \alpha$ and the world interest rate to $i^{*\prime} < i^*$, where $i^{*\prime}$ is close to zero.

The fall in the demand parameter to α' does not affect the $\dot{X} = 0$ locus, but it shifts the $\dot{Y} = 0$ locus upward (hence leftward). Indeed, by (3.9),

$$\frac{\partial X}{\partial \alpha}\Big|_{\dot{Y}=0, Y \text{ fixed}} = -\frac{D_\alpha}{D_x \frac{P^*}{P}} < 0,$$

so that

$$dX = -\frac{D_\alpha}{D_x \frac{P^*}{P}} d\alpha > 0,$$

for $d\alpha < 0$. The interpretation is that a fall in α induces a lower output demand; then, for demand to still match the fixed Y, a depreciation (higher X) is needed.

On the other hand, the fall in the world interest rate does not affect the $\dot{Y} = 0$ locus, but it shifts the $\dot{X} = 0$ locus to the left. Indeed, by (3.10), where Y can be replaced by $\bar{Y}$, we have

$$\frac{\partial \bar{Y}}{\partial i^*}\Big|_{\dot{X}=0, M \text{ fixed}} = \frac{1}{i_Y} = -\frac{1}{L_Y/L_i} > 0,$$

so that

$$d\bar{Y} = -\frac{1}{L_Y/L_i} di^* < 0,$$

for $di^* < 0$. The interpretation is that a lower i^* requires a lower domestic interest rate for UIP to hold with $\dot{X} = 0$. In turn, given M/P, a lower domestic interest rate requires a lower transaction demand for money, i.e., a lower output level.

The phase diagram portraying the dynamics for $t < t_0$ is shown in Fig. 3.2. The two shocks have opposite effects on the long-run exchange rate. We are told that circumstances are such that the sign of the long-run net effect on X is dominated by the influence from the fall in the world interest rate. Hence, the new steady state level, $\bar{X}'$, of the exchange rate must be below the old, $\bar{X}$. In consequence, the new steady state, E', is South-West of the old, as indicated in the figure. Fig. 3.3 shows the time profiles of Y_t, X_t, and r_t.

The long-run effect of the two shocks is thus an economic recession in the SOE combined with (in the present case) *appreciation* of its currency. The impact effect on the exchange rate is, however, a *larger* appreciation than in the long run - the phenomenon called "overshooting". Indeed, immediately after time t_0 the economy must be on the new saddle path, i.e., at the point A in Fig. 3.2. Then Y gradually falls in response to the low output demand generated by the low real exchange rate x_t (low competitiveness); along with the falling Y the interest rate gradually falls as well, because of lower demand for transaction balances (M is unchanged). During the adjustment we have $i_t > i^{*'}$ with interest parity maintained through gradual depreciation of the exchange rate until the new steady state is "reached".

The intuition behind the overshooting is the following. The downward jump in the world interest rate triggers an inflow of financial capital sufficiently large to cause immediate appreciation of the domestic currency to a level from which it is expected to *depreciate* at a sufficient rate to re-establish and maintain interest parity. Very fast arbitrage brings about the new short-run equilibrium where the demand for domestic bonds is again satisfied by the given supply because the higher domestic interest rate is counterbalanced by expected depreciation.

e) Under "normal circumstances" an upward shift in M would stimulate output demand by lowering the interest rate and triggering off a depreciation. But we are told that

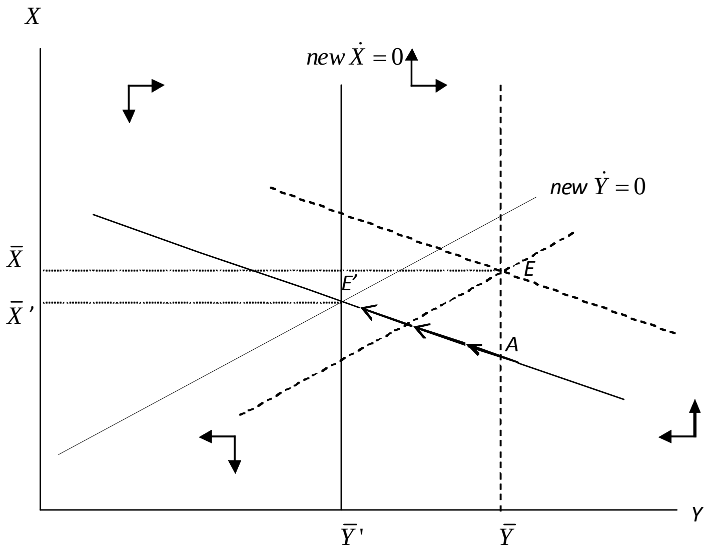

Figure 3.2:

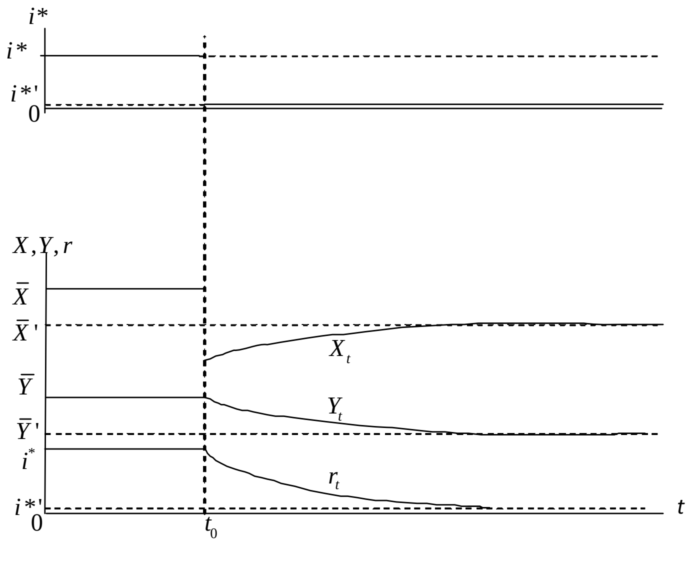

Figure 3.3:

$i^{*\prime}$ is close to zero. The adjustment described under d) therefore brings i_t close to zero.

Conventional monetary policy. A plausible reason that conventional monetary policy might not work could be that the economy is locked in a *liquidity trap* in the sense that raising M by open market operations cannot decrease the interest rate further. Noting this is definitely enough. (The details are complicated and the model is not sufficiently specified or microfounded to rule out multiple self-fulfilling expectations equilibria in this situation. The complicating factor is that, in the phase diagram, for the given M, the usual smooth dynamics of X are not operative at $\bar{Y}'$ or to the left of $\bar{Y}'$. Indeed, every point in the region $Y \leq \bar{Y}'$ of Fig. 3.2 will have the property that $\dot{X}_t = 0$ in view of $i(Y, M/P) = i^{*\prime}$. Only the Y dynamics are operative, albeit in a modified form because the interest rate is insensitive to Y when $Y \leq \bar{Y}'$.)

Conventional fiscal policy. The following gives a plausible reason for conventional fiscal policy in the form of an upward shift in G (or reduced taxation) being impotent. When capital mobility is perfect and domestic and foreign bonds are perfect substitutes, as assumed by the model, then expansionary fiscal policy is immediately counteracted by appreciation of the domestic currency. The role of a higher G is simply the opposite of that of a lower α under d). Thus a rise in G does not affect the $\dot{X} = 0$ locus, but it shifts the $\dot{Y} = 0$ locus downward (hence rightward). Indeed, by (3.9),

$$\frac{\partial X}{\partial G}\Big|_{\dot{Y}=0, Y \text{ fixed}} = -\frac{1}{D_x \frac{P^*}{P}} < 0,$$

so that

$$dX = -\frac{1}{D_x \frac{P^*}{P}} dG < 0,$$

for $dG > 0$. Hence, if an (unanticipated) upward shift in G is implemented at time $t_1 > t_0$, the saddle path and the long-run exchange rate shift to a lower position, whereas long-run output remains at the same depressed level, $\bar{Y}'$.

If this steady-state level is already (almost) reached at time t_1, output will stay there. If it is not yet reached, output will just continue its convergence towards the steady-state level along the new lower saddle path, corresponding to a lower expected long-run value of the exchange rate. The demand effect of the higher G is counteracted by the lower exchange rate (lower competitiveness). As before, during the adjustment we have $i_t > i^{*\prime}$ with interest parity maintained through gradual depreciation of the exchange rate towards its new steady-state level $\bar{X}' < \bar{X}$.

f) Yes, the outlook is improved if international coordination of expansionary fiscal policy is possible. Then one can regard the international economy as a large closed economy, where the problem of offsetting appreciation does not arise.

There is a further advantage of a coordinated fiscal stimulus. A unilateral fiscal stimulus in a small open economy (even under a fixed exchange rate) faces the problem that a lot of the stimulus goes to other countries through the imports leakage. This restrains the spending multiplier and can make the needed fiscal expansion problematic for a country with already high public debt or current account deficit problems. With a coordinated fiscal expansion these problems are eased.

4. Solution to Problem 4

a) False. The condition is necessary, not sufficient. Take the simple version of the Barro model, where technical progress is ignored. Let R be the (pure) intergenerational discount rate and r_D the real interest rate in the associated Diamond economy. Then the bequest motive in the Barro economy is operative if and only if $R < r_D$. Knowing that the associated Diamond economy is dynamically efficient only ensures that $r_D > n$, where n is the constant rate of growth in the number of young (the labor force). In Barro's model it is assumed that $R > n$. But $r_D > n$ and $R > n$ do not ensure $r_D > R$.

b) False. A neoclassical model (optimizing agents, market clearing through perfectly flexible prices) with perfect competition may include convex capital installation costs as presumed in Tobin's q-theory of investment. In such a setup the net marginal product of capital will only be one of the determinants of the real interest rate.

c) Money is said to be *superneutral* if in steady state the real variables such as consumption and investment are independent of the growth rate of the money supply

Seigniorage is the revenue obtained by the public sector by printing money.

Propagation mechanism is a term used in business cycle theory and refers to the mechanism through which effects of a shock are spread out in the economic system (more or less synonymous with "transmission mechanism").

Liquidity trap is said to be present if the short-term nominal interest rate has come so close to its lower bound (zero) that conventional monetary policy is no longer effective.

Precautionary saving is the increase in saving resulting from increased uncertainty.

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