Advanced Macroeconomics. Exercise solutions

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A brief solution of Problem IV.4

IV.4 The economy is described by the two differential equations

$$\begin{aligned} \dot{\tilde{k}}_t &= f(\tilde{k}_t) - \frac{\omega + n + p}{n + p}\tilde{c}_t - (\delta + g + n)\tilde{k}_t, \qquad \tilde{k}_0 > 0 \text{ given}, \\ \dot{\tilde{c}}_t &= \left[f'(\tilde{k}_t) - \delta - \rho + \omega - g\right]\tilde{c}_t - (n+p)(\rho+p)\tilde{k}_t, \end{aligned}$$

where $\tilde{k}_t \equiv K_t/(T_t L_t)$ etc. We assume the economy has been in steady state until time 0. Then unanticipatedly $g \curvearrowright g' > g$.

a) Note that $r_0 = f'(\tilde{k}_0) - \delta$, where $\tilde{k}_0 \equiv K_0/(T_0 L_0)$. Since $\tilde{k}_0$ does not depend on g, r_0 is unaffected by the shock. The new long-run interest rate, $r^{*\prime}$, will be higher than the old, r^*, because $\tilde{k}^{*\prime} < \tilde{k}^*$ in the new situation. The argument is given at b) below.

b) The phase diagram is like that on p. 293 in Lecture Notes. The shift to $g' > g$ causes the $\dot{\tilde{k}}_t = 0$ locus to move downwards, whereas the $\dot{\tilde{c}}_t = 0$ locus is turned counter-clockwise. Hence, the new steady state is to the left of the old. Therefore $\tilde{k}^{*\prime} < \tilde{k}^*$. Whether the old steady state point, E, is below or above the new saddle path can not be established apriori. This ambiguity reflects that the impact effect on consumption has ambiguous sign. Indeed, $C_0 = (\rho + p)(K_0 + H_0)$, where H_0 tends to be higher than without the shock because there will be a higher w in the future, but on the other hand H_0 tends to be smaller because of more heavy discounting due to r being higher in the future.

c) Labor productivity is $y \equiv Y/L = \tilde{y}T = f(\tilde{k})T$ and in the long run $y_t^* = f(\tilde{k}^*)T_t$. Let $y_A \equiv Y_A/L_A$ and $y_B \equiv Y_B/L_B$. Then $y_{Bt}^* = f(\tilde{k}_B^*)T_t > f(\tilde{k}_A^*)T_t = y_{At}$, because a lower p, everything else equal, implies a higher $\tilde{k}^*$ in that the $\dot{\tilde{k}}_t = 0$ locus is moved downwards, whereas the $\dot{\tilde{c}}_t = 0$ locus is turned clockwise.

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