Advanced Macroeconomics. Exercises

07.12.2008. Christian Groth

Problem set XI

XI.1 *Precautionary saving* Consider a given household facing uncertainty about future labor income. For simplicity, assume the household supplies one unit of labor inelastically. The household never knows for sure whether it will be able to sell that amount of labor in the next period. Given the time horizon $T \geq 2$, the decision problem is:

$$\max E_0(U_0) = E_0[\sum_{t=0}^{T-1} u(c_t)(1+\rho)^{-t}] \quad \text{s.t.} \tag{1}$$

$$c_t \geq 0, \tag{2}$$

$$a_{t+1} = (1+r_t)a_t + w_t\ell_t - c_t, \qquad a_0 \text{ given,} \tag{3}$$

$$a_T \geq 0. \tag{4}$$

where $u' > 0$ and $u'' < 0$. Think of "period t" as the time interval $[t, t+1)$; the last period within the planning horizon T is thus period $T-1$. Real financial wealth is denoted a_t, and w_t (> 0) is the real wage, whereas ℓ_t is the exogenous offered amount of employment in period t, $0 \leq \ell_t \leq 1$. The real rate of return on financial wealth is called r_t, and E_0 is the expectation operator, conditional on the information available in period 0. This information includes knowledge of all variables up to period 0, including that period. There is uncertainty about future values of r_t, w_t, and ℓ_t, but the household knows the stochastic processes that these variables follow.

a) Interpret (1) - (4).

b) Derive the Euler equation. *Hint:* consider maximization of $E_t\tilde{U}_t$ for $t = 0, 1, 2, ...,$ where $\tilde{U}_t \equiv (1+\rho)^t U_t$. Comment.

c) Determine the consumption in period $T-1$, given the financial wealth a_{T-1}. Comment.

d) With a CRRA utility function, what is the sign of u'''?

From now, assume our $u(c)$ satisfies $u''' > 0$, that is, marginal utility is strictly convex ("prudence").

e) Draw a graph in (c, u') space illustrating how marginal utility of consumption depends on the consumption level c.

From now, suppose that there is no uncertainty about the future value of r_t, only about future employment and therefore labor income.

f) Consider the decision problem as seen from period 1 and assume period 2 is the last period (i.e., $T = 3$). The consumption level chosen in period 1 will determine a_2. Let there be two possible outcomes for labor income in period 2, say y_L and y_H, each with probability $\frac{1}{2}$. Write down c_2 as a function of a_2 for each of the possible labor income outcomes.

g) Let the diagram from d) represent the situation and enter the two possible values, c_L and c_H, of c_2 on the c_2 axis and indicate how the expected marginal utility, $E_1 u'(c_2)$, conditional on a_2, can be found graphically.

h) To find out the effect of an increased uncertainty, consider a mean-preserving spread in y_a and y_b. Let the two new possible values of c_2, conditional on the same a_2, be called c_L^* and c_H^*, respectively and indicate their position on the c_2 axis. Further, indicate how the new expected marginal utility, $E_1 u'(c_2^*)$, conditional on a_2, can be found graphically.

i) Use the Euler equation relating c_1 and c_2 to establish how the increased uncertainty affects saving in period 1. *Precautionary saving* is defined as the additional saving that may result from higher uncertainty. Is precautionary saving present here? Why or why not?

j) Let $u(c)$ be a quadratic utility function:

$$u(c) = \eta c - \frac{1}{2}c^2, \qquad \eta > 0, \eta \text{ "large"}.$$

Will increased uncertainty about future labor income result in precautionary saving in this case? Why or why not?

XI.2 *The IS-LM-CC model*[1]

[1] See Short Note 5.

Consider the static IS-LM-CC model. Notation as in Short Note 5. A refresher of some of the symbols is here:

$$
\begin{aligned}
D &= \text{ demand deposits (earn no interest),} \\
\sigma &= \text{ required reserve-deposit ratio, } \sigma \in [0,1)\,, \\
M_0 &= \text{ monetary base,} \\
E &\equiv M_0 - \sigma D = \text{ excess reserves (earn no interest),} \\
L^s &= \text{ supply of bank loans (credit),} \\
\rho &= \text{ a shift parameter measuring perceived riskiness of supplying bank loans,} \\
B &= B_b + B_p = \text{ nominal stock of government bonds held by the private sector.}
\end{aligned}
$$

The consolidated commercial banks face the constraint

$$E + L^s + B_b = (1-\sigma)D. \tag{(1)}$$

The model leads to the following equilibrium conditions,

$$mm(i_B)M_0 = M(i_B, Y), \qquad mm_{i_B} > 0, M_{i_B} < 0, M_Y > 0, \tag{MM}$$

$$\ell(i_B, i_L)(1-\sigma)mm(i_B)M_0 = C(i_B, i_L, Y), \quad \ell_{i_B} < 0, \ell_{i_L} > 0, \ell_\rho < 0, \; C_{i_B} > 0, C_{i_L} < 0, C_Y > 0, \tag{CC}$$

and

$$Y = Y^d(Y, i_B, i_L) + G, \qquad 0 < Y^d < 1, Y^d_{i_B} < 0, Y^d_{i_L} < 0. \tag{YY}$$

a) Briefly interpret these four equations.

b) Equation (CC) gives i_L as an implicit function of $Y, i_B, \rho,$ and M_0 :

$$i_L = f(Y, i_B, \rho, M_0), \tag{5}$$

with partial derivatives

$$
\begin{aligned}
f_Y &> 0, \\
f_{i_B} &> 0, \quad \text{(if } mm_{i_B} \text{ is not ``too large'', which we assume)} \\
f_\rho &> 0, \\
f_{M_0} &< 0.
\end{aligned}
$$

Briefly, interpret these signs.

c) In (Y, i_B) space give a graphical illustration of the general equilibrium of the model. Briefly explain why the MM curve has positive slope and the YC curve negative slope.

d) Suppose M_0 is increased by an open market operation. Illustrate in (Y, i_B) space how the MM curve and the YC curve are affected. Sign the effects on i_B and Y. Are the signs unambiguous? Why or why not?

e) Suppose G is increased. Illustrate in (Y, i_B) space what happens. Sign the effects on i_B and Y. Explain.

f) Suppose an economic crisis is on the way and that an increased riskiness of making loans is perceived. Sign the effects on i_L, i_B, and Y. Explain.

g) Relate to your knowledge about what has happened and is happening during the present financial crisis.

h) In continuation of f), what happens to the money supply? Why? Relate to your knowledge about what actually happened to M_1^s in the US in the early part of the Great Depression.

—