

Problem set X

X.1 *Short questions* Illustrate graphically the following alternative definitions of booms and recessions:

- a) Some authors use as a defining characteristic for booms and recessions the criterion that the output *level* has in at least, say, five consecutive quarters been above or below, respectively, the trend growth path of the economy.
- b) Other authors consider the *growth rate* of GDP as the key variable and define a boom or recession to be present if in at least five consecutive quarters the growth rate of GDP has been above or below, respectively, the long-run growth rate of GDP.
- c) A definition of a recession popular in the newspapers: A recession is present if GDP has been declining in two consecutive quarters.

Suppose we want the demarcation line between booms and recessions to be such that during booms the employment rate tends to rise and during recessions it tends to fall. Which of the three above definitions is then appropriate?

X.2 *Models and business cycle facts* Consider a decision problem in discrete time for a given household facing uncertainty. To begin with we assume, in accordance with new classical theory, that the household never expects having to face the problem of getting less employment than desired at the going wage. As seen from period 0, the decision problem is:

$$\max E_0(U_0) = E_0\left[\sum_{t=0}^{T-1} (\log c_t - \gamma \frac{\sigma}{1+\sigma} \ell_t^{(1+\sigma)/\sigma})(1+\rho)^{-t}\right] \quad \text{s.t.} \quad (1)$$

$$c_t > 0, 0 \leq \ell_t \leq 1, \quad (2)$$

$$a_{t+1} = (1+r_t)a_t + w_t \ell_t - c_t, \quad a_0 \text{ given}, \quad (3)$$

$$a_T \geq 0, \quad (4)$$

where c = consumption, ℓ = labor supply, a = financial wealth, r = real rate of return on financial wealth, and w = real wage. The parameters γ , σ , and ρ are all positive. We assume the upper boundary, 1, to labor supply is large enough so as to be never binding, given the environment in which the household acts. The symbol E_0 (generally E_t) denotes the mathematical expectation conditional on the information available in period 0 (generally t). This information includes knowledge of all realizations of the variables up to period 0, including that period. There is uncertainty about future values of r_t and w_t , but the household knows the stochastic processes which these variables follow.

- a) Derive two first-order conditions, the first of which (call it (*)) describes the trade-off between consumption and labor supply in, say, period t , and the second of which (call it (**)) describes the trade-off between consumption in period t and consumption in period $t + 1$, both conditions as seen from period t ($t = 0, 1, \dots$). *Hint:* consider maximization of $E_t \tilde{U}_t$ for $t = 0, 1, 2, \dots$, where $\tilde{U}_t \equiv (1 + \rho)^t U_t$.
- b) Interpret the two first-order conditions.

Among the “stylized facts” of business cycle fluctuations (based on time series data after detrending) are the following:

- (i) Employment (aggregate labor hours) is procyclical and fluctuates almost as much as GDP.
 - (ii) Aggregate consumption and employment are positively correlated.
 - (iii) Real wages are weakly procyclical and do not fluctuate much.
- c) Are these facts supportive or the opposite for the RBC theory in the light of the condition (*)? Discuss. *Hint:* it will prove convenient to rewrite (*) such that w_t is isolated on one side of the equation.
 - d) In order to simplify the discussion, suppose for a moment there is no uncertainty. Then find ℓ_t/ℓ_{t+1} as a function of w_t/w_{t+1} . From this expression, give an interpretation of the parameter σ . Relate this, together with your empirical knowledge about the elasticity of intertemporal substitution in labor supply, to the discussion under c).

- e) Within the market-clearing framework of the RBC approach, if fluctuations in the real wage are almost negligible, is it then likely that fluctuations in r_{t+1} could be a driving force behind fluctuations in employment? Relate your answer to your result under d), the condition (*), and the stylized facts above. *Hint:* given that fluctuations in the real wage are almost negligible, we can on the basis of (*) sign the expected correlation between consumption and employment and compare with fact (ii).

We now reintroduce the existence of uncertainty and reconsider the household's decision problem under the hypothesis that there is also uncertainty as to the prospect of *employment* in the future. That is, for $t = 0, 1, 2, \dots$, we replace (1) by the constraint $c_t > 0$, $0 \leq \ell_t \leq \min(z_t, 1)$, where $z_t \geq 0$ is the exogenous maximum employment offered to the household in period t . The current z_t is known by the household, but not the future values.

- f) It can be shown that when z_t is binding, the equality sign in (*) is replaced by a weak inequality sign. Write down the new (*) and interpret.
- g) Is it possible to reconcile theory with the stylized facts within this framework? Why or why not?

X.3 In continuation of Problem X.2, g) suppose an increase in uncertainty occurs. Discuss how this is likely to affect the current consumption of the household, given the specified period utility function in (1).

X.4 Some empirical studies (e.g., Danthine and Donaldson, EER, 1993) find a negative correlation between productivity and employment (hours) in many OECD countries, but not in the US, where the correlation is weakly positive. Briefly relate this finding to alternative business cycle theories.

X.5 List some leading indicators for aggregate economic activity.

X.6 *Short questions.* What is meant by the following terms?

- a) Precautionary saving.
- b) Certainty equivalence.

c) Liquidity trap.

X.7 Briefly describe the difference between the concepts “wage curve” and “Phillips curve”. Is it possible to unify them theoretically? Empirically? Comment.

X.8 Commenting on the fact that the Danish Welfare Commission had proposed a decrease in taxation on labour income without a corresponding simultaneous increase in other taxes, a journalist made the claim: “Because the proposed decrease in taxation on labour income is not accompanied by simultaneous increases in other taxes, the positive effect on labour supply is likely to be considerable.” Give your evaluation of the claim in terms of the different relevant “effects” (substitution effect etc.).

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