Advanced Macroeconomics. Exercises

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Problem set VII

VII.1 *A fiscal sustainability gap indicator with the Danish economy in mind, October 2005.*[1] Consider the government budget in a small open economy (SOE) with perfect mobility of financial capital, but no mobility of labor. The real rate of interest at the world capital market is a positive constant $r > g + n > 0$, where g is a constant rate of (Harrod-neutral) technical progress and n is a constant rate of growth of the labor force. The aggregate production function, $Y = F(K, TL)$ has CRS. Time is continuous. Let

$$\begin{aligned} Y_t &= \text{GDP}, \\ G_t &= \text{government spending on goods and services including} \\ &\quad\ \text{elder care and health services}, \\ X_t &= \text{transfer payments including public pensions}, \\ \tilde{T}_t &= \text{gross tax revenue}, \\ B_t &= \text{public debt}, \end{aligned}$$

all at time t and in real terms (i.e., measured with the output good as numeraire). Assume the future is known with certainty and that budget deficits are exclusively financed by debt issue (no money financing). Initial public debt, B_0, is positive.

a) Ignoring business cycle fluctuations, what is the growth rate of GDP? Write down the time path of Y_t.

b) Write down an expression for the real primary budget surplus, S_t, at time t.

c) Write down a condition that the time path of S_t must satisfy, as seen from time 0, for fiscal policy to be solvent. Translate this to a condition for the time path of the primary surplus-income ratio, $s_t \equiv S_t/Y_t$. It is convenient to introduce $\tilde{r} \equiv r-(n+g)$.

Let $\tau_t \equiv \tilde{T}_t/Y_t$, $x_t \equiv X_t/Y_t$ and $\gamma_t \equiv G_t/Y_t$. Suppose G_t grows at the same rate as Y_t.

[1] This problem is based on a suggestion by Mads Diness Jensen, October 2005.

d) What can we conclude about the time path of γ_t?

It is well-known that many industrialized countries feature an increasing elderly dependency ratio due to longer life span and lower fertility. Fig. 1 shows this for Denmark. Fig. 2 shows projected paths of the primary budget deficit and the total budget deficit in Denmark if current (2005) fiscal policy rules, including welfare arrangements, are maintained. A rough formalization of this expected development is:

$$\tau_t = \tau_0 - (1 - e^{-\lambda t})\Delta\tau, \tag{*}$$

$$x_t = x_0 + (1 - e^{-\lambda t})\Delta x, \tag{**}$$

where $\Delta\tau$ and Δx are some positive numbers such that $\Delta\tau + \Delta x > s_0$, and λ (the adjustment speed) is positive.

e) Interpret. Find a formula showing the movement of s_t over time and find the limiting value of s_t for $t \to \infty$ if there is no change in fiscal policy rules. Illustrate the time profile of s_t in a diagram.

Numerical projections for Denmark indicate that the present discounted value of the future primary surpluses with unchanged fiscal policy is not very far from zero. Assume it is exactly zero.

f) Is current (2005) fiscal policy, which we may call $\mathcal{P}$, sustainable? Why or why not?

Suppose a suggested new policy design, $\mathcal{P}'$, implies that the path of G_t remains unchanged, but the path $(\tau_t, x_t)_{t=0}^{\infty}$ is replaced by the path $(\tau'_t, x'_t)_{t=0}^{\infty}$ with time profiles

$$\tau'_t = \tau'_0 - (1 - e^{-\lambda t})\Delta\tau,$$

$$x'_t = x'_0 + (1 - e^{-\lambda t})\Delta x,$$

g) Write down an expression for the primary surplus-income ratio at time t according to the new policy $\mathcal{P}'$.

h) Find the minimum initial primary surplus-income ratio, $\bar{s}'_0$, required for the fiscal policy $\mathcal{P}'$ to be sustainable as seen from time 0. *Hint:* $\int_0^\infty e^{-at}dt = 1/a$ for any constant $a \neq 0$.

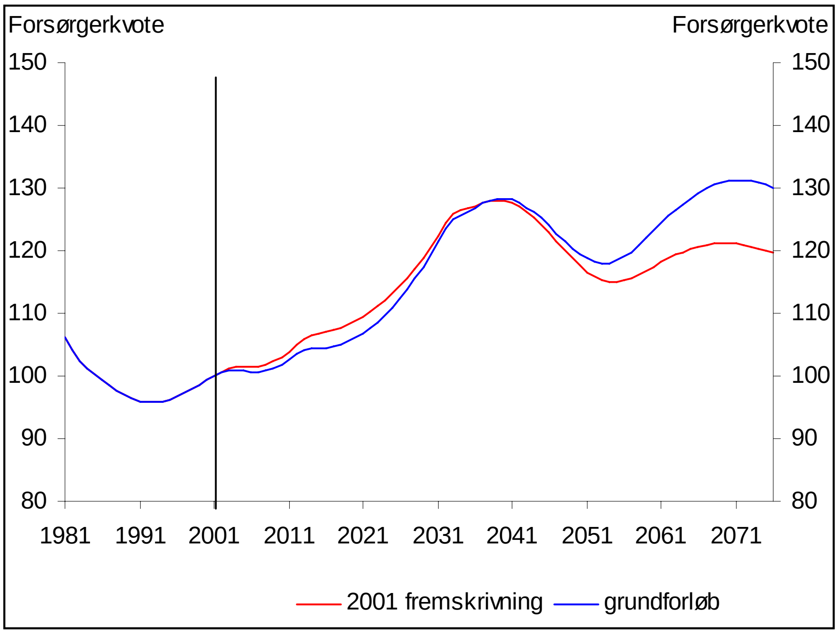

Figure 1: Elderly dependency ratio, Denmark. Simulation based on DREAM. Source: The Danish Welfare Commission, 2005.

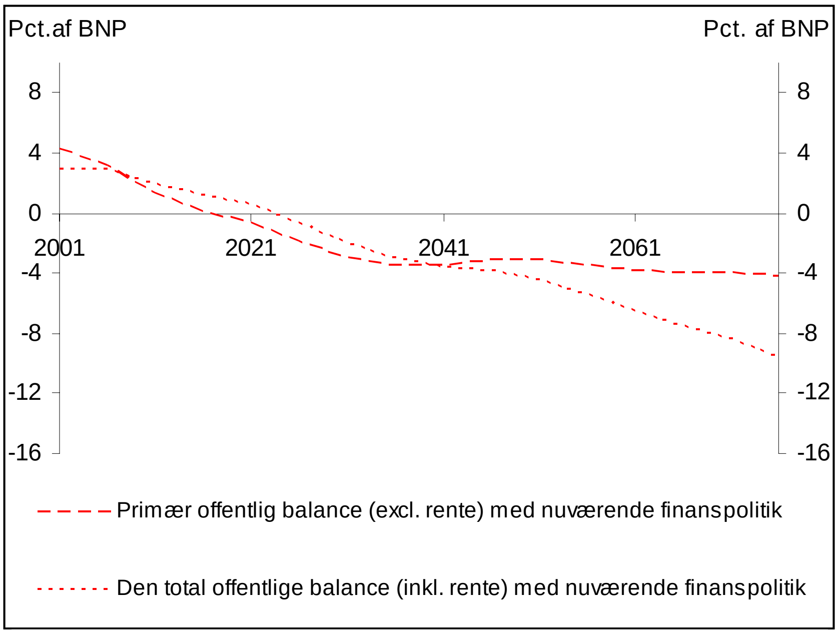

Figure 2: Primary budget deficit and total budget deficit if current fiscal policy is maintained. Denmark. Simulation based on DREAM. Source: The Danish Welfare Commission, 2005.

As a sustainability gap indicator at time 0 we choose $gap_0 \equiv \bar{s}'_0 - s_0$.

i) Illustrate the gap in the diagram from question e). How does gap_0 depend on:

1. the debt-income ratio at time 0?
2. the adjustment speed λ?
3. the growth-corrected interest rate presupposing Δx, $\Delta \tau$ and λ are independent of the growth-corrected interest rate?

Hint: If no unambiguous answer as to the sign of the effect can be given, write down a criterion in the form of an inequality on which the sign depends.

Comment.

j) In fact an increase in the interest rate *is* likely to affect $\Delta \tau$, namely be reducing $\Delta \tau$ partly through the postponed taxation of labor market pensions and partly through the induced increase in wealth accumulation, which implies higher future tax revenue; there are also potential counteracting factors such as a possible increase in tax deductability due to increased interest payments. Can this matter for the conclusion to i.3)? Comment.

VII.2 *A partial equilibrium model of the housing service market and the house market.* We consider the housing sector in a small open economy facing an exogenous constant risk-free real interest rate $r > 0$. Time is continuous. Let

$$\begin{aligned} H_t &= \text{the aggregate stock of houses at time } t, \\ S_t &= \text{the aggregate housing services at time } t \end{aligned}$$

The *stock* of houses is measured in terms of some basic unit, i.e., as the number of houses of "normal size" at a given point in time. The housing *services* at time t is a *flow*, that is, something measured *per time unit*, say per year: so and so many square meter-months are at the disposal for utilization (accommodation) for the public during the year. The two concepts are related through

$$S_t = \alpha H_t, \qquad \alpha > 0,$$

where α is the rate of utilization of the stock. Ignoring all problems associated with differences in quality, we can assume α to be a constant, which only depends on the measurement unit for housing services. If these are measured in square meter-months, α equals the number of square meters of a normal sized house times 12. Notice that there are two goods, houses and housing services, and therefore also two markets and two prices:

$$\begin{aligned} R_t &= \text{the (real) price of housing services at time } t, \\ p_t &= \text{the (real) price of houses at time } t. \end{aligned}$$

The price R_t of housing services is called the *rental rate* at the housing market. Buying a housing service means *renting* the apartment or the house for a certain period. Or, if we consider an owner-occupied house (or apartment), R_t is the owner's opportunity cost of occupying the house. The prices R_t and p_t are measured in *real* terms, that is, deflated by the consumer price index. We assume perfect competition on both markets.

In the short run the stock of houses is historically given. It takes time to change H_t. Hence, also the supply S_t of housing services is given in the short run. Suppose the aggregate demand for housing services at time t is

$$S_t^d = D(R_t, A, PDV(wL)), \qquad D_1 < 0, D_2 > 0, D_3 > 0,$$

where A is aggregate financial wealth and $PDV(wL)$ is the present discounted value of expected future labor income (after tax) for those alive at time t. In our partial equilibrium framework, we consider A and $PDV(wL)$ as exogenous and we assume they are approximately constant (i.e., economic growth is ignored).

a) Set up an equation which determines the equilibrium rental rate at time t as an implicit function of S_t, A, and $PDV(wL)$. Sign the partial derivatives of this function. Illustrate in (S, R) space for given A and $PDV(wL)$. Comment.

b) Given A and $PDV(wL)$, we may express the equilibrium rental rate at time t as a function R of H_t, i.e., $R_t = R(H_t)$. Sign the derivative of this function. Comment.

Assume houses depreciate physically at a constant rate $\delta > 0$. Suppose there is a constant tax rate $\tau_R \in [0, 1)$ applied to rental income after allowance for depreciation. In case of an owner-occupied house the owner still has to pay the tax $\tau_R(R_t\alpha - \delta p_t)$ out of the imputed income $(R_t\alpha - \delta p_t)$ per house per year. Assume further there is a constant property tax rate $\sigma \geq 0$ applied to the market value of houses. Finally, suppose that a constant tax rate $\tau_r \in [0,1)$ applies to interest income. There is symmetry in the sense that if you are a debtor and have negative interest income, then the tax acts as a subsidy. We assume capital gains are not taxed and we ignore all complications arising from the fact that most countries have nominal-income based tax systems rather than real-income-based tax systems. In a low-inflation world this limitation may not be serious.[2]

Now, suppose housing services are valued independently of whether you own or rent. Assume further there is no uncertainty and that there are no transaction costs. Then the above leads to the no-arbitrage equation

$$\frac{(1-\tau_R)(\alpha R(H_t) - \delta p_t) - \sigma p_t + \dot{p}_t^e}{p_t} = (1-\tau_r)r, \qquad (*)$$

where $\dot{p}_t^e$ denotes the expected capital gain per time unit.

c) Interpret (*).

d) Let $\dot{p}_t^e$ be given and assume it is non-negative or, if it is negative, at least not "too negative". Determine the market price p_t of a house of normal size. How does p_t depend on r, τ_r, σ, H_t, and $\dot{p}_t^e$? It is enough to sign the partial derivatives by inspection. Comment.

As mentioned it takes time for the stock H to change. Marginal building costs (per time unit) are rising because it is costly to adjust the stock of houses abruptly (it takes time to plan and execute building activity, there are capacity constraints, haste makes waste etc.). We will model these adjustment costs by considering house building as a two-stage process. At the first stage, a kind of "intermediate goods for building", I_H, are produced using the same technology (with labor and capital) as that applied to the production of the general all-purpose aggregate output Y. That is, we imagine the non-consumed part of Y is used for not only net exports, NX, and investment, I_K, in business fixed capital, but also for investment, I_H, in building intermediates. Therefore, the real price per unit of I_H is 1. For simplicity, from now we call I_H just I.

At the second stage, the building intermediates are instantly transformed into new houses (without further use of labor and capital). The key assumption here is that the

[2] Notice, however, that if all capital income should be taxed at the same rate, capital gains should also be taxed at the rate τ_r, and τ_R should equal τ_r. In Denmark, a few years ago, the rental value tax τ_R on owner-occupied houses was, for political reasons, replaced by what amounts to an upward shift in σ. Since then, due to the "tax stop", σ has been gradually decreasing.

rate at which an increase in I is transformed into new houses becomes smaller and smaller the higher is I. That is, we assume a transformation function T which is strictly concave in the level of building activity, I. On the other hand, the larger is H, the larger is the accumulated experience (learning); hence, for larger H the productivity of I is higher:[3]

$$\begin{aligned} \dot{H} &= T(I,H) - \delta H, \quad \text{where} \\ T(0,H) &= 0, \qquad T_I > 0, T_I(0,H) = 1, T_{II} < 0, \; T_H > 0. \end{aligned}$$

(An example satisfying this, at least for $I > 0$, is the CES function $T(I,H) = \frac{1}{2}(\frac{1}{2}I^{-1} + \frac{1}{2}H^{-1})^{-1}$.)

Let the amount of new houses per time unit be denoted B, i.e., $B = \dot{H} + \delta H = T(I,H)$. Hence, given the market price p, the revenue to the representative building firm is $pT(I,H)$ and costs are $1 \cdot I$.

e) For given H, draw the graph of B considered as a function of I only.

f) Considering building activity I as the control variable, derive the first-order condition for the profit maximizing representative building firm. Express the profit maximizing I as an implicit function of p and H. Sign the partial derivative wrt. p of this function.

From now, assume the transformation function T is homogeneous of degree one. As an implication we have

$$\dot{H}_t = \left[T(\frac{I_t}{H_t}, 1) - \delta \right] H_t.$$

g) Show that building activity can in this case be written

$$I_t = m(p_t) H_t, \qquad m(1) = 0, m' > 0.$$

Hint: by Euler's theorem, $T_I(I,H)$ is now homogeneous of degree 0; if the continuously differentiable function $y = f(x)$ is invertible (has $f' \neq 0$), then the inverse function, $x = f^{-1}(y)$, has the derivative $x' = 1/f'(f^{-1}(y))$.

[3] It would be more logical to use $\int_{-\infty}^{t} I_s ds$ as an indicator of accumulated learning. It is considerably more simple to use H, however. At least if the model were embedded in an economic growth context where H is almost never decreasing, this simplified modelling of the learning effect would seem an acceptable first approximation.

Illustrate the building activity function m in $(p, I/H)$ space.

The real price p has affinity to Tobin's q. In what sense?

h) Show that we can write

$$\dot{H}_t = (b(p_t) - \delta)H_t. \tag{**}$$

where $b(p_t)$ is a function satisfying $b(1) = 0$, $b' > 0$.

i) Assuming perfect foresight, i.e., $\dot{p}_t^e = \dot{p}_t$ for all t, write (*) on the standard form for a first-order differential equation. Observing that (*) and (**) now constitute two coupled differential equations in H and p, sketch the phase diagram in (H, p) space. Comment.

j) Let the steady state values of H and p be denoted H^* and p^*, respectively. Set up the equations determining p^* and H^*, respectively. What can be said about p^* relative to 1?

k) Suppose the house building sector has been in steady state until time t_1. Then there is an unanticipated downward shift in the property tax σ to a new constant level σ' expected to last forever in the future. Assuming divergent paths can be ruled out, sketch in a phase diagram what happens and comment.

ℓ) Comment on the dichotomy between the price and quantity adjustment mechanism in the short run and the longer run.

VII.3 *Interpreting building behavior in a marginal cost framework.* We may look at the building behavior found under f) in Problem VII.2 from the point of view of increasing marginal costs in the building sector. Indeed, total costs associated with building B $(= T(I, H))$ houses per time unit are $\mathcal{TC} = I$. These total costs are an implicit function of B and H, i.e., $I = \mathcal{TC}(B, H)$.

a) Find an expression for the marginal costs, $\mathcal{MC}$, of building and show that marginal costs are increasing.

b) Given that the building sector produces new houses up the point where $\mathcal{MC} = p$, characterize this production level. Illustrate by drawing the $\mathcal{MC}$ curve and the price line in (B, p) space. What is the marginal cost of building "the first" new house?

VII.4 *Short questions.*

a) Define the concept of "money".

b) What are the essential functions of money?

c) "The Sidrauski model contains a satisfactory micro-based theory of money demand". Do you agree? Why or why not?

d) Is it possible to establish a connection between "money in the utility function" and the cash-in-advance constraint (= Clower constraint)? *Hint:* If c_t is desired consumption, and $\bar{c}_t$ is actual consumption, then the cash-in-advance constraint implies $\bar{c}_t = \min(c_t, M_t/P_t)$, where M_t is money holding, and P_t is the nominal price.

e) "Neoclassical macroeconomics suggests that money is neutral and superneutral. Hence, according to neoclassical macroeconomics, the rate of money growth is irrelevant." Discuss whether these statements are valid.

f) "According to the Sidrauski model, hyperinflation driven by expectations cannot occur." Evaluate this statement.

g) List some cases where money is not superneutral.

h) "According to New Keynesian Economics, money is neutral, but not superneutral." Evaluate this statement.

i) What is meant by the "Friedman zero interest rule" or the "Friedman money satiation result"?

j) A controversial question in monetary theory is whether the Sidrauski model and similar neoclassical models are a suitable or not a suitable framework for the design of monetary policy. Briefly discuss this question!

VII.5 *Some quotations.*

a) "When the growth rate of money supply is high, liquidity is low." Can you make sense of this?

b) *Oscar Wilde* once said: “When I was young, I used to think that money is the most important thing in life; now that I am old, I know it is.” Abstracting from his legitimate literary point of view, how would you, from a portfolio choice point of view, suggest the statement to be modified?

c) A riddle asked by Paul Samuelson (Nobel Prize winner 1970): “A physicist, a chemist and an economist are stranded on an island, with nothing to eat. A can of soup washes ashore. The physicist says, 'Lets smash the can open with a rock'. The chemist says, 'Lets build a fire and heat the can first' ... ”. Guess what the economist says? Comment.

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