Advanced Macroeconomics.   Exercises

21.10.2008.   Christian Groth

Problem set VI: Fiscal sustainability[1]

VI.1   *Fiscal sustainability.* Consider the government budget in a small open economy (SOE) with perfect mobility of financial capital, but no mobility of labour. The real rate of interest at the world capital market is a positive constant r. Time is continuous. Let

$$
\begin{aligned}
Y_t &= \text{GDP at time } t, \\
G_t &= \text{government spending on goods and services at time } t, \\
T_t &= \text{net tax revenue } (= \text{ gross tax revenue} - \text{transfer payments}) \text{ at time } t, \\
B_t &= \text{public debt at time } t.
\end{aligned}
$$

All variables are in real terms (i.e., measured with the output good as numeraire). Taxes and transfers are lump-sum. Assume there is no uncertainty and that the budget deficit is exclusively financed by debt issue (no money financing).

a) Write down an equation describing how the budget deficit and the increase per time unit in public debt are linked.

Suppose Y grows at a constant rate equal to $g + n$, where g is the rate of (Harrod-neutral) technical progress and n is the growth rate of the labour force ($=$ employment). Suppose $r > g + n > 0$. Assume $T_t = \tau Y_t$ and $G_t = \gamma Y_t$, where τ and γ are constant over time, $0 < \gamma < 1$. Let initial debt, B_0, be positive.

b) Find the minimum initial primary surplus S_0 required for fiscal sustainability. *Hint:* one possible approach is to derive an expression for $\dot{b}_t$, where $b_t \equiv B_t/Y_t$; another approach is based on the fact that $\int_0^\infty e^{-at} dt = 1/a$ for a given constant $a \neq 0$.

c) Suppose $\tau > \gamma$. Is debt explosion possible?

d) How does S_0 depend on the growth-corrected interest rate?

Suppose instead that B_0 is negative.

e) Is debt explosion possible?

f) Answer question b) again. Comment.

g) Answer question d) again. Comment.

VI.2   *Welfare arrangements and fiscal sustainability in the "ageing society".* Consider a small open economy (henceforth SOE) with a government sector. For simplicity, assume:

1. Perfect goods and financial capital mobility across borders.

2. Domestic and foreign financial claims are perfect substitutes.

3. No labour mobility across borders.

4. No uncertainty.

5. Perfect competition on all markets.

There is at the world market for financial capital a constant (real) rate of interest $r > 0$. The SOE has (adult) population equal to N and a labour force equal to L, where both N and L are constant. Due to retirement we have $L < N$. The technology of the representative firm is given by

$$Y_t = F(K_t, E_t L) \equiv E_t L f(\hat{k}_t),$$

where $\hat{k}_t \equiv K_t/(E_t L)$, F is a neoclassical production function with CRS, and Y_t and K_t are output and capital input, respectively. The whole labour force is employed. We treat time t as continuous, and the time unit is one year. The symbol E_t represents a technology factor ("E" for "efficiency of labour") growing at the constant rate $g > 0$, that is, $E_t = e^{gt}$, by choosing measurement units such that $E_0 = 1$. There are no capital adjustment costs. The rate of physical capital depreciation is $\delta \geq 0$ and is constant. Firms maximize profit.

a) Find an expression showing how the capital intensity $\hat{k}$ chosen by the firm is determined. Comment.

b) Show how the equilibrium real wage w_t is determined and that it can be written $w_t = w_0 e^{gt}$.

Let G_t denote government spending on goods and services. Suppose G_t is primarily eldercare including health services. Specifically, assume

$$G_t = \gamma(N - L)w_t, \qquad \gamma > 0,$$

where the factor of proportionality, γ, is a constant. Let X_t denote transfer payments including pensions. Assume

$$X_t = \alpha w_t(N - L), \qquad 0 < \alpha < 1,$$

where α is the "degree of compensation", a constant. Further, let $\tilde{T}_t$ denote gross tax revenue and assume

$$\tilde{T}_t = \tau(w_t L + X_t), \qquad 0 < \tau < 1,$$

where the tax rate τ is constant (capital income taxation, consumption taxes etc. are ignored). Finally, let B_t denote real public debt and assume that the budget deficit (whether positive or negative) is exclusively financed by changes in B (no money financing). Initial debt, B_0, is positive.

c) Write down an equation describing how the budget deficit and the increase per time unit in public debt are linked.

d) Determine the primary surplus, S_t, and its growth rate. How does S_t depend on L?

Assume $r > g$. Let $\bar{S}_0$ denote the minimum size of the initial primary surplus consistent with fiscal sustainability.

e) Find $\bar{S}_0$. What is the sign of $\bar{S}_0$? Comment. *Hint:* if a is a positive constant, then $\int_{t_0}^{\infty} e^{-a(t-t_0)} dt = \frac{1}{a}$.

From now, suppose $S_0 = \bar{S}_0$.

f) Find τ. Comment.

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g) Determine the path over time of the debt-income ratio $b_t \equiv B_t/Y_t$. Illustrate the time profile of b_t in a diagram. Comment. *Hint:* the differential equation $\dot{x}+ax = c$, where a and c are constants, $a \neq 0$, has the solution $x_t = (x_{t_0} - x^*)e^{-a(t-t_0)} + x^*$, where $x^* = \frac{c}{a}$.

Suppose that, in analogy with the Blanchard OLG model with age-dependent labour supply,
$$L = \frac{p}{\omega + p}N,$$
where ω is a constant "retirement rate" (prescribed by law), and p is a constant "death rate", so that $1/p$ is a rough indicator of "life expectancy", i.e., expected life time (as adult). As a crude representation of the much debated supposed increase in life expectancy of future generations, imagine that the government at time $t_0 > 0$ becomes aware that from time $t_1 = t_0 + 35$ years, life expectancy for a young person just entering the labour force will be $1/p'$ instead of $1/p$, where $p' < p$ (of course, in the real world this demographic change will not be a once for all change, but a gradual change, but for simplicity this is ignored). Population size remains equal to the constant N.

h) In a diagram draw the time profile of $\ln S_t$ as it would be in case there is no change in fiscal policy. Is the current fiscal policy sustainable? *Hint:* consider either the present discounted value of future primary surpluses as seen from time t_1 or the time path of the debt-income ratio.

Let τ' denote the minimum size of the (constant) tax rate required for fiscal sustainability from time t_1, assuming γ and α to be unchanged for ever and no change in taxation before time t_1 (Policy I).

i) Find τ'. Determine the sign of $\tau' - \tau$. Comment.

Now assume instead that at time t_0 the government decides to incur a budget surplus (including interest payments) until time t_1 such that the debt-income ratio in the time interval (t_0, t_1) gradually falls according to
$$\dot{b}_t = -c,$$
where c is a positive constant large enough such that at time t_1 one has $b = 0$. The plan is to accomplish this not by changing τ, but by temporary and gradual adjustments of γ and/or α.

j) Find the required value of c. *Hint:* if $\dot{x} = a$, a constant, then $x_t = x_{t_0} + \int_{t_0}^{t} \dot{x}_\tau d\tau$ $= x_{t_0} + a(t - t_0)$.

Further, the plan is, for $t \geq t_1$, to let γ and α be back at their pre t_0 level and to let τ take the minimum value, τ'', now needed to obtain fiscal sustainability from time t_1 (Policy II).

k) Find τ''. Determine the sign of $\tau'' - \tau'$. Comment.

Suppose that at time t_0 an alternative policy is proposed, namely to let τ, γ, and α stay at their pre t_0 level forever and at time t_1 adjust ω such that fiscal sustainability is obtained (Policy III).

l) Find the required ω.

m) Assuming $\tau'' > \tau$, compare Policy II and Policy III w.r.t. the implied intergenerational "burden" and "benefit" distributions.

VI.3   *Short questions.*

a. In the fiscal policy model in Lecture Notes, Chapter 9.4, the total of government expenditure on goods and services is (implicitly or explicitly) public consumption. Could you imagine a class of richer models modifying the conclusion as to the consequences for future generations of government expenditure and budget deficits?

b. Consider a small open economy with perfect mobility of goods and financial capital, but no mobility of labour. Can you imagine an aggregate production function, with three inputs, that would allow a lump-sum-tax financed increase in public investment to stimulate private wealth formation?

c. "Public debt is a strain on future generations and should always be avoided or at least be reduced as fast as possible." Discuss.

d. "When the real interest rate remains above the GDP growth rate of the economy, then the NPG condition for the government is a necessary and sufficient condition for fiscal sustainability." True or false? Comment.