Advanced Macroeconomics 2. Exercises

19.10.2008. Christian Groth

Problem set V

V.1 *Short questions about standard national accounting related to the public sector.* For a closed economy with a government sector, taxes and transfers, let

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\begin{aligned}
\tilde{T} &= \text{ total tax revenue,} \\
X &= \text{ total transfers to the private sector,} \\
T = \tilde{T} - X &= \text{ net tax revenue,} \\
C_g &= \text{ public consumption,} \\
I_g &= \text{ public investment,} \\
C_p &= \text{ private consumption,} \\
I_p &= \text{ private investment,} \\
Y &= \text{ GDP} \\
Y^n &= Y - \text{capital depreciation} = \text{ net national product (``national income'')} \\
S^n &= Y^n - C = \text{ net aggregate saving,} \\
B &= \text{ public debt,} \\
r &= \text{ rate of interest,}
\end{aligned}
$$

where all variables are in real terms. We assume that transfers and interest income are taxable (as, e.g., in Denmark).

a. Often, in popular discussions and even by official authorities, the "tax burden" is measured as $\tilde{T}/Y$. But if by the "tax burden" is meant an indicator of what the tax rate *would* be in a proportional income tax system, can you then propose a better measure of the "tax burden" than $\tilde{T}/Y$? Comment.

b. Propose an appropriate measure of the public absorption of value added?

c. What is meant by a balanced government budget? Use symbols from the above list to make your answer precise?

d. Using symbols from the above list, write down what is the government budget deficit, what is the government budget surplus, what is the primary government budget deficit and what is the primary government budget surplus?

e. If we care about future generations, which of the above listed variables are particularly important? Why?

f. Using your knowledge about the specific items actually included in the official national accounting category "public consumption", the answer to question e. might be modified. Why?

V.2 *Capital installation costs.* Consider a firm with production function

$$Y_t = F(K_t, L_t),$$

where F is a neoclassical production function with CRS in the "normal" range for (K, L), and Y_t, K_t, and L_t are output, capital input, and labour input per time unit, respectively, at time t. Apart from points of discontinuity, if any, the increase per time unit in the firm's capital stock is given by

$$\dot{K}_t \equiv \frac{dK_t}{dt} = I_t - \delta K_t, \qquad \delta \geq 0,$$

where I_t is gross investment per time unit at time t and δ is the capital depreciation rate. There is perfect competition in all markets and the real interest rate faced by the firm is a constant $r > 0$. To begin with, we assume there are no capital installation costs, i.e., cash flow at time t is

$$R_t = F(K_t, L_t) - w_t L_t - I_t,$$

where w_t is the real wage. Given the real rate of interest, r, the firm wants to maximize its value, i.e., the present discounted value of its cash flow (dividends):

$$V_0 = \int_0^\infty R_t e^{-rt} dt.$$

a) Defining current profit as $\Pi_t = F(K_t, L_t) - w_t L_t - (r_t + \delta)K_t \equiv \Pi(K_t, L_t)$, show that $R_t = \Pi_t + rK_t - \dot{K}_t$.

b) Show that, given K_0, V_0 is maximized if and only if K_t and L_t are at each t chosen such that Π_t is maximized. *Hint:* integration by parts gives $\int_0^T \dot{K}_t e^{-rt} dt$

$= K_t e^{-rt}\big|_0^T + \int_0^T K_t r e^{-rt} dt$ so that $\int_0^T R_t e^{-rt} dt = \int_0^T \Pi_t e^{-rt} dt + r \int_0^T K_t e^{-rt} dt - (K_T e^{-rT} - K_0 + \int_0^T K_t r e^{-rt} dt) = \int_0^T \Pi_t e^{-rt} dt - K_T e^{-rT} + K_0$. You may assume that circumstances are always that the firm is motivated to plan such that $\lim_{T\to\infty} K_T e^{-rT} = 0$.

Comment!

Now, we modify the firm's decision problem by introducing capital installation costs (in a simple form). Let total capital installation costs be $C(I_t)$, where

$$C(0) = C'(0) = 0, \; C''(I) > 0.$$

c) Write down the firm's problem as a standard optimal control problem.

d) Derive the first-order conditions for the solution of the firm's problem.

e) Show that the optimal capital intensity at time t is an increasing function of w_t. *Hint:* use the intensive production function f.

Suppose, from now, that $w_t = w$ for all $t \geq 0$, where w is a positive constant. Further, assume that for K below a certain positive threshold, unit production costs are prohibitively large.

f) Show that "marginal q", q, is a constant along the optimal plan. *Hint:* the differential equation $\dot{x} - ax = -b$, where a and b are constants, $a \neq 0$, has the solution $x(t) = [x(0) - b/a]\, e^{at} + b/a$. Combine this with the relevant TVC.

g) Characterize in words the implied investment rule.

h) Show that, along the optimal plan, the capital stock gradually approaches a constant, say K^*, and find an expression for this constant.

i) Show that optimal net investment, $I_t - \delta K_t$, equals $\delta(K^* - K_t)$. Comment!

j) Let F be a Cobb-Douglas production function and let $C(I) = \frac{1}{2}\beta I^2$, $\beta > 0$. Find q, I, and K^* along the optimal plan.

k) Briefly assess the model.

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