

## Problem set IX

### IX.1 *Short questions*

- a) “The expectations theory of the term structure predicts that the long-term interest rate tends to be higher than the short-term interest rate.” True or false? Comment.
- b) We consider a Blanchard-Fischer-style short-run model of a small open economy with fixed prices and floating exchange rate. It takes time for output to adjust to changes in output demand. The central bank pursues the policy  $i_t = i^0 + i^1 Y_t$ , where  $i_t$  is the short-term nominal interest rate,  $Y_t$  is aggregate output,  $i^0$  and  $i^1$  are constants and  $i^1 > 0$ . Suppose the economy has been in steady state until time  $t_0$ . Then the central bank unexpectedly changes  $i^0$  to a lower constant,  $i^{0'}$ . After  $t_0$  everybody rightly expects the monetary policy to remain  $i_t = i^{0'} + i^1 Y_t$ . “There will be no exchange rate overshooting phenomenon generated by this policy shift.” True or false? Comment.

### IX.2 We consider a small open economy (SOE) satisfying (approximately):

1. Perfect mobility across borders of financial capital, but no mobility of labour.
2. Domestic and foreign financial claims are perfect substitutes (no uncertainty).
3. Domestic and foreign output goods are imperfect substitutes.
4. Nominal prices are fixed.

Suppose the short-term behaviour of the economy can be described by the following model in continuous time. Given the function  $D(Y_t, R_t, \frac{XP^*}{P}, \tau)$ , where  $0 < D_Y < 1$ ,  $D_R <$

$0, D_{\frac{XP^*}{P}} > 0$  and  $-1 < D_\tau < 0$ , the model is:

$$\begin{aligned} \dot{Y}_t &= \lambda(D(Y_t, R_t, \frac{XP^*}{P}, \tau) + G - Y_t), & \lambda > 0, \\ i_t &= i^*, \\ \frac{M_t}{P} &= L(Y_t, i_t), & L_Y > 0, L_i < 0, \\ R_t &= \frac{1}{Q_t}, \\ \frac{1 + \dot{Q}_t^e}{Q_t} &= r_t, \\ r_t &\equiv i_t - \pi_t^e. \end{aligned}$$

The endogenous variables are:  $Y_t \equiv$  output,  $R_t \equiv$  long-term real interest rate,  $i_t \equiv$  short-term nominal interest rate,  $M_t \equiv$  money supply,  $Q_t \equiv$  real price of a long-term bond (a consol),  $r_t \equiv$  short-term real interest rate and  $\pi_t^e \equiv$  expected (forward-looking) rate of inflation, where  $t$  is time. The superscript  $e$  signifies an expected value. A dot over a variable denotes the time derivative. The variables  $X, P^*, P, G, \tau$ , and  $i^*$  are exogenous and constant; their interpretation is as follows:  $X \equiv$  nominal exchange rate,  $P^* \equiv$  foreign price level,  $P \equiv$  domestic price level,  $G \equiv$  government spending on goods and services,  $\tau \equiv$  tax parameter and  $i^* \equiv$  foreign short-term nominal interest rate. The parameter  $\lambda$  is constant. The initial value,  $Y_0$ , of  $Y$  is predetermined. We assume that expectations are rational and speculative bubbles never occur.

- a) Briefly interpret the model.
- b) To characterize the movement over time of the economy, derive two coupled differential equations in  $Y$  and  $R$ .
- c) Construct the corresponding phase diagram and illustrate the path the economy follows. Comment.
- d) Determine the long-term real interest rate in steady state.
- e) How does  $Y$  in steady state depend on  $i^*$  and  $G$ ?

Suppose that the economy has been in its steady state until time  $t_0$ .

- f) At time  $t_0$  an unanticipated upward shift in the foreign short-term nominal interest rate occurs. But after this shift everybody rightly expects the foreign short-term nominal interest rate to remain unchanged for a very long time. Illustrate by a phase diagram and by graphical time profiles what happens to  $R_t, Y_t, r_t$ , and  $M_t$  for  $t \geq t_0$ . Comment.

- g) Assume instead that at time  $t_0$ , people in the SOE become aware that a monetary tightening in the leading countries in the world economy is underway. As a crude representation of this, suppose the agents rightly expect an upward shift in the foreign short-term nominal interest rate to take place at time  $t_1 > t_0$ . After this shift everybody rightly expects the foreign short-term nominal interest rate to remain unchanged for a very long time. Illustrate by a phase diagram and by graphical time profiles what happens to  $R_t$ ,  $Y_t$ ,  $r_t$ , and  $M_t$  for  $t \geq t_0$ . Comment.
- h) Now imagine the scenario is somewhat different from that described in g). Until time  $t_2 > t_1$  everything is as described in g). But at time  $t_2$ , due to now foreseeable unemployment problems, the government of the SOE credibly announces an upward shift in  $G$  to take place at time  $t_3 > t_2$ . After this shift everybody rightly expects  $G$  to remain unchanged for a very long time. The size of the shift in  $G$  is such as to reestablish, in the long run, an output level equal to that attained at time  $t_1$ . Illustrate by a phase diagram and by graphical time profiles what happens to  $R_t$ ,  $Y_t$ ,  $r_t$ , and  $M_t$  for  $t \geq t_2$ . Comment.

**IX.3** We consider a small open economy (SOE) satisfying (approximately) the conditions 1-4 in Problem IX.2. Suppose the short-term behaviour of the economy can be described by the following model in continuous time. Given the function  $D(Y_t, R_t, \frac{XP^*}{P}, \tau)$ , where  $0 < D_Y < 1$ ,  $D_R < 0$ ,  $D_{\frac{XP^*}{P}} > 0$  and  $-1 < D_\tau < 0$ , the model is:

$$\begin{aligned}
Y_t^d &= D(Y_t, R_t, \frac{XP^*}{P}, \tau) + G, \\
\dot{Y}_t &= \lambda(Y_t^d - Y_t), \quad \lambda > 0, \\
i_t &= i^*, \\
\frac{M_t}{P} &= L(Y_t, i_t), \quad L_Y > 0, L_i < 0, \\
R_t &= 1/Q_t, \\
\frac{1 + E_t \dot{Q}_t}{Q_t} &= r_t, \\
r_t &\equiv i_t - E_t \pi_t.
\end{aligned}$$

Notation:  $Y_t^d \equiv$  output demand,  $Y_t \equiv$  output,  $R_t \equiv$  long-term real interest rate,  $i_t \equiv$  short-term nominal interest rate,  $M_t \equiv$  money supply,  $Q_t \equiv$  real price of a long-term bond (a consol),  $r_t \equiv$  short-term real interest rate, and  $\pi_t \equiv$  (forward-looking) rate of inflation. The variables  $X$ ,  $P^*$ ,  $P$ ,  $G$ ,  $\tau$ , and  $i^*$  are exogenous and constant; their interpretation is as follows:  $X \equiv$  nominal exchange rate,  $P^* \equiv$  foreign price level,  $P \equiv$  domestic price level,  $G \equiv$  government spending on goods and services,  $\tau \equiv$  tax parameter and  $i^* \equiv$  foreign

short-term nominal interest rate. The parameter  $\lambda$  is constant. The initial value,  $Y_0$ , of  $Y$  is given. The symbol  $E_t$  denotes expectation conditional on information available at time  $t$ . Expectations are rational and there are never speculative bubbles.

- a) Briefly interpret the model.
- b) To characterize the movement over time of the economy, derive two differential equations in  $Y$  and  $R$ .
- c) Construct the corresponding phase diagram and illustrate the path the economy follows. Comment.
- d) Determine the long-term real interest rate in steady state.
- e) How does  $Y$  in steady state depend on  $G$  and  $\tau$ , respectively? Considering alternative fiscal tightening policies, compare the effect (on  $Y$  in steady state) of  $dG = -1$  to that of increasing  $\tau$  by an amount,  $d\tau$ , so that net tax revenue,  $\mathcal{T}$ , is increased by 1, i.e.,  $d\mathcal{T} = 1$ , assuming  $\mathcal{T} = \tau + T(Y)$ , where  $0 < T'(Y) < 1$ . Comment.

Suppose that the economy has been in its steady state until time  $t_0$ .

- f) At time  $t_0$  an unanticipated upward shift in  $\tau$  occurs, but apart from this shift, everybody expects  $\tau$  to remain unchanged forever. Illustrate by a phase diagram and by graphical time profiles what happens to  $Y_t$ ,  $M_t$ ,  $r_t$ , and  $R_t$  for  $t \geq t_0$ . Comment.
- g) Assume instead that at time  $t_0$ , due to foreseeable problems of fiscal sustainability arising in the “aging society”, the government credibly announces an upward shift in  $\tau$  to take place at time  $t_1 > t_0$ . Illustrate by a phase diagram and by graphical time profiles what happens to  $Y_t$ ,  $M_t$ ,  $r_t$ , and  $R_t$  for  $t \geq t_0$ . Comment.

*Hint:* the answer to this question may be easier than one might immediately think.

**IX.4** Consider a small open economy satisfying (approximately) the conditions 1-4 in Problem IX.2. Suppose the short-term behaviour of the economy can be described by the following model in continuous time. Given the function  $D(Y_t, r_t, x_t)$ , where  $0 < D_Y <$

1,  $D_r < 0$  and  $D_x > 0$ , the model is:

$$\begin{aligned}
 Y_t^d &= D(Y_t, r_t, x_t), \\
 \dot{Y}_t &= \lambda(Y_t^d - Y_t), & \lambda > 0, \\
 \frac{M}{P} &= L(Y_t, i_t), & L_Y > 0, L_i < 0, \\
 i_t &= i^* + \frac{\dot{X}_t^e}{X_t}, \\
 r_t &\equiv i_t - \pi_t^e, \\
 x_t &\equiv \frac{X_t P^*}{P}.
 \end{aligned}$$

The endogenous variables are:  $Y_t^d$  = output demand,  $Y_t$  = output,  $r_t$  = real interest rate,  $x_t$  = real exchange rate,  $i_t$  = nominal interest rate,  $X_t$  = nominal exchange rate,  $\pi_t^e$  = expected (forward-looking) rate of inflation, all at time  $t$ ; the superscript  $e$  denotes expectation. The variables  $M, P, P^*$ , and  $i^*$  are exogenous and constant; their interpretation is as follows:  $M$  = money supply,  $P$  = domestic price level,  $P^*$  = foreign price level and  $i^*$  = foreign nominal interest rate. The parameter  $\lambda$  is constant. The initial value,  $Y_0$ , of  $Y$  is given. Expectations are rational and speculative bubbles never occur.

- a) Briefly interpret the model.
- b) To characterize the movement over time of the economy, derive two differential equations in  $Y$  and  $X$ .
- c) Construct the corresponding phase diagram and illustrate the path that the economy follows. Comment.

Suppose that the economy has been in steady state until time  $t_0$ .

- d) At time  $t_0$  an unanticipated tightening of monetary policy (downward shift in  $M$ ) occurs. After  $t_0$  everybody rightly expects the money supply to remain at the new lower level,  $M'$ , forever. Illustrate by a phase diagram and a separate figure with time profiles what happens to  $Y_t$ ,  $X_t$ , and  $r_t$  for  $t \geq t_0$ . Comment.
- e) Assume instead that at time  $t_0$ , due to foreseeable overheating problems everybody become aware that the monetary authority will at time  $t_1 > t_0$  carry into effect a shift in money supply to the level  $M' < M$ . Illustrate by a phase diagram and a separate figure with time profiles what happens to  $Y_t$ ,  $X_t$ , and  $r_t$  for  $t \geq t_0$ . Comment.

f) Briefly discuss the model.

**IX.5** Consider a small open economy satisfying (approximately) the conditions 1-4 in Problem IX.2. Suppose the short-term behaviour of the economy can be described by the following model in continuous time:

$$\begin{aligned}\dot{Y}_t &= \lambda(D(Y_t, r_t, x_t) - Y_t), & \lambda > 0, 0 < D_Y < 1, D_r < 0, D_x > 0, \\ \frac{M}{P} &= L(Y_t, i_t), & L_Y > 0, L_i < 0, \\ i_t &= i^* + \frac{\dot{X}_t^e}{X_t}, \\ r_t &\equiv i_t - \pi_t^e, \\ x_t &\equiv \frac{X_t P^*}{P}.\end{aligned}$$

The endogenous variables are:  $Y_t$  = output,  $r_t$  = real interest rate,  $x_t$  = real exchange rate,  $i_t$  = nominal interest rate,  $X_t$  = nominal exchange rate,  $\pi_t^e$  = expected (forward-looking) rate of inflation, all at time  $t$ ; the superscript  $e$  denotes expectation. The variables  $M, P, P^*$ , and  $i^*$  are exogenous and constant; their interpretation is as follows:  $M$  = money supply,  $P$  = domestic price level,  $P^*$  = foreign price level and  $i^*$  = foreign nominal interest rate. The parameter  $\lambda$  is constant. The initial value,  $Y_0$ , of  $Y$  is given. Expectations are rational and speculative bubbles never occur.

- a) Briefly interpret the first three equations of the model.
- b) Derive two differential equations in  $Y$  and  $X$  that characterize the movement over time of the economy.
- c) Construct the corresponding phase diagram and illustrate the path that the economy follows for  $t \geq 0$ . Comment.

Suppose that the economy has been in steady state until time  $t_0$ .

- d) At time  $t_0$  an unanticipated upward shift in  $M$  occurs. After  $t_0$  everybody rightly expects the money supply to remain at the new higher level,  $M'$ , forever. Illustrate by a phase diagram and a separate figure with time profiles what happens to  $Y_t$ ,  $X_t$  and  $r_t$  for  $t \geq t_0$ . Explain in detail by words the economic intuition behind what happens.

- e) Assume instead that at time  $t_0$  everybody become aware that the monetary authority will at time  $t_1 > t_0$  carry into effect a shift in money supply to the level  $M' > M$ . Illustrate by a phase diagram and a separate figure with time profiles what happens to  $Y_t$ ,  $X_t$  and  $r_t$  for  $t \geq t_0$ . Comment.

**IX.6** *Monopolistic competition.*<sup>1</sup> Suppose we want to extend the Blanchard-Kiyotaki model with fiscal policy. Let the government's utility index for public consumption be

$$G = m \left( m^{-1} \sum_{i=1}^m G_i^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1, \quad (*)$$

where  $G_i$  is public consumption of commodity  $i$ ,  $i = 1, 2, \dots, m$ . Consider the decision problem: For a given nominal tax revenue  $\tilde{T} > 0$

$$\begin{aligned} & \max G \quad \text{s.t.} \\ & (*) \text{ and} \\ & \sum_{i=1}^m P_i G_i = \tilde{T}, \end{aligned}$$

where  $P_i > 0$  is a given price of commodity  $i$ ,  $i = 1, 2, \dots, m$ . Solve the problem. *Hint:* set up the Lagrange function and derive the first-order conditions wrt.  $G_i$ ; use that the Lagrange multiplier is inversely related to the ideal price index,

$$P = \left( m^{-1} \sum_{i=1}^m P_i^{1-\theta} \right)^{\frac{1}{1-\theta}}.$$

**IX.7** (From the exam Jan. 2005) Give a brief account of the main points of the menu cost theory. Doing so, relate your account to:

- the concepts of “first-order effects” and “second-order effects”;
- the problem that in models like the Blanchard-Kiyotaki model a low elasticity of marginal disutility of labor is needed to make menu costs operative;
- the fact that the problem mentioned under b) can in principle be overcome by modelling the labor market differently.

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<sup>1</sup>This problem illustrates the level of technicalities concerning CES gymnastics you are supposed to master at the exam.

**IX.8** (Almost identical to Problem 4 at the exam, January 2004) *Short questions*

- a) “If leisure is a ‘normal good’, then the income effect as well as the wealth effect on leisure of an increase in the wage rate are capable at offsetting the substitution effect.” True or not true? Comment.
- b) “According to the theory of the term structure, if there is no uncertainty, the difference between the short-term and the long-term interest rate is determined only by expectations.” True or not true? Comment.
- c) “In the Blanchard-Kiyotaki model with monopolistic competition and menu costs, there may be more than one equilibrium.” True or not true? Comment.

**IX.9** *Keynesian concepts*

- a) State and compare the “technical” definitions of the phenomena “underemployment” and “involuntary unemployment”.
- b) On the basis of your general macroeconomic knowledge, mention some models where the phenomenon of “involuntary unemployment” can occur and some models where it cannot occur.
- c) What does “nominal rigidities” mean and what does “real rigidities” mean?
- d) Both nominal and real rigidities are important for persistence of real effects of changes in the money supply. Give a brief intuitive explanation.
- e) Give a brief account of the “minimum transaction rule” and the Keynesian concept of effective demand.

**IX.10** *Short questions*

- a) Let  $c_t \equiv$  consumption per member of the labor force,  $k_t \equiv$  capital per member of the labor force and  $k_{GR} \equiv$  the golden rule capital intensity. Consider the statement: “In the Barro model of parental altruism (without technical progress) a technically feasible path  $\{c_t, k_t\}_{t=0}^{\infty}$  such that for  $t \rightarrow \infty$ ,  $k_t \rightarrow k^* < k_{GR}$ , must be dynamically efficient.” True or not true? Comment.



- b) “In the Blanchard OLG model, the income effect on current consumption of an increase in the rate of interest exactly offsets the substitution effect.” True or not true? Comment.
- c) “In any macro model with rational expectations wage setting in advance implies that not only unanticipated changes in money supply, but also anticipated changes, have real effects.” True or not true? Comment.

**IX.11**      *Short questions*

- a) What is the conceptual difference between “white noise fluctuations” and “business cycle fluctuations”?
- b) In the theory of business cycle fluctuations a lot of terms fly around. What is meant by the following terms?  
Impulse. Response. Propagation. Amplification. Persistence. Comovement.
- c) Say in a few words, what are the shocks and the mechanisms that drive business cycle fluctuations,
  - 1. according to the RBC theory?
  - 2. according to Keynesian thinking?
- d) How do real wages behave over the business cycle?
- e) How has, at least in the US, productivity behaved over the business cycle?
- f) Briefly compare and discuss the two theories mentioned under c).
- g) “International and historical evidence can add to the understanding of business cycles.” Do you agree? Why or why not?

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