

## Problem set III: OLG in discrete and continuous time

### III.1 *Uncertainty about the time of death in a three-period OLG model.*

- a) In the simple Diamond OLG model without technical progress, if not in reality, all are born with the same work ability and the same work willingness – within generations as well as across generations. Nevertheless, as long as the economy has not reached a steady state, the members of different generations get different labor incomes. Explain why.
- b) Through what channel does the behavior of one generation affect the economic conditions for the next generation?

We now extend the model by adding uncertainty about the time of death to the model. We also assume that people may live *three* periods. But they always work only in the two first. The individual's labor supply is inelastic and equals one unit of labor in each period. There is a probability  $p \in (0, 1)$  of dying already at the time of retirement. Hence, for members of generation  $t$ , the unconditional probability of staying alive three periods is  $1 - p$ . Suppose the young individual born at the beginning of period  $t$  maximizes expected utility,  $u(c_{1t}) + (1 + \rho)^{-1}u(c_{2t+1}) + (1 - p)(1 + \rho)^{-2}u(c_{2t+2})$ .

- c) Given the pure rate of time preference  $\rho$ , in what direction does a decrease in  $p$  affect the “effective” degree of impatience?
- d) Suppose there are no life annuity markets and that the young knows the inheritance before deciding the saving in the first period of life. Assume there is a constant real interest rate,  $r$ . For a young belonging to generation  $t$  whose parent dies at the end of period  $t$  with financial wealth  $a_t$ , the period budget constraints are

$$\begin{aligned} c_{1t} + s_t &= w_t + \frac{1}{1+n}a_t, \\ c_{2t+1} + a_{t+1} &= w_{t+1} + (1+r)s_t, \\ c_{2t+1} &= (1+r)a_{t+1}. \end{aligned}$$

Explain.

- e) Suppose that for some unexplained reason, all members of generation  $t$  happen to inherit equally much at the end of period  $t$ . Yet, after some periods, an in-egalitarian distribution of wealth within generations tends to arise although all individuals have the same utility function, including the same  $\rho$ . Explain in a few words why.
- f) Suppose now that there exist competitive private life annuity companies. The annuity companies operate by accepting deposits from middle-aged individuals. These place part or all their saving  $s_t$  in the life annuity companies that use the deposits to buy capital goods which are rented out to the production firms. At the beginning of next period these pay back a return,  $1+r$ , per unit of account invested. At the same time, the annuity company distributes its holdings (with interest) to its surviving depositors in proportion to their initial deposits. Suppose that the annuity companies have no operating costs. Their aim is to maximize expected profit. Then, 1) given free entry and exit, in equilibrium what will expected profit in the annuity industry be? 2) In equilibrium, how much will each surviving depositor receive per unit of account initially deposited?
- g) “Assuming middle-aged individuals have no bequest motive, they will choose to hold all their financial wealth in the form of such life annuities.” True or false? Why?

### **III.2**     *Short questions relating to Blanchard’s OLG model in continuous time.*

- a) Why is a market for life annuities likely to arise when individuals have no bequest motive and face uncertainty as regards the length of their remaining lifetime?
- b) Suppose production is riskless and that the production firms can finance their activities by loans at the risk-free interest rate  $r$ . Society is large so that the life insurance (or pension) companies face no aggregate uncertainty. Assume further that these companies have no administration costs, and that there is free entry to the insurance industry. Consider an individual (“depositor”) who at some point in time buys a life annuity contract for one unit of account. Why must the annuity the depositor receives as long as he/she is alive (per unit of account deposited) from the insurance or pension company, over and above  $r$ , in equilibrium be equal to the death intensity  $p$ ?

Consider the infinite horizon consumer problem in the Blanchard OLG model (standard notation):

$$\begin{aligned} \max_{(c_t)_{t=0}^{\infty}} U_0 &= \int_0^{\infty} (\ln c_t) e^{-(\rho+p)t} dt \quad \text{s.t.} \\ c_t &> 0, \\ \dot{a}_t &= (r_t + p)a_t + w_t - c_t, \quad \text{where } a_0 \text{ is given,} \\ \lim_{t \rightarrow \infty} a_t e^{-\int_0^t (r_s + p) ds} &\geq 0. \end{aligned}$$

- c) Briefly, interpret the objective function and the constraints.
- d) Solve the problem, i.e., find the consumption function. *Hint:* combine the Keynes-Ramsey rule with strict equality in the intertemporal budget constraint.
- e) How will a rise in the interest rate level affect current consumption and saving? Comment in terms of the Slutsky effects.

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