Advanced Macroeconomics. Exercises

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Problem set II: More on overlapping generations models in discrete time

II.1 *Technical progress in the diamond model.* Consider a Diamond OLG model with period utility specified as

$$u(c) = \frac{c^{1-\theta} - 1}{1-\theta}, \qquad \theta > 0.$$

a) How is it possible to give a meaning to the case $\theta = 1$?

b) Derive the saving function of the young. Comment.

c) Let the aggregate production function be a general neoclassical production function with CRS and assume Harrod-neutral (i.e., labor-augmenting) technical progress at a constant exogenous rate, $g > 0$. Derive the fundamental difference equation of the model.

d) Assuming that $\theta = 1$, illustrate examples of the dynamics in a diagram. Is the model necessarily well-behaved? Comment.

e) Assuming the production function is Cobb-Douglas, illustrate the possible dynamics in a diagram. Comment.

From now we stay within this special case ($\theta = 1$, f Cobb-Douglas).

f) Suppose the economy is in steady state. Then suddenly a permanent positive shift in the level of total factor productivity occurs. Assuming the future rate of technical progress is not affected, illustrate the dynamics in a diagram.

g) How, if at all, is the real wage per working hour in the long run affected by the shock?

h) How, if at all, is the real interest rate in the long run affected by the shock?

II. 2 *A three-period OLG model.* Consider an extension of the Diamond OLG model such that people live for three periods. For an individual born at time t, let c_{1t}, c_{2t+1} and c_{3t+2} be the consumption in the first period of life ("youth"), the second period of life ("middle age period") and the third period of life ("retirement period"), respectively. The utility function of a young born at time t is $U(c_{1t}, c_{2t+1}, c_{3t+2}) = \ln c_{1t} + (1+\rho)^{-1} \ln c_{2t+1} + (1+\rho)^{-2} \ln c_{3t+2}$, where $\rho > -1$. Individuals supply inelastically $\bar{x}$ units of labor in the first period of life, one unit of labor in the second period of life, while people don't work in the third period of life. The technology side of the model is as in the two-period Diamond model, people have perfect foresight and all markets are competitive. The rate of population growth is a constant $n > -1$.

a) From now, assume $0 < \bar{x} < 1$. Is this assumption natural? Why or why not?

b) For notational simplicity, disregard for a moment the time indices t, $t+1$ and $t+2$. Set up the optimization problem of the young.

c) Derive the intertemporal budget constraint and find the optimal c_1, c_2 and c_3. Comment.

d) Find the financial wealth a_1 held by the young at the end of the first period and the financial wealth a_2 of the middle aged individual at the end of the second period ($a_2 = a_1 + s_2$, where s_2 is the saving of the middle aged individual) as functions of the relevant parameters. Do a_1 and a_2 depend on the rate of interest between the first and the second period? Comment!

e) Give a list of conditions that could make $a_{1t} < 0$. Comment!

f) Ignoring technical progress, show that the fundamental difference equation of the model is a second order non-linear difference equation.

g) If, instead of the log function, a general CRRA utility function is used, the fundamental difference equation would be a third order equation. Guess why.

II.3 *Government, income taxes and capital accumulation.* Consider a Diamond OLG model extended with government. Let G_t denote the government's purchases of goods in period t. Assume $G_t = G_0(1+n)^t$, where G_0 is a given positive number and n is the given constant rate of population growth, $n \geq 0$. The government uses G_t to provide free public consumer services, say free broadcasting services or free exhibitions and museum services.

To finance its purchases, the government levies taxes. To begin, assume that only labor income is taxed. Let the labor income tax rate be denoted τ_t. The government budget is balanced every period. Assume that G_0 is "small" enough in relation to K_0 and L_0 (standard notation) so that for all t we have $0 \leq \tau_t < 1$. Let the aggregate production function satisfy the Inada conditions and ignore technical progress.

a) Assume from now that an individual born at time t has the utility function $U(c_{1t}, c_{2t+1}, G_t, G_{t+1}) = \ln c_{1t} + \beta \ln G_t + (1+\rho)^{-1} [\ln c_{2t+1} + \beta \ln G_{t+1}]$, where β is a positive parameter (an indicator of how strongly the public good is desired). List at least three *special* features of this two-goods-two-periods utility function.

b) Derive the saving function of the young.

c) Write down 1) the balance sheet for the utilization of output and 2) the government budget constraint. Find the tax rate τ_t, given k_t (standard notation). Derive the fundamental difference equation of the model and illustrate the dynamics in a diagram.

d) Is it true or not true that in the present model, labor income taxes are similar to lump-sum taxes in their effect on the behaviour of the young? Why?

Let the aggregate production function be Cobb-Douglas.

e) How does the long-run capital intensity depend on the level of G_0? A qualitative answer based on a phase diagram is enough.

f) Assume that the economy has been in its steady state until period 0. Then there is an unanticipated change in government tax policy so that also capital income is taxed, i.e., from period 0 capital income is taxed at the same proportional rate as labor income, this common rate being denoted σ_t. The path of government expenditure is unchanged and the budget is still balanced. Find the new tax rate, σ_t, for $t = 0, 1, 2, \ldots$. *Hint:* Repeat the steps b) and c) in this new situation.

g) Show that the long-run capital intensity will be higher than with pure labor income taxation. Explain in words.

h) Assume that the economy has been in its new steady state for a long time. Then there is an unanticipated change in government tax policy so that *only* capital income is taxed, i.e., from now capital income is taxed at the rate required for a balanced budget, this rate being denoted $\tilde{\sigma}_t$. How is the long-run capital intensity affected by this?

II.4 *Realized and non-realized expectations.* Consider an OLG model of a small open economy with endogenous retirement and a voluntary early retirement scheme as in Section 3.2.2. Suppose a shock changes the circumstances in period t compared to what was expected. Then the old individual re-optimizes at the beginning of period t, facing the *one-period* problem $\max\left[\ln c_{2t} + \gamma \ln(1-\ell_t)\right]$ s.t. $c_{2t} = (1+r)\bar{s}_{t-1} + \hat{w}_{2t}\ell_t + \hat{m}_t(1-\ell_t)$, where $\bar{s}_{t-1}$ denotes the now exogenous saving undertaken by this person in the previous period.

a) Solve the problem.

b) Does the solution (c_{2t}, ℓ_t) to this one-period-problem coincide with the as-young-in-period-$t-1$ *planned* actions, which would have been realized in the absence of the shock? Why or why not?

c) Suppose the shock mentioned is a policy shock, implying a lower $\hat{m}_t$ than expected. Compare actual labor supply of the old in period t to planned labor supply. Comment.

II.5 *Senior policy.* Consider an OLG model of a small open economy with endogenous retirement and a voluntary early retirement scheme as in Section 3.2.2. The rate of technical progress is g and the real interest rate at the international market for financial capital is r (both g and r constant). The tax rate, τ, required to finance the early retirement compensation, given the degree of compensation is equal to μ, is given by

$$\tau = \frac{\mu\gamma(\frac{1+r}{1+g}+1)}{(1-\mu)\left[(2+n)(2+\rho+\gamma) - \gamma(\frac{1+r}{1+g}+1)\right]},$$

where n is the constant rate of population growth and γ is the weight on leisure in the utility function of a young person: $U = \ln c_1 + (1+\rho)^{-1}\left[\ln c_2 + \gamma \ln(1-\ell)\right]$. Suppose, a senior policy can improve working conditions for elderly people such that γ is decreased. Derive a formula showing the effect on τ of such a senior policy. Is the sign of the effect unambiguous? Comment.

II.6 *Intertemporal substitution in labor supply.* Consider an individual who lives two periods (as young and as old) and works and consumes in both periods. For simplicity, assume the individual has no inherited financial wealth and that there are no taxes and no uncertainty. Let $\rho > -1$, $\gamma > 0$ and $\sigma > 0$ be given parameters. The optimization problem of the young is:

$$\max U = \ln c_1 - \gamma \frac{\sigma}{\sigma+1} \ell_1^{\frac{\sigma+1}{\sigma}} + \frac{1}{1+\rho}\left[\ln c_2 - \gamma \frac{\sigma}{\sigma+1} \ell_2^{\frac{\sigma+1}{\sigma}}\right] \qquad \text{s.t.}$$

$$\begin{aligned} c_1 + \frac{c_2}{1+r} &= w_1 \ell_1 + \frac{w_2 \ell_2}{1+r}, \quad \text{where} \\ c_1 &> 0, \quad c_2 > 0, \quad 0 \leq \ell_1 \leq 1, \quad 0 \leq \ell_2 \leq 1. \end{aligned}$$

Here, c_1 and ℓ_1 are consumption and labor supply, respectively, in the first period and c_2 and ℓ_2 are planned consumption and labor supply, respectively, in the second period, w_1 = real wage in period 1, w_2 = (expected) real wage in period 2 and r = (expected) real interest rate. Time available in each period is 1 time unit so that $1 - \ell_t$ is leisure in period t. We assume parameter values are such that the restriction $\ell_t \leq 1$, $t = 1, 2$, is never binding.

a) Interpret and solve the optimization problem, i.e., determine c_1, c_2, ℓ_1, and ℓ_2 as functions of variables that are exogenous to the individual. In fact, in this problem it is possible to find *explicit* solutions. *Hint:* Choosing the substitution method (recommended), you may use the budget constraint to substitute for c_1 in U, considering only c_2, ℓ_1 and ℓ_2 as decision variables. Calculate the three first-order conditions. Substituting these into the intertemporal budget constraint you find an explicit solution for c_1. Finally, the-first order conditions give c_2, ℓ_1 and ℓ_2 in terms of this solution.

b) Determine the intertemporal elasticity of substitution in labor supply.

c) For interpretation purposes, rewrite the intertemporal budget constraint such that the right-hand-side is exogenous to the individual.

d) What is the effect on c_1 and ℓ_1 of a rise in the interest rate? Comment in terms of the three Slutsky effects.

e) What is the effect on saving of a rise in the interest rate? Comment.

f) What is the effect on ℓ_1, ℓ_2 and ℓ_1/ℓ_2, respectively, of a rise in the interest rate? Comment in terms of the three Slutsky effects.

g) The intertemporal elasticity of substitution in labor supply for men is typically estimated (with period length equal to one year) to be quite small, say in the range from 0.00 (or even negative) to 0.45, see Course Material, p. 139, last column in Table 1.22. This has relevance for the issue whether observed business cycle fluctuations in employment can be explained by intertemporal substitution in labor supply. What relevance?

II.7 *Employer's contribution, taxation, wages, and labor supply in a small open economy.*[1] Consider a two-period OLG model for a small open economy (SOE), where people work and consume in both periods (no mobility of labor across borders). There is at the world market for financial capital a given constant real interest rate, $r > 0$. Let w_t denote employers' total labor costs per unit of labor, that is, $w_t = (1 + \eta_t)\bar{w}_t$, where $\bar{w}_t$ is the real wage that the worker receives (before tax) in period t and $\eta_t \geq 0$ is employers' contribution rate (representing non-wage labor costs paid to the public sector) in period t. There is a flat tax rate, τ_t, on labor income, $0 \leq \tau_t < 1$, so that the after-tax real wage is $\hat{w}_t = (1 - \tau_t)\bar{w}_t$. There are no taxes on capital income and no corporate taxation.

The aggregate production function is $Y_t = F(K_t, T_t L_t)$, where F is a neoclassical production function with CRS and Y_t, K_t and L_t are output, capital and labor input, respectively, in period t, while T_t is the technology level, which grows at a constant rate $g > 0$. Call the capital depreciation rate δ, a constant, $0 \leq \delta \leq 1$. There is perfect competition on all markets and no uncertainty.

a) Determine employers' total labor costs per unit of labor, w_t, in general equilibrium. Relate your result to Figure 1 below and to the current public debate about globalisation and competition from low-wage countries.

b) Determine the real-wage, $\bar{w}_t$, in general equilibrium. How does an increase in employer's contribution rate, η_t, affect w_t and $\bar{w}_t$, respectively? Comment.

c) Determine the after-tax real wage, $\hat{w}_t$, in general equilibrium. How does an increase in the tax rate τ_t affect w_t, $\bar{w}_t$, and $\hat{w}_t$, respectively? Comment.

We assume from now that the employers' contribution rate and the tax rate are constant over time at the levels η and τ, respectively, and that they are known by all agents.

[1] This problem is based on a suggestion by Mads Diness Jensen.

Let $\rho > -1$, $\gamma > 0$ and $\varepsilon > 0$ be given parameters. The utility function of the young in period t is:

$$U_t = \ln c_{1t} - \gamma \frac{1}{\varepsilon + 1} \ell_{1t}^{\varepsilon+1} + \frac{1}{1+\rho} \left[\ln c_{2t+1} - \gamma \frac{1}{\varepsilon+1} \ell_{2t+1}^{\varepsilon+1} \right],$$

where c_{1t} and ℓ_{1t} are consumption and labor supply, respectively, of the young in period t; further, c_{2t+1} and ℓ_{2t+1} are the planned consumption and labor supply, respectively, as old in period $t+1$. Time available as young is 1 time unit and so is time available as old. We assume parameters are such that corner solutions never arise. The number of young people in period t is N_t and the population growth rate is $n > -1$, a constant.

d) Write down the intertemporal budget constraint (IBC) for the young in period t.

e) By the method of Section 3.3, it can be shown that maximizing U_t s.t. IBC gives

$$\begin{aligned} c_2 &= \frac{1+r}{1+\rho} c_1, \\ \ell_1 &= \left(\frac{w_1}{\gamma c_1} \right)^{1/\varepsilon}, \\ \ell_2 &= \left(\frac{(1+\rho) w_2}{\gamma c_1 (1+r)} \right)^{1/\varepsilon}. \end{aligned}$$

Find the intertemporal elasticity of substitution in labor supply.

f) On the basis of your general knowledge, assess whether it is likely that observed business cycle fluctuations in employment can be explained by intertemporal substitution in labor supply.

By the method of Section 3.3, the aggregate labor supply can be shown to be

$$\begin{aligned} L_t &= \ell_{1t} N_t + \ell_{2t} N_{t-1}, \\ &= \left(\gamma \frac{1+\rho}{2+\rho} \right)^{\frac{-1}{1+\varepsilon}} \left[1 + (1+\rho)^{1/\varepsilon} \left(\frac{\hat{w}_{t+1}}{\hat{w}_t (1+r)} \right)^{1+1/\varepsilon} \right]^{\frac{-1}{1+\varepsilon}} N_t \\ &\quad + \left(\frac{\gamma}{2+\rho} \right)^{\frac{-1}{1+\varepsilon}} (1+\rho)^{\frac{1}{\varepsilon(1+\varepsilon)}} \left[\left(\frac{\hat{w}_{t-1}(1+r)}{\hat{w}_t} \right)^{1+1/\varepsilon} + (1+\rho)^{1/\varepsilon} \right]^{\frac{-1}{1+\varepsilon}} \frac{N_t}{1+n}. \end{aligned}$$

g) Find the aggregate capital stock in period t.

h) Comment on the role of η and τ for the aggregate labor supply and the aggregate capital stock. *Hint:* compare two countries that differ only wrt. η and τ.[2]

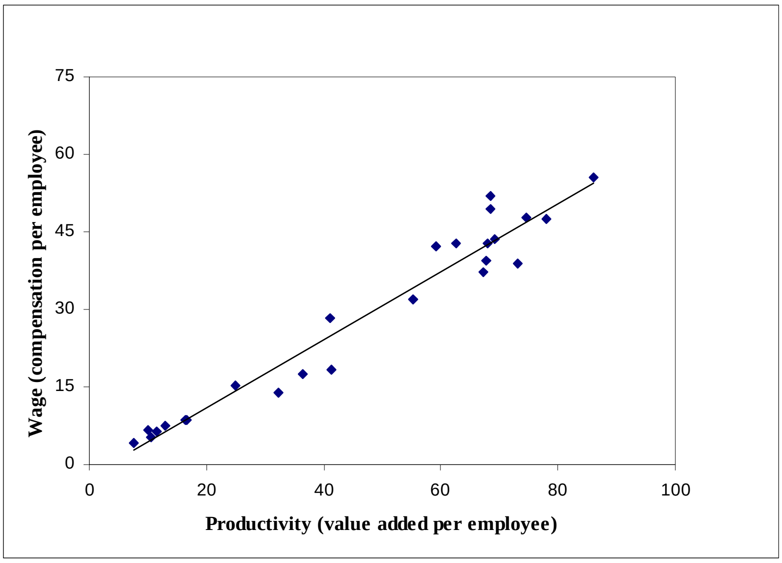

Figure 1: Wage (total labour cost) and value added per employee in manufacturing industry across OECD countries 2003. Source: AMECO (the database of the European Commission).

i) The conclusion at h) is crucially dependant on several questionable assumptions. Mention some. Comment.

II.8 *Social security, the bequest motive, and the question of neutrality of public finance.* Consider a Diamond OLG model extended with a standard pay-as-you-go social security program (tax-financed pensions): $p_t = (1+n)d_t$. Let the utility function be $U(c_{1t}, c_{2t+1}) = \ln c_{1t} + (1+\rho)^{-1} \ln c_{2t+1}$ and let the aggregate production be Cobb-Douglas. Ignore technical progress.

a) Set up and solve the decision problem of the young.

b) Is a small increase in the social security tax, d_t, neutral as regards the resource allocation in general equilibrium? Use either your general knowledge or the model to derive an answer.

c) Set up an extended model with a bequest motive as in the Barro model, i.e., the utility of the children enters the utility function of the parent with an effective discount rate $\bar{R} > 0$. Suppose the bequest motive is operative in the steady state.

d) Answer question b) for the extended model.

[2] This is generally more simple than considering a shift at time t.

e) In the extended model suppose the government decides to reduce the social security tax d_t by a small amount and use some debt-financing instead (without violating the intertemporal government budget constraint). Will this have an effect on the allocation in the economy? Why or why not?

II.9 Commenting on the fact that the Danish Welfare Commission had proposed a decrease in taxation on labor income without a corresponding simultaneous increase in other taxes, a journalist said: "Because the proposed decrease in taxation on labor income is not accompanied by simultaneous increases in other taxes, the positive effect on labor supply is likely to be considerable." Give your evaluation of this claim in terms of the different relevant "effects" (substitution effect etc.).

II.10 *Short questions.* The term "likelihood" is here used in a quite free and loose sense.

a) "When the Diamond OLG model is extended with a bequest motive (such that the utility of the children enters the utility function of the parents), the likelihood of dynamic inefficiency arising becomes smaller than in the model without a bequest motive." True or false? Explain.

b) "Let R denote a given positive number. Extending the Diamond OLG model with parental altruism and an intergenerational discount rate equal to R increases the likelihood of the long-run interest rate being equal to the modified-golden-rule value corresponding to this R." True or false? Explain. *Hint*: consider what happens in each of the three possible cases, $R < r_D$, $R = r_D$ and $R > r_D$.

c) "Extending the Diamond OLG model with a bequest motive increases the likelihood that the market equilibrium is equal to what a social planner (with the same effective intergenerational discount rate as the private individuals) would accomplish." True or false? Explain.

d) "Extending the Diamond OLG model with a bequest motive increases the likelihood of Ricardian Equivalence to hold." True or false? Provide economic intuition.

II.11 Consider a small open economy, SOE, with perfect mobility of goods and financial capital across borders, but no mobility of labour. Domestic and foreign financial claims are perfect substitutes. The real rate of interest at the world credit market is a constant, r. Time is discrete. People live for two periods, as young and as old. As young they supply one unit of labour inelastically. As old they do not work. As in the Barro dynasty model we consider single-parent families with a bequest motive. Each parent belonging to generation t has $1+n$ descendants, $n<r$ and n constant. There is perfect competition on all markets, no uncertainty, and no technical progress. Notation is

$$\begin{aligned}
L_t &= \text{ number of young in period } t,\\
\tilde{T}_t &= \text{ real gross tax revenue in period } t,\\
\sigma_t &= \tilde{T}_t/L_t = \text{ a lump-sum tax levied on the young in period } t,\\
B_t &= \text{ real government debt as inherited from the end of period } t-1.
\end{aligned}$$

In every period each old receives the same pension payment, π, from the government. From time to time the government runs a budget deficit (surplus) and in such cases the deficit is financed by bond issue (withdrawal). That is,

$$B_{t+1} - B_t = rB_t + \pi L_{t-1} - \tilde{T}_t,$$

where B_0, L_0, and π are given (until further notice, π is constant). Thus, the pension payments are, along with interest payments on government debt, the only government expenses. The government always preserves solvency in the sense that sooner or later tax revenue is adjusted to satisfy the intertemporal government budget constraint (more about his below).

An individual belonging to generation t chooses saving, s_t, and bequest, b_{t+1}, to each of the descendants so as to maximize

$$U_t = \sum_{i=0}^{\infty}(\frac{1+n}{1+R})^i \left[u(c_{1t+i}) + \frac{1}{1+\rho}u(c_{2t+i+1})\right] \qquad (*)$$

s.t.

$$\begin{aligned}
c_{1t} + s_t &= w_t - \sigma_t + b_t,\\
c_{2t+1} + (1+n)b_{t+1} &= (1+r)s_t + \pi, \qquad b_{t+1} \geq 0,
\end{aligned}$$

and taking into account the optimal responses of the descendants. Here $R > n \geq 0$ (both R and n constant). Also $\rho > -1$ is constant. The period utility function u satisfies the "no fast" assumption and $u' > 0, u'' < 0$. Negative bequests are forbidden by law.

a) How comes that the preferences of the single parent can be expressed as in (*)?

b) Derive the first-order conditions for the decision problem, taking into account that two cases are possible, namely that the constraint $b_{t+1} \geq 0$ is binding and that it is not binding. Comment.

Suppose it so happens that $r = R$ and that, at least for a while, circumstances are such that the agents are at an interior solution (i.e., $b_{t+1} > 0$). We define a steady state of this economy as a path along which c_{1t} and c_{2t} do not change over time.

c) Is the economy in a steady state? Why or why not? *Hint:* combine the first-order conditions and use that $r = R$.

As seen from the beginning of period t the intertemporal government budget constraint is:

$$\begin{aligned}
\sum_{i=0}^{\infty} \pi L_{t+i-1}(1+r)^{-i-1} &= \sum_{i=0}^{\infty} \tilde{T}_{t+i}(1+r)^{-i-1} - B_t \Rightarrow \\
L_t \frac{1}{1+r} \sum_{i=0}^{\infty} \sigma_{t+i} (\frac{1+n}{1+r})^i &= L_t \frac{1}{1+r} \sum_{i=0}^{\infty} \frac{\pi}{1+n} (\frac{1+n}{1+r})^i + B_t \Rightarrow \\
L_t \frac{1}{1+r} \sum_{i=0}^{\infty} (\frac{1+n}{1+r})^i \left[\sigma_{t+i} - \frac{\pi}{1+n} \right] &= B_t .
\end{aligned}$$

d) Explain in economic terms what each row here expresses.

e) The intertemporal budget constraint of the representative dynasty is

$$L_{t-1} \sum_{i=0}^{\infty} \frac{(1+n)^i}{(1+r)^{i+1}} \left[c_{2t+i} + (1+n)c_{1t+i} \right] = A_t + H_t,$$

where A_t is aggregate financial wealth in the economy and H_t is aggregate human wealth (after taxes):

$$H_t = L_t \sum_{i=0}^{\infty} \frac{(1+n)^i}{(1+r)^{i+1}} (w_{t+i} - \sigma_{t+i} + \frac{\pi}{1+n}).$$

Explain.

f) Suppose that in period $t+1$, π is increased (a little) to a higher constant level, before the bequest b_{t+1} is decided. Is the consumption path $(c_{2t+i}, c_{1t+i})_{i=0}^{\infty}$ affected? Why or why not?

g) Given π, suppose that for some periods there is a (small) tax cut so that $\tilde{T}_{t+i} < \pi L_{t+i-1} + r B_{t+i}$, that is, a budget deficit is run. Is the consumption path $(c_{2t+i}, c_{1t+i})_{i=0}^{\infty}$ affected? Why or why not?

Now suppose instead that $r < R$ (but still $r > n$) and that the economy is, at least initially, in steady state.

h) Will the bequest motive be operative? Why or why not?

i) Suppose π is increased (a little) to a higher level without σ_t being immediately adjusted correspondingly. Is resource allocation affected? Why or why not?

j) Given π, suppose a tax cut occurs so that for some periods a budget deficit is run. Is resource allocation affected? Why or why not?

k) If $r > R$, a model like this runs into trouble as a model for a small open economy. Why?

ℓ) In a few words give your opinion of the Barro model of infinitely-lived families linked through bequests.

II.12 *Can a fall in labor supply increase aggregate labor income?* Consider a closed economy with two factors of production, K and L, that is, capital and labor, respectively. Assume a neoclassical aggregate production function with CRS: $Y = F(K,L) = LF(k,1) \equiv Lf(k)$, where $k \equiv K/L$. Assume perfect competition.

a) Find the equilibrium real wage, w, as a function of k and show that the gross capital income share is $f'(k)k/f(k)$.

b) Define $\alpha(k) \equiv f'(k)k/f(k)$. If the production function is Cobb-Douglas, $\alpha(k)$ equals a key parameter of the function. What parameter?

c) Show that the elasticity of w wrt. k equals $\alpha(k)/\sigma(k)$, where $\sigma(k)$ is the elasticity of substitution between K and L. *Hint:* Let MRS denote the marginal rate of substitution of K for L, i.e., $MRS = F_L(K,L)/F_K(K,L)$ and the elasticity of substitution between K and L generally be called $\eta_{k,MRS}$ (i.e., the elasticity of k wrt. MRS). Then, for a general production function $F(K,L)$,

$$\eta_{k,MRS} \equiv \frac{MRS}{k}\frac{dk}{dMRS}_{|Y=\bar{Y}} = -\frac{F_K F_L(KF_K + LF_L)}{KL\left[(F_L)^2 F_{KK} - 2F_K F_L F_{KL} + (F_K)^2 F_{LL}\right]}.$$

(For derivation of this formula, see for example Sydsaeter and Hammond (2002).) When $F(K,L)$ has CRS, the formula simplifies to

$$\eta_{k,MRS} = \frac{F_K(K,L)F_L(K,L)}{F(K,L)F_{KL}(K,L)} = -\frac{f'(k)\left(f(k) - f'(k)k\right)}{kf(k)f''(k)} \equiv \sigma(k).$$

d) Apply your result and your general empirical knowledge to assess whether a lower L (i.e., a rise in the scarcity of labor) is likely to decrease or increase aggregate labor income, $w(K/L)L$.