Written exam for the M. Sc. in Economics 2009-I

Advanced Macroeconomics 2

Master's Course

January 8, 2009

(4-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The weighting of the problems is:

Problem 1: 30 %, Problem 2: 30 %, Problem 3: 30 %, Problem 4: 10 %.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 A debated issue both in economics and in the general public is whether the demographic development in industrialized countries will lead to a fall in the long-run rate of return. Those who believe in such a fall refer, among other things, to what is by them considered a likely decrease of the long-run growth rate of GDP, both as a direct consequence of a lower growth rate in the labor force and as a result of a lower rate of technical progress induced by the slowdown of labor force growth. The issue is a concern for several reasons, including the prospect of lower future pensions, everything else equal.

In this problem you are asked to evaluate some of the arguments in this debate on the basis of Blanchard's OLG model for a closed economy.

The fundamental differential equations of the model are:

$$\begin{aligned} \dot{\tilde{k}}_t &= f(\tilde{k}_t) - \frac{\omega + n + p}{n + p}\tilde{c}_t - (\delta + g + n)\tilde{k}_t, \qquad \tilde{k}_0 > 0 \text{ given}, \\ \dot{\tilde{c}}_t &= \Big[f'(\tilde{k}_t) - \delta - \rho + \omega - g\Big]\,\tilde{c}_t - (n + p)(\rho + p)\tilde{k}_t. \end{aligned}$$

Notation: $\tilde{k}_t \equiv K_t/(T_t L_t)$ and $\tilde{c}_t \equiv C_t/(T_t N_t) \equiv c_t/T_t$, where K_t and C_t are aggregate capital and aggregate consumption, respectively, N_t is population, L_t is labor supply, and T_t is the technology level, all at time t. Finally, f is a production function on intensive form, satisfying $f(0) = 0$, $f' > 0$, $f'' < 0$ and the Inada conditions. The remaining symbols stand for parameters and we assume all these are positive. Furthermore, $\rho \geq n$ and $\omega < \delta + \rho + g$.

a) Briefly interpret the two equations, including the parameters.

b) Draw a phase diagram and illustrate the path the economy follows, given some arbitrary positive initial value of $\tilde{k}$. Comment.

c) Consider two economies, A and B, that can be described by the model and are completely alike, except that A has higher n and g than B. Can we unambiguously conclude whether A or B has the higher long-run rate of return? Relate to the debate mentioned above. (*Hint:* In steady state we have two expressions for $\tilde{c}$ in terms of $\tilde{k}$ and parameters. Equate these expressions and reorder such that p and $\tilde{k}$ do not appear on the same side. Then either a graphical argument or implicit differentiation can be used.)

d) Another feature of the demographic development is that p is likely to change in the future. In what direction? Suppose the two economies, A and B, are completely alike, except that A has higher n, g, and p than B. Can we unambiguously conclude whether A or B has the higher long-run rate of return? Why or why not?

e) Yet another feature of the development is that ω is likely to change in the future. In what direction? Suppose the two economies, A and B, are completely alike, except that A has higher n, g, p, and ω than B. Can we unambiguously conclude whether A or B has the higher long-run rate of return? Why or why not?

f) Do you know of other models that might have something to say in relation to the debate referred to in the introductory paragraph? Briefly discuss.

Problem 2 Consider the government budget in a small open economy (SOE) with perfect mobility of financial capital, but no mobility of labor. The real interest rate in the world financial market is a positive constant r. Time is continuous. Let

$$\begin{aligned} Y_t &= \text{GDP at time } t, \\ T_t &= \text{net tax revenue } (= \text{ gross tax revenue} - \text{transfer payments}) \text{ at time } t, \\ G_t &= \text{government spending on goods and services at time } t, \\ B_t &= \text{public debt at time } t, \\ b_t &\equiv B_t/Y_t \ = \text{debt-income ratio at time } t, \\ S_t &= \text{primary budget surplus at time } t. \end{aligned}$$

All variables are in real terms (i.e., measured with the output good as numeraire). All government debt is short-term. Taxes and transfers are lump sum. Assume there is no uncertainty and that the budget deficit is exclusively financed by issuing debt (no money financing).

a) Write down two equations, the first describing how the budget deficit and the increase per time unit in public debt are linked, and the second how the primary budget surplus at time t is defined

Suppose Y grows at a constant rate equal to $g+n > 0$, where g is the rate of (Harrod-neutral) technical progress and n is the growth rate of the labor force (= employment). Assume $G_t = \gamma Y_t$ and, at least to begin with, $T_t = \tau Y_t$, where γ and τ are constant over time and $0 < \gamma < 1, 0 < \tau < 1$. Furthermore, assume $r > g + n$ and $B_0 > 0$.

b) Given γ, find the minimum initial primary surplus, $\bar{S}_0$, and the corresponding τ consistent with fiscal sustainability. (*Hint:* one possible approach is to consider an expression for $\dot{b}_t$; another approach is based on the fact that $\int_0^\infty e^{-at}dt = 1/a$ for any given constant $a \neq 0$.)

We now change the setup as far as the net tax revenue is concerned. We assume that the expected demographic development is such that with unchanged transfer and taxation rules the net tax rate, T_t/Y_t, will be a time-dependent rate,

$$\tau_t = \tau_0 - (1 - e^{-\lambda t})\alpha. \qquad (*)$$

Here λ and α are positive parameters and $\alpha > \bar{S}_0/Y_0$, where $\bar{S}_0$ refers to the result under b). Government spending on goods and services still satisfies $G_t = \gamma Y_t$.

c) Interpret (*). What will the limiting value of $s_t \equiv S_t/Y_t$ for $t \to \infty$ be, assuming $\tau_0 = \gamma + \bar{S}_0/Y_0$? Illustrate the time profile of s_t for $t \geq 0$ in a diagram.

Suppose that numerical projections for our SOE indicate that the present discounted value of the stream of future primary surpluses is close to zero. For analytical purposes we assume it is exactly zero.

d) Is current fiscal policy, which we will call $\mathcal{P}$, sustainable? Why or why not?

Suppose a suggested new policy, $\mathcal{P}'$, implies that the path $(\tau_t)_{t=0}^{\infty}$ is replaced by the path $(\tau'_t)_{t=0}^{\infty}$ with time profile

$$\tau'_t = \tau'_0 - (1 - e^{-\lambda t})\alpha,$$

whereas the path of G_t remains unchanged.

e) Write down an expression showing the evolution of $s'_t \equiv S_t/Y_t$ under the new policy $\mathcal{P}'$.

f) Find the minimum initial primary surplus-income ratio, $\bar{s}'_0$, required for the fiscal policy $\mathcal{P}'$ to be sustainable as seen from time 0.

As a sustainability gap indicator we choose $gap \equiv \bar{s}'_0 - s_0$, where s_0 equals S_0/Y_0 from policy $\mathcal{P}$.

g) Indicate the sustainability gap in the diagram from question c). How does *gap* depend on the growth-corrected interest rate $\tilde{r} \equiv r - g - n$, presupposing α and λ are independent of $\tilde{r}$? (*Hint:* if no unambiguous sign can be given, write down a criterion in the form of an inequality on which the sign depends.) Comment.

Problem 3 In this problem we consider short-run aspects of a small open economy (SOE) satisfying:

(i) Perfect mobility across borders of financial capital, but no mobility of labor.

(ii) Domestic and foreign financial claims are perfect substitutes.

(iii) Domestic and foreign output goods are imperfect substitutes.

More specifically, it is assumed that:

$$\dot{Y}_t = \lambda(D(Y_t, r_t, x_t, \alpha) + G - Y_t), \quad \lambda > 0, 0 < D_Y < 1, D_r < 0, D_x > 0, D_\alpha > 0, \tag{1}$$

$$\frac{M}{P} = L(Y_t, i_t), \qquad L_Y > 0,\ L_i < 0, \tag{2}$$

$$i_t = i^* + \frac{\dot{X}_t^e}{X_t}, \tag{3}$$

$$r_t \equiv i_t - \pi_t^e, \tag{4}$$

$$x_t \equiv \frac{X_t P^*}{P}. \tag{5}$$

Time is continuous. The endogenous variables are: Y_t = output, i_t = nominal interest rate, X_t = nominal exchange rate, π_t^e = expected (forward-looking) rate of inflation, all at time t; the superscript e denotes expectation. The variables α, M, P, P^*, and i^* are exogenous and constant; their interpretation is as follows: α = a demand shift parameter, G = government spending on goods and services, M = money supply, P = domestic price level, P^* = foreign price level, and i^* = nominal world market interest rate. The parameter λ is constant. The initial value, Y_0, of Y is given. Expectations are rational. We assume speculative bubbles never occur.

a) Briefly interpret the model.

b) Derive two key differential equations and construct a phase diagram portraying the dynamics of the economy in (Y, X) space. Indicate the path that the economy follows for $t \geq 0$. Comment.

Suppose that the SOE is initially in steady state. Then, unexpectedly, a recession in the leading economies in the world comes about and gives rise to an offsetting monetary policy in these countries. As a crude representation of these events vis-a-vis our SOE we "translate" them into two unanticipated parameter shifts occurring at time $t_0 > 0$: a shift in the demand shift parameter to $\alpha' < \alpha$ and a shift in the world interest rate to $i^{*\prime} < i^*$, where $i^{*\prime}$ is close to zero.

c) Suggest an interpretation of the fall in the demand shift parameter.

d) Assume that after time t_0 the public in the SOE rightly expects that the mentioned two new parameter values will remain in force for a long time and that no policy change in the SOE will occur. Under these circumstances illustrate how the SOE evolves for $t \geq t_0$, using a phase diagram as well as a figure with time profiles of Y_t, X_t, and r_t, presupposing that the sign of the long-run effect on X is dominated by the influence from the fall in the world interest rate. Explain the economic intuition.

e) As an alternative scenario imagine that at time $t_1 > t_0$ the monetary and fiscal authorities in the SOE find the situation unsatisfactory and contemplate monetary and fiscal measures to stimulate economic activity. It is soon realized, however, that neither conventional monetary policy (upward shift in M) nor conventional fiscal policy (upward shift in G) will work. Give plausible reasons for this unfavorable outlook.

f) If international coordination of fiscal policy is possible, can this then improve the outlook? Why or why not?

Problem 4 *Short questions*

a) "In Barro's OLG model the bequest motive is operative when the associated Diamond economy is dynamically efficient." True or false? Why?

b) "In all neoclassical macromodels with perfect competition (at least in this macro course) the real interest rate equals the net marginal product of capital." True or false? Why?

c) What is meant by the following terms? Money super neutrality. Seigniorage. Propagation mechanism. Liquidity trap. Precautionary saving.

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