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A suggested solution to the problem set at the exam in **Advanced Macroeconomics** January 14, 2008

(4-hours closed book exam)[1]

As formulated in the course description, a score of 12 is given if the student's performance demonstrates precise understanding of the concepts and methods needed to solve the questions raised and demonstrates reliable ability to use these tools with no or only minor weaknesses.

1. Solution to Problem 1

In the introduction we are told that the budget deficit is financed exclusively by debt issue so that

$$\dot{D}_t = iD_t + P_t(G_t - T_t). \tag{1.1}$$

We are then asked to consider the following deficit rule:

$$\frac{iD_t + P_t(G_t - T_t)}{P_t Y_t} \leq \alpha, \tag{*}$$

where $\alpha > 0$.

a) In view of $b \equiv B/Y = D/(PY)$, we have

$$\begin{aligned}\frac{\dot{b}}{b} &= \frac{\dot{D}}{D} - \frac{\dot{P}}{P} - \frac{\dot{Y}}{Y} = \frac{iD + P(G-T)}{D} - (\pi + g + n) \\ &= \frac{\alpha PY}{D} - (\pi + g + n) = \frac{\alpha}{b} - (\pi + g + n),\end{aligned}$$

from (*) with equality. Thus, the law of movement for b_t is

$$\dot{b}_t = \alpha - (\pi + g + n)b_t. \tag{1.2}$$

[1]The solution below contains *more* details and more precision than can be expected at a four hours exam.

b) It is natural to assume that $\pi + g + n > 0$. From the hint, the solution of (1.2) is

$$b_t = (b_0 - b^*)e^{-(\pi+g+n)t} + b^*, \tag{1.3}$$

where

$$b^* = \frac{\alpha}{\pi + g + n}. \tag{1.4}$$

c) With $-(\pi + g + n) < 0$, (1.3) gives $b_t \to b^*$ for $t \to \infty$. Hence, the long-run value of b_t is the steady-state value, b^*. With $\alpha = 0.04, g + n = 0.02$ and $\pi = 0.02$ we get

$$b^* = \frac{0.04}{0.04} = 1.00.$$

d) Yes, the SGP is a special case of (*), namely the case $\alpha = 0.03$. One comment could be: If in addition to $\alpha = 0.03$ we have $g + n = 0.03$ and $\pi = 0.02$, then $b^* = 0.03/0.05$ $= 0.60$, which corresponds to the ceiling for the debt-income ratio in the Maastricht Treaty and the SGP.

e) We now consider the rule

$$\frac{iD_t + P_t(G_t - T_t)}{P_t\tilde{Y}_t} \leq \alpha. \tag{**}$$

The motivation for imposing such a rule rather than (*) could be to give more scope for counter-cyclical fiscal policy, while at the same ensuring fiscal sustainability and a long-run debt-income ratio at most equal to b^* in (1.4). Thus, (**) implies

$$\frac{iD_t + P_t(G_t - T_t)}{P_tY_t} \leq \alpha\frac{\tilde{Y}_t}{Y_t},$$

where the right-hand-side is higher (lower) than α during a recession (boom).

f) In view of $\tilde{b} \equiv B/\tilde{Y} = D/(P\tilde{Y})$, where $\dot{\tilde{Y}}_t/\tilde{Y}_t = g + n$, (1.3) and (1.4) hold approximately with b replaced by $\tilde{b}$. Thus, the approximate long-run value of $\tilde{b}$ is

$$\tilde{b}^* = \frac{0}{\pi + g + n} = 0.$$

g) No. If the rule mentioned under f) is followed, public investment is on average over the business cycle fully financed by taxes and there is no debt financing. But according to the benefit principle, only the depreciation of the public capital should be financed by taxes, the remainder by debt.

h) We now assume that the rule

$$rB_t + C_t^g + \delta K_t - (T_t + \rho K_t) = 0 \tag{***}$$

should hold "on average over the business cycle". Further, public investment is such that $K_t/\tilde{Y}_t$ equals a positive constant, h, for all t. To find I_t^g/K_t, note that

$$\dot{K}_t = I_t^g - \delta K_t$$

implies

$$\frac{I_t^g}{K_t} = \frac{\dot{K}_t}{K_t} + \delta = g + n + \delta,$$

since, when $K_t/\tilde{Y}_t = h$, K must grow at the same rate as $\tilde{Y}$, the rate $g+n$.

i) There are two alternative interpretations, equally acceptable. One goes as follows:

$$\begin{aligned}\frac{\dot{B}}{B} &= \frac{\dot{D}}{D} - \frac{\dot{P}}{P} = \frac{iD + P(G - T - \rho K)}{D} - \pi \text{ (if } \rho K_t \text{ enters the public budget)} \quad (1.5)\\ &= i - \pi + \frac{G - T - \rho K}{B} = \frac{rB + C^g + I^g - (T + \rho K)}{B}.\end{aligned}$$

Substituting $T + \rho K = rB + C^g + \delta K$, from (***), we now get

$$\frac{\dot{B}}{B} = \frac{rB + C^g + I^g - (rB + C^g + \delta K)}{B} = \frac{I^g - \delta K}{B} = \frac{(I^g/K - \delta)K}{B} = \frac{(g+n)K}{B}. \quad (1.6)$$

Log-differentiating $\tilde{b} \equiv B/\tilde{Y}$ w.r.t. t gives

$$\frac{\dot{\tilde{b}}}{\tilde{b}} = \frac{\dot{B}}{B} - \frac{\dot{\tilde{Y}}}{\tilde{Y}} = (g+n)\frac{K}{B} - (g+n), \qquad \text{(from (1.6))}$$

and so

$$\dot{\tilde{b}} = \frac{(g+n)K}{B}\frac{B}{\tilde{Y}} - (g+n)\tilde{b} = (g+n)h - (g+n)\tilde{b} = (g+n)(h - \tilde{b}).$$

The solution to this differential equation is

$$\tilde{b}_t = (\tilde{b}_0 - \tilde{b}^*)e^{-(g+n)t} + \tilde{b}^*, \qquad \text{where } \tilde{b}^* = h.$$

A key step in this derivation is highlighted by the remark in brackets at (1.5). To reconcile that ρK here enters as revenue in the budget with its absence in (1.1), one can assume that there is no public investment up to and including f).

An alternative interpretation can be based on the assumption that the financial return on public capital is always kept separate and never enters the official government budget.

Then (1.5) is replaced by

$$\begin{aligned}\frac{\dot{B}}{B} &= \frac{\dot{D}}{D} - \frac{\dot{P}}{P} = \frac{iD + P(G-T)}{D} - \pi = i - \pi + \frac{G-T}{B} = \frac{rB + C^g + I^g - T}{B} \\ &= \frac{rB + C^g + I^g - (rB + C^g + \delta K - \rho K)}{B} \qquad \text{(from (***))} \\ &= \frac{I^g - (\delta - \rho)K}{B} = \frac{(I^g/K - \delta + \rho)K}{B} = \frac{(g+n+\rho)K}{B}.\end{aligned}$$

Then

$$\dot{\tilde{b}} = \frac{(g+n+\rho)K}{B}\frac{B}{\tilde{Y}} - (g+n)\tilde{b} = (g+n+\rho)h - (g+n)\tilde{b},$$

which has the solution

$$\tilde{b}_t = (\tilde{b}_0 - \tilde{b}^*)e^{-(g+n)t} + \tilde{b}^*, \qquad \text{where } \tilde{b}^* = \frac{g+n+\rho}{g+n}h.$$

j) Given that $g + n > 0$, which is a natural assumption, we get, under the first interpretation,

$$\tilde{b}_t \to \tilde{b}^* = h \quad \text{for } t \to \infty. \tag{1.7}$$

The alternative interpretation gives

$$\tilde{b}_t \to \tilde{b}^* = \frac{g+n+\rho}{g+n}h \quad \text{for } t \to \infty. \tag{1.8}$$

In both cases, the fiscal rule precludes run-away debt dynamics and is thus consistent with fiscal sustainability. Considering the ratio of public debt to public capital, we get

$$\frac{B_t}{K_t} = \frac{B_t/\tilde{Y}_t}{K_t/\tilde{Y}_t} = \frac{\tilde{b}_t}{h} \to \begin{cases} 1, & \text{if (1.7) holds,} \\ \frac{g+n+\rho}{g+n}, & \text{if (1.8) holds.} \end{cases}$$

Thus, either all or least a fraction of the public debt remains in the long run backed by public capital.

2. Solution to Problem 2

Given the function $D(Y_t, R_t, \frac{XP^*}{P}, \tau)$, where $0 < D_Y < 1, D_R < 0, D_{\frac{XP^*}{P}} > 0$ and $-1 < D_\tau < 0$, the model is:

$$\begin{aligned} Y_t^d &= D(Y_t, R_t, \frac{XP^*}{P}, \tau) + G, && (2.1) \\ \dot{Y}_t &= \lambda(Y_t^d - Y_t), \qquad \lambda > 0, && (2.2) \\ i_t &= i^*, && (2.3) \\ \frac{M_t}{P} &= L(Y_t, i_t), \qquad L_Y > 0,\ L_i < 0, && (2.4) \\ R_t &= \frac{1}{Q_t}, && (2.5) \\ \frac{1 + \dot{Q}_t^e}{Q_t} &= r_t, && (2.6) \\ r_t &\equiv i_t - \pi_t^e. && (2.7) \end{aligned}$$

a) Evidently, the model is Blanchard's dynamic IS/LM model extended to a SOE with a fixed exchange rate. It is a short-run model, since the price level P is an exogenous constant. Equation (2.1) gives aggregate output demand, which naturally depends negatively on the real long-term interest rate R (high R means high costs of investment and low consumption because of low wealth in present value terms). Aggregate output demand depends positively on the real exchange rate, XP^*/P, which is an indicator of competitiveness. Equation (2.2) says that the adjustment of output to demand takes time; the parameter λ is the speed of adjustment. Equation (2.3) is a no-arbitrage condition, given the fixed exchange rate and the perfect substitutability and mobility of financial capital.

Equation (2.4) expresses equilibrium at the money market. Naturally, real money demand depends positively on Y (because of the "transaction motive") and negatively on the short-term nominal interest rate, the opportunity cost of holding money.

The inverse relationship between the real long-term interest rate and the real market price of a long-term bond in equation (2.5) comes from the definition of the real long-term rate as the internal rate of return on a consol paying one unit of account (the output good) per time unit forever. This internal rate of return is the solution in R_t to

$$Q_t = \int_t^\infty 1 \cdot e^{-R_t(s-t)} ds.$$

Since this integral is $1/R_t$, we get (2.5). Equation (2.6) is a no-arbitrage condition saying that, absent uncertainty, the real rate of return on the long-term bond is at any time equal to the real rate of return on the short-term bond. Finally, equation (2.7) defines r_t as the short-term nominal interest rate minus the expected rate of inflation.

We are told that expectations are rational, that there is no uncertainty, and that speculative bubbles never occur.

b) The assumptions of rational expectations and no uncertainty imply $\dot{Q}_t^e = E_t\dot{Q}_t = \dot{Q}_t$. Since the price level P is an exogenous constant in the model, we have $\pi_t^e = E_t\pi_t = \pi_t = 0$ for all t. Therefore, equation (2.7) reduces to

$$r_t = i_t = i^* > 0, \tag{2.8}$$

in view of (2.3). Further, (2.5) gives $1/Q_t + \dot{Q}/Q = R_t - \dot{R}_t/R_t = r_t$. Ordering and using (2.8), gives the differential equation

$$\dot{R}_t = (R_t - i^*)R_t. \tag{2.9}$$

The other differential equation is immediately obtained from (2.2), which, inserting (2.1), can be written

$$\dot{Y}_t = \lambda(D(Y_t, R_t, x, \tau) + G - Y_t), \tag{2.10}$$

where, for convenience, we have substituted $x \equiv XP^*/P$.

The differential equations (2.9) and (2.10) in R and Y constitute the dynamic system of the model.

c)

To draw the phase diagram, note that (2.9) implies

$$\dot{R} = 0 \quad \text{for} \quad R = i^*.$$

Hence, the $\dot{R} = 0$ locus (the "LM line") is horizontal, cf. Fig. 2.1. Similarly, (2.10) implies

$$\dot{Y} = 0 \quad \text{for} \quad D(Y, R, x, \tau) + G = Y. \tag{2.11}$$

Totally differentiating this gives $D_Y dY + D_R dR = dY$, implying

$$\frac{dR}{dY}\Big|_{\dot{Y}=0} = \frac{1 - D_Y}{D_R} < 0. \tag{2.12}$$

Thus, the $\dot{Y} = 0$ locus (the "IS curve") is downward-sloping as shown in Fig. 2.1. The figure also shows the direction of movement in the different regions, as determined by (2.9)

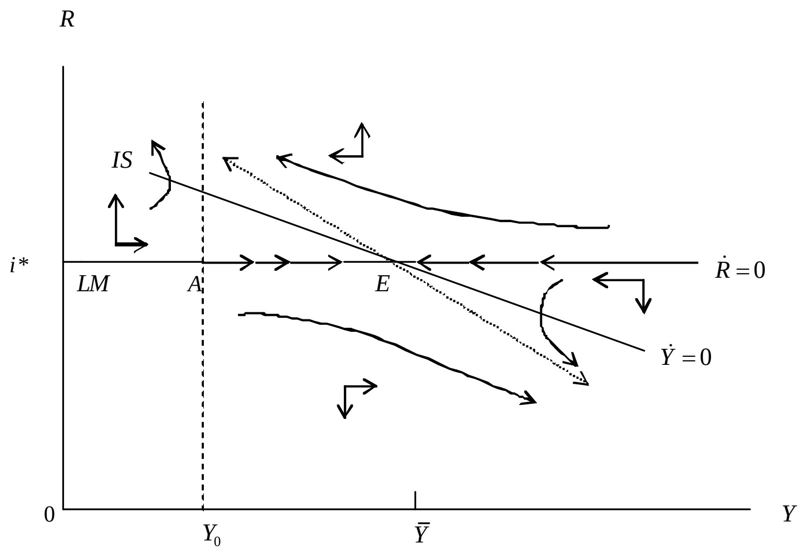

Figure 2.1:

and (2.10). We see that the steady-state point, E, is a saddle point.[2] This implies that two and only two solution paths – one from each side – converges towards E. These two saddle paths coincide with the $\dot{R} = 0$ locus. Since Y is (in this model) a predetermined variable, and R is a jump variable, our two-dimensional dynamic system has one predetermined variable and one jump variable. Finally, the saddle path is not parallel with the jump variable axis. Thus, the steady state is saddle-point stable.

At time $t = 0$, the economy must be somewhere on the vertical line $Y = Y_0$. In view of the absence of speculative bubbles, the explosive or implosive paths of R (corresponding to implosive and explosive paths of the asset price Q) in Fig. 2.1 can not arise. Hence, we are left with the saddle path, the path AE in Fig. 2.1, as the unique solution to the model.

d) In view of (2.9), we have in steady state

$$R = \bar{R} = i^*. \tag{2.13}$$

e) In steady state $Y = \bar{Y}$, where $\bar{Y}$ is the solution in Y of (2.11) with $R = i^*$. That is, the equation

$$D(\bar{Y}, i^*, x, \tau) + G = \bar{Y} \tag{2.14}$$

defines $\bar{Y}$ as an implicit function of i^* and the other exogenous variables, $\bar{Y} = \varphi(i^*, x, \tau, G)$.

[2] More formally, the determinant of the Jacobian matrix for the right hand sides of the two differential equations, evaluated in the steady state point ($\bar{Y}$, $\bar{R}$), is $\bar{R}\lambda(D_Y - 1) < 0$.

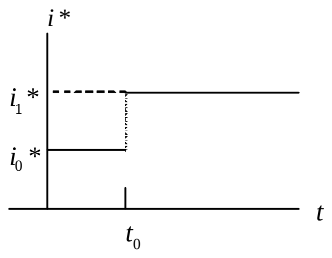

Figure 2.2:

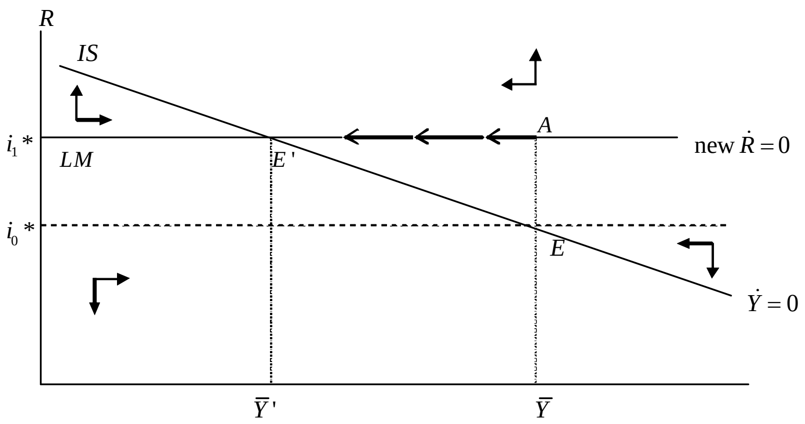

Figure 2.3:

To find the partial derivative of $\bar{Y}$ w.r.t. i^* and G, respectively, we totally differentiate the equation (2.14) to get

$$\begin{aligned} D_Y d\bar{Y} + D_R di^* + dG &= d\bar{Y} \Rightarrow \\ (1 - D_Y)d\bar{Y} &= D_R di^* + dG. \end{aligned}$$

This gives

$$\begin{aligned} \frac{\partial \bar{Y}}{\partial i^*} &= \frac{D_R}{1 - D_Y} < 0, \quad \text{and} \\ \frac{\partial \bar{Y}}{\partial G} &= \frac{1}{1 - D_Y} > 0. \end{aligned} \tag{2.15}$$

f) Fig. 2.2 illustrates the rise in i^* at time t_0. This upward shift moves the $\dot{R} = 0$ locus upwards, whereas, in view of (2.11), the $\dot{Y} = 0$ locus does not move. The lower output demand implied by higher $R = \bar{R}' = i_1^*$ results in a gradual decline in production, which further lowers demand and so on. The system approaches the new steady state at E' in Fig. 2.3.

Fig. 2.4 shows the time profiles of R_t, Y_t, r_t and M_t for $t \geq t_0$. At time t_0 the long-term interest rate jumps to the higher level i_1^* and stays there. To understand this intuitively, note that the real long-term interest rate is a kind of average of the expected future real short-term interest rates, r_t. Indeed, due to the absence of speculative bubbles, Q_t $(= 1/R_t)$ equals the fundamental value of the consol (the present discounted value of the future revenue from owning the consol):

$$Q_t = \int_t^\infty 1 \cdot e^{-\int_t^s r_\tau d\tau} ds. \tag{2.16}$$

Hence, from (2.5)

$$R_t = \frac{1}{Q_t} = \frac{1}{\int_t^\infty 1 \cdot e^{-\int_t^s r_\tau d\tau} ds}. \tag{2.17}$$

Now, the international interest rate, i^*, is rightly expected to stay at the level i_1^* for all $t > t_0$. And so is, therefore, the home interest rate, i_t, and, in the absence of inflation, the expected r_t, for all $t > t_0$. Thus, immediately after the shock

$$Q_{t_0} = \int_{t_0}^\infty 1 \cdot e^{-i_1^*(s - t_0)} ds = \frac{1}{i_1^*}.$$

Therefore, by (2.17), $R_t = i_1^*$ for all $t > t_0$.

The upward shift in R reduces output demand and so Y_t begins to gradually fall towards the new steady-state level, $\bar{Y}'$. From (2.8) follows $r_t = i_1^*$ for all $t > t_0$.

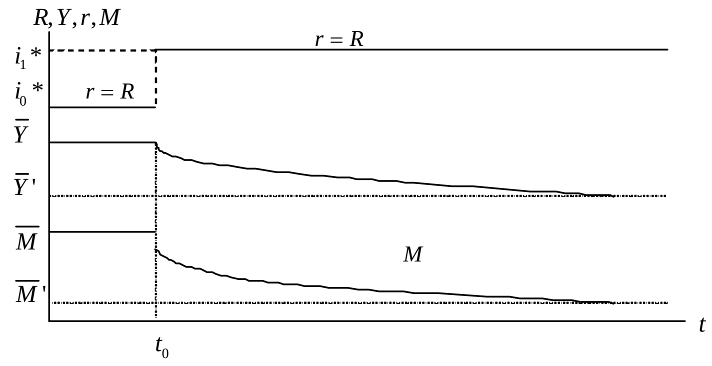

Figure 2.4:

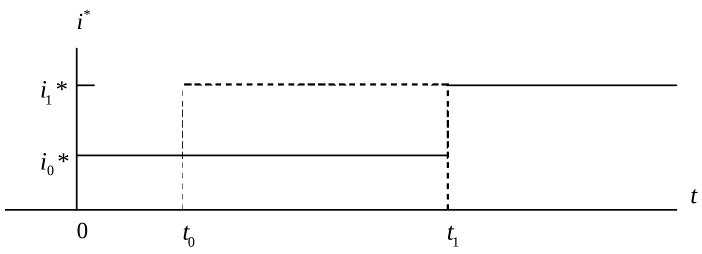

Figure 2.5:

The upward shift in i^* causes a downward jump in the money supply. This comes about because financial investors rush to convert home currency to foreign currency to buy foreign bonds and take advantage of the higher foreign interest rate. The downward jump in the money supply is exactly so large as to induce a rise in the home nominal short-term interest rate, i, up to the level i_1^*. This whole process takes place instantaneously. After time t_0, when output gradually falls, so does money demand (reduced transaction motive). The lower money demand generates an incipient tendency for the home nominal short-term interest rate, i, to fall. But this tendency is immediately counteracted by a decline in M_t due to financial investors converting home currency to foreign currency to take advantage of the slightly higher foreign interest rate. Thus, for $t > t_0$, M_t will gradually fall along with the falling Y_t.

Fig. 2.4 also illustrates that r and R remain unaffected by the decrease in output and money demand. This is due to the no-arbitrage condition (2.3) combined with the fact that the foreign short-term nominal interest rate after time t_0 remains at the constant level i_1^*.

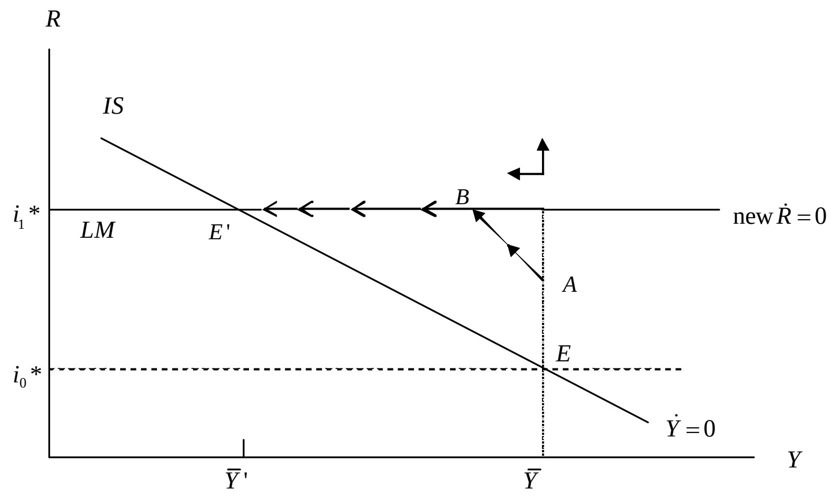

Figure 2.6:

g) Fig. 2.5 illustrates the new scenario. The phase diagram illustrating the response to the events is shown in Fig. 2.6. At time t_0, people in the SOE become aware that an upward shift in the foreign short-term nominal interest rate will take place at time $t_1 > t_0$. Already at time t_0, the real long-term rate jumps upward, anticipating the later permanent increase in the short-term rate. This dampens demand, and output begins to fall already before t_1, that is, before the foreign nominal interest rate has actually changed. In the time interval (t_0, t_1) the dynamics are determined by the "old" phase diagram and the economy follows that path (AB in Fig. 2.6) which, starting from a point on the vertical line $Y = \bar{Y}$, takes precisely $t_1 - t_0$ units of time to reach the new saddle path. The basic principle behind this is the rule that arbitrage prevents an *expected* jump in an asset price to occur. Thus, arbitrage ensures that exactly at the time t_1 the economy reaches the new saddle path generated by the now higher foreign nominal interest rate (cf. the point B in Fig. 2.6). The reason that R is rising in the time interval (t_0, t_1) is, again, that R is a kind of average of the future $r's$ and as t approaches t_1, the expected rise in r is getting closer. And then, for $t > t_1$, the economy follows the new saddle path, approaching the new steady state, E'.

In short, rational expectations and arbitrage determine the size of the upward jump in R at time t_0, that is, where exactly the point A in Fig. 2.6 is. After t_1, output Y continues falling towards its new and lower steady-state level, because the low output demand, due to high R, pulls actual output downwards. These time profiles of R and Y are shown in

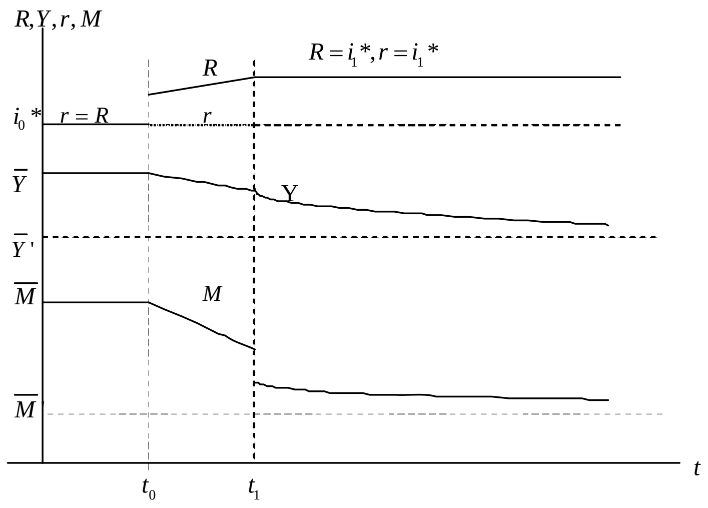

Figure 2.7:

Fig. 2.7.

This figure also shows the time profile of the real short-term rate, r, which is identical to that of the nominal short-term rate, i. In the time interval (t_0, t_1), the nominal interest rate remains unchanged, while output gradually falls. Hence, money demand falls (reduced transaction motive). This generates an incipient tendency for the home nominal short-term interest rate, i, to fall. But this tendency is immediately counteracted by a decline in M_t due to financial investors converting home currency to foreign currency to take advantage of the slightly higher foreign interest rate. Thus M_t will gradually fall in the time interval (t_0, t_1). At time t_1, the upward shift in i^* causes a downward jump in the money supply as explained under f). The downward jump in the money supply is exactly so large as to induce a rise in the home nominal short-term interest rate, i, up to the level i_1^*. This whole process takes place instantaneously. After time t_1, since output continues its downward movement, money demand gradually falls further (reduced transaction motive) and so does, then, money supply, as indicated in Fig. 2.7.

h) The described time path of g is shown in Fig. 2.8. The response of the economy is depicted in the phase diagram in Fig. 2.9. Until time $t_2 > t_1$ everything is as described in g). At time t_2, new information arrives, namely that an expansionary fiscal policy will be implemented from time t_3. But this anticipation itself of a future rise in G does not affect the forward-looking variable, R, which remains unchanged. The explanation for this is that the foreign short-term nominal interest rate remains unchanged at the level i_1^*, hence implying unchanged domestic short-term nominal and real interest rates i and

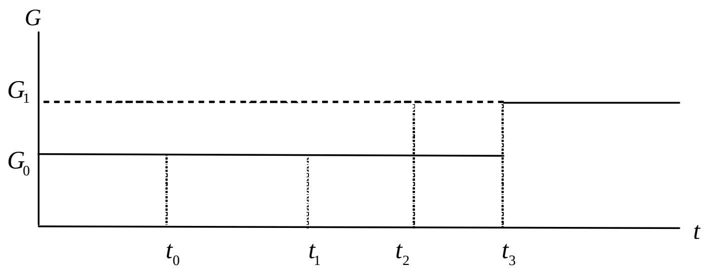

Figure 2.8:

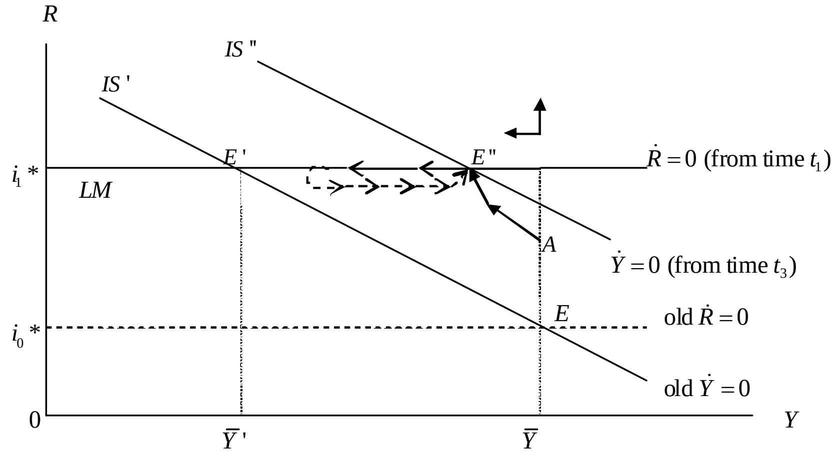

Figure 2.9:

r. Since there is thus no anticipating response in the time interval (t_2, t_3), the evolution of the economy until time t_3 is as under g).

But at time t_3 the expected upward shift in G *takes place*, and then output demand is stimulated. This implies that the $\dot{Y} = 0$ curve is shifted rightward. Since the size of the shift in G is assumed to be such as to reestablish, in the long run, an output level equal to that attained at time t_1, the final steady state, E", is at the point, which in Fig. 2.6 was called B. Thus, the Y-path will be inverted, and Y will gradually rise again towards the final steady-state level, which is Y_{t_1}. This reversal of the Y movement must take place exactly at time t_3, as depicted in the Fig. 2.10. This is because at that time the sign of $D(Y, i_1^*, x, \tau) + G - Y$ shifts from negative to positive. The sign was negative immediately before t_3, because otherwise Y could not have been decreasing towards $\bar{Y}'$, which it was. That the sign is positive immediately after t_3, is implied by the fact that $Y_{t_3} < Y_{t_1}$. Indeed, in view of $0 < D_Y < 1$, from $Y_{t_3} < Y_{t_1}$, where Y_{t_1} is the final steady-state level of Y, follows

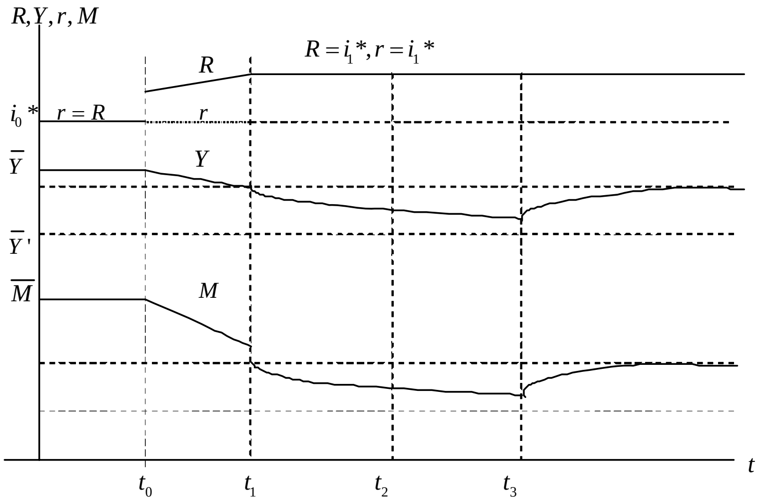

Figure 2.10:

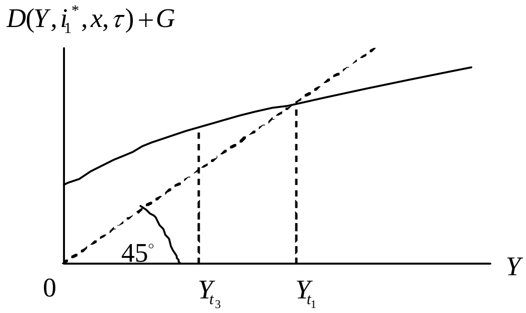

Figure 2.11:

that $Y_{t_3} < D(Y_{t_3}, i_1^*, x, \tau) + G_1$, as illustrated in Fig. 2.11.

Until time t_3, the time profile of M in Fig. 2.10 is as under g). But from time t_3, along with the gradual restitution of Y, the transaction-motivated demand for money will gradually rise. This implies an incipient tendency for the home nominal short-term interest rate, i, to rise. But this tendency is immediately counteracted by an increase in money supply, M_t, due to financial investors converting foreign currency to home currency to take advantage of the slightly higher home interest rate. Thus M_t is gradually rising until the final steady state is “reached”. Here we must have $M = PL(Y_{t_1}, i_1^*) = M_{t_1}$, as indicated in Fig. 2.10.

3. Solution to Problem 3

a) Money is said to be *superneutral* if in steady state the real variables such as consumption and investment are independent of the growth rate of the money supply. In our syllabus the following examples are mentioned as cases, within a neoclassical model framework (optimizing agents, flexible prices), where money is *not* superneutral:

1. Any model where taxes are based on nominal incomes and where there is inflation.
2. Representative agent models.
    1. A Sidrauski model extended with endogenous labour supply.
    2. A Sidrauski model extended with "money in the production function".
3. Overlapping generations (OLG) models. Here two effects may imply absence of superneutrality.
    1. The *Tobin effect.* Higher money growth implies in the long run higher inflation and nominal interest rate. This implies higher opportunity costs of holding money and thus induces agents to hold less money and more capital in their portfolios. This may stimulate capital accumulation in the economy. And the resulting lowering of the real interest rate will *not* be neutralized by lower saving, since in an OLG model the Keynes-Ramsey rule only holds at the individual level, not at the aggregate level, due to generation replacement effects.
    2. The *transfers effect.* To the extent money growth is used to finance government transfers to the private sector, the real value of these transfers will generally depend on the money growth rate (both directly and indirectly, through inflation). Thus consumption tends to be affected. And again there is no aggregate Keynes-Ramsey rule to neutralize this effect.

b) "White noise fluctuations" is a statistical concept and refers to the stochastic behavior of so-called *white noise*, which is a sequence of stochastic variables with zero expected value, constant variance and zero covariance.

In contrast, "business cycle fluctuations" is an economic concept and refers to fluctuations in aggregate economic variables, which together exhibit much more composite stochastic regularities than white noise. Some business cycles analysts emphasize the following regularities:

1. GDP and employment exhibit *fluctuations around trend.*
2. The ups and downs (expansions and contractions) exhibit *persistence* (duration) in that, for some time, positive deviations are likely to be followed by further positive deviations and negative deviations are likely to be followed by further negative deviations (positive autocorrelation).
3. The ups and downs tend to be *hump-shaped* rather than saw-tooth shaped.
4. The fluctuations are recurrent, but *neither periodic nor predictable.* The distance from peak to peak may be, say, 3-8 years.
5. The fluctuations exhibit systematic *co-movement* across production sectors, across GDP components and across countries.

4. Solution to Problem 4

a) True. In our syllabus we have for example a two-period OLG model with a voluntary early retirement scheme and where leisure is a normal good. The intertemporal budget constraint of the young is:

$$c_1 + \frac{c_2}{1+r} + \frac{\hat{w}_2 - \hat{m}}{1+r}(1-\ell) = \hat{w}_1 + \frac{\hat{w}_2}{1+r}, \tag{IBC}$$

where ℓ is planned senior-working time, $\hat{m}$ is the after-tax retirement compensation and $\hat{w}_i$ is the after-tax wage rate, $i = 1, 2$. A decrease in $\hat{m}$ increases the opportunity cost of leisure, $1 - \ell$, hence the substitution effect and the income effect on leisure are both negative. Wealth (the right-hand-side of (IBC)) is not affected, hence there is no wealth effect.

b) If we think of the *simple* version of the Blanchard OLG model (the "perpetual youth" version), the statement is definitely true. According to the *Keynes-Ramsey rule*, individual consumption grows at the rate $r^* - \rho$ (utility is logarithmic). And in that version of the Blanchard model, aggregate consumption per capita, $c \equiv C/N$, grows according to

$$\frac{\dot{c}}{c} = r^* - \rho - b(\rho + p)\frac{a}{c},$$

where $a \equiv A/N > 0$ (average financial wealth). This growth rate is lower, and the reason is the *generation replacement* effect: when the old die and are replaced by the young, they are replaced by people with less financial wealth, but the same human wealth, hence lower consumption. This pulls down the growth rate of average consumption.

If we think of the Blanchard OLG model extended with gradual retirement, whether the statement is true or not depends on the specific parameter values, because now the old are replaced by young with less financial wealth, but more human wealth. From an empirical point of view, the first factor seems liely to dominate, and so the statement can in this context be defended empirically.

c) False. Given $r^* > g+n$, the NPG condition is only necessary for fiscal sustainability, not in itself sufficient. Indeed, if B_t is the real value of the public debt at time t, we could have

$$g + n < \lim_{t\to\infty} \dot{B}_t/B_t < r^*.$$

Here, the NPG condition is satisfied (in view of the second inequality), yet the debt-income ratio explodes.

d) False. The statement holds for the Taylor model, but not for the Fischer model. The reason is easiest to explain, if we start with the Taylor model. Here one half of the labour force, group A, presets at the end of period $t-1$ its nominal wage level for period t and period $t+1$. The essential feature of the Taylor model is that this level is *the same for the two periods*, i.e., $w_{t-1,t} = w_{t-1,t+1} = x_t$ (period t's contract wage). At the end of period $t+1$, group A resets the wage level for the next two periods, i.e., chooses x_{t+2}, and so on. The other half, group B, presets at the end of period $t-2$ its nominal wage level for period $t-1$ and period t at some level x_{t-1}, i.e., $w_{t-2,t-1} = w_{t-2,t} = x_{t-1}$. At the end of period t, group B resets its wage to some level, x_{t+1}, for the next two periods and so on.

The crucial feature is that here output has always a *backward* link. This is because output depends on demand, which depends negatively on the price level, which depends positively on the average wage level. But this wage level is formed as an average of x_t and x_{t-1}, where x_t is set in advance with a view of how x_{t-1} was set a period ago. That is, not only do the expected circumstances in period t matter, but also (in contrast to the Fischer model, see below) the *expected circumstances in period* $t-1$ as seen from the end of period $t-2$. And so on backward in time. The system never gets completely free from its previous history. Therefore, the effects of changes in the money supply last longer than the time during which each nominal wage is fixed, i.e., longer than the contract period.

It is different in the Fischer model. Here, "group A", presets at the end of period $t-1$ its nominal wages for period t and period $t+1$, respectively, but not necessarily at the same level. Indeed, the decided $w_{t-1,t}$ and $w_{t-1,t+1}$ may differ, depending on differences in the expectations relating to period t and period $t+1$. At the end of period $t+1$, group A resets wages for the next two periods and so on. The other half of the labour force, "group B", presets at the end of period $t-2$ its nominal wages for period $t-1$ and period t, respectively, i.e., $w_{t-2,t-1}$ and $w_{t-2,t}$. At the end of period t, group B resets wages for the next two periods and so on.

Thus, in the Fischer model, when group A sets $w_{t-1,t}$ in advance, it does so with a view of (a) own expectations about period t and (b) at what level $w_{t-2,t}$ was set by group B a period ago. But when $w_{t-2,t}$ was set a period ago, it was also only expectations *about period t* which mattered, because group B could set $w_{t-2,t}$ independently of its $w_{t-2,t-1}$. Thus, there is no *backward* link.

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