Written exam for the M. Sc. in Economics 2008-I

Advanced Macroeconomics

Master's Course

January 14, 2008

(4-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The weighting of the problems is:

Problem 1: 30 %, Problem 2: 40 %, Problem 3: 10 %, Problem 4: 20 %.

Problem 1 Consider the government budget in a small open economy (SOE) where domestic and foreign financial claims are perfect substitutes. There is perfect mobility of financial capital. The real rate of interest at the world credit market is a constant $r > 0$. Time is continuous with one year as the time unit. A dot over a variable denotes the time derivative. There is no uncertainty. Let

$$
\begin{aligned}
Y_t &\equiv \text{ real GDP at time } t,\\
G_t &\equiv \text{ real government spending on goods and services at time } t,\\
T_t &\equiv \text{ real net tax revenue (= gross tax revenue} - \text{transfer payments) at time } t,\\
B_t &\equiv \text{ real public debt at time } t,\\
b_t &\equiv \frac{B_t}{Y_t},\\
D_t &\equiv \text{ nominal public debt at time } t,\\
P_t &\equiv \text{ price level at time } t,\\
\pi &\equiv \frac{\dot{P}_t}{P_t} \equiv \text{ rate of inflation, assumed a non-negative constant,}\\
i &\equiv r + \pi \equiv \text{ nominal rate of interest.}
\end{aligned}
$$

All debt is short-term. To begin with we assume that $Y_t = Y_0 e^{(g+n)t}$, where g is a constant growth rate of technology and n is a constant growth rate of the labour force. Assume that the budget deficit is financed exclusively by debt issue so that $\dot{D}_t = iD_t + P_t(G_t - T_t)$. Suppose that, by law, the following deficit rule must be followed:

$$\frac{iD_t + P_t(G_t - T_t)}{P_t Y_t} \leq \alpha, \qquad (*)$$

where $\alpha > 0$.

a) Derive the law of movement (differential equation) for b_t, assuming the deficit rule (*) is always binding.

b) Find the time path of b_t (still assuming (*) is always binding). *Hint:* the differential equation $\dot{x} + ax = k$, where a and k are constants, $a \neq 0$, has the solution $x_t = (x_0 - x^*)e^{-at} + x^*$, where $x^* = k/a$.

c) Find the long-run value of b_t, when $\alpha = 0.04, g + n = 0.02$ and $\pi = 0.02$ (still assuming (*) is always binding).

d) Is the deficit rule (concerning the year-by-year deficit) of the SGP of the EMU a special case of (*)? Comment.

From now we make a distinction between the actual (fluctuating) year-by-year output, Y_t, and trend output, $\tilde{Y}_t$, growing at the rate $g + n$.

e) Suppose that instead of (*) the SOE follows the rule

$$\frac{iD_t + P_t(G_t - T_t)}{P_t \tilde{Y}_t} \leq \alpha. \qquad (**)$$

What could be the motivation for imposing such a rule rather than (*)?

f) Among the rules contained in the SGP of the EMU is a rule that "on average over the business cycle" the government budget "should be close to balance or in surplus". In practice, obedience to this rule is often interpreted as implying that B_t in the medium term moves approximately as if (**) holds for all t with "$\leq$" replaced by "="and α replaced by 0. What will, according to this interpretation, the approximate long-run value of $\tilde{b}_t \equiv B_t/\tilde{Y}_t$ be?

Let public consumption and public investment be denoted C_t^g and I_t^g, respectively. Thus, $G_t = C_t^g + I_t^g$. Since we shall not consider private capital explicitly, we will simply call the public real capital (infrastructure etc.) K_t. We have

$$\dot{K}_t = I_t^g - \delta K_t,$$

where δ is a constant capital depreciation rate. Let ρ = the real gross "direct" rate of return on public capital (for simplicity ρ is assumed constant). By "direct" rate of return

on public capital is meant the financial rate of return arising from user payment for the associated public service. We may plausibly assume that

$$0 < \rho < r + \delta.$$

Yet public investment may be worthwhile because its *social* rate of return, due to indirect effects on productivity etc. in the economy, is likely to exceed the "direct" rate of return.

g) The so-called benefit principle implies that public consumption and transfers should be financed out of current taxes and public investment be financed over the life of the public capital goods. Is the rule mentioned under f) consistent with this principle? Why or why not?

Suppose that instead of (*) and (**) the rule is that

$$rB_t + C_t^g + \delta K_t - (T_t + \rho K_t) = 0 \qquad (***)$$

should hold "on average over the business cycle". Suppose further that public investment is such that $K_t/\tilde{Y}_t$ equals a positive constant, h, for all t.

h) As a preparation for question i), find I_t^g/K_t. *Hint:* first find the growth rate of K_t.

i) We interpret the rule that (***) should hold "on average over the business cycle" as a requirement that B_t in the medium term should move approximately as if (***) holds for all t. Find the implied approximate time path of $\tilde{b}_t \equiv B_t/\tilde{Y}_t$. *Hint:* a possible approach is to start by finding an expression for $\dot{B}_t/B_t$ and then use that if $z = x/y$, then the law of movement for z can be found by considering $\dot{z}/z = \dot{x}/x - \dot{y}/y$.

j) Comment on the long-run behaviour of $\tilde{b}_t$ and B_t/K_t. Relate to the issue of fiscal sustainability.

Problem 2 We consider a small open economy (SOE) satisfying (approximately):

1. Perfect mobility across borders of financial capital, but no mobility of labour.
2. Domestic and foreign financial claims are perfect substitutes (no uncertainty).
3. Domestic and foreign output goods are imperfect substitutes.
4. Nominal prices are fixed.

Suppose the short-term behaviour of the economy can be approximately described by the following model in continuous time. Given the function $D(Y_t, R_t, \frac{XP^*}{P}, \tau)$, where $0 < D_Y < 1, D_R < 0, D_{\frac{XP^*}{P}} > 0$ and $-1 < D_\tau < 0$, the model is:

$$
\begin{aligned}
Y_t^d &= D(Y_t, R_t, \frac{XP^*}{P}, \tau) + G, && (1)\\
\dot{Y}_t &= \lambda(Y_t^d - Y_t), \qquad \lambda > 0, && (2)\\
i_t &= i^*, && (3)\\
\frac{M_t}{P} &= L(Y_t, i_t), \qquad L_Y > 0,\ L_i < 0, && (4)\\
R_t &= \frac{1}{Q_t}, && (5)\\
\frac{1 + \dot{Q}_t^e}{Q_t} &= r_t, && (6)\\
r_t &\equiv i_t - \pi_t^e. && (7)
\end{aligned}
$$

The endogenous variables are: $Y_t^d \equiv$ output demand, $Y_t \equiv$ output, $R_t \equiv$ long-term real interest rate, $i_t \equiv$ short-term nominal interest rate, $M_t \equiv$ money supply, $Q_t \equiv$ real price of a long-term bond (a consol), $r_t \equiv$ short-term real interest rate and $\pi_t \equiv$ (forward-looking) rate of inflation, where t is time. A dot over a variable denotes the time derivative. The superscript e signifies an expected value. The variables X, P^*, P, G, τ and i^* are exogenous and constant; their interpretation is as follows: $X \equiv$ nominal exchange rate, $P^* \equiv$ foreign price level, $P \equiv$ domestic price level, $G \equiv$ government spending on goods and services, $\tau \equiv$ tax parameter and $i^* \equiv$ foreign short-term nominal interest rate. The parameter λ is constant. The initial value, Y_0, of Y is predetermined. We assume that expectations are rational and speculative bubbles never occur.

a) Briefly interpret the model.

b) To characterize the movement over time of the economy, derive two coupled differential equations in Y and R.

c) Construct the corresponding phase diagram and illustrate the path the economy follows. Comment.

d) Determine the long-term real interest rate in steady state.

e) How does Y in steady state depend on i^* and G?

Suppose that the economy has been in its steady state until time t_0.

f) At time t_0 an unanticipated upward shift in the foreign short-term nominal interest rate occurs. But after this shift everybody rightly expects the foreign short-term nominal interest rate to remain unchanged for a very long time. Illustrate by a phase diagram and by graphical time profiles what happens to R_t, Y_t, r_t and M_t for $t \geq t_0$. Comment.

g) Assume instead that at time t_0, people in the SOE become aware that a monetary tightening in the leading countries in the world economy is underway. As a crude representation of this, suppose the agents rightly expect an upward shift in the foreign short-term nominal interest rate to take place at time $t_1 > t_0$. But after this shift everybody rightly expects the foreign short-term nominal interest rate to remain unchanged for a very long time. Illustrate by a phase diagram and by graphical time profiles what happens to R_t, Y_t, r_t and M_t for $t \geq t_0$. Comment.

h) Now imagine the scenario is somewhat different from that described in g). Until time $t_2 > t_1$ everything is as described in g). But at time t_2, due to now foreseeable unemployment problems, the government of the SOE credibly announces an upward shift in G to take place at time $t_3 > t_2$. After this shift everybody rightly expects G to remain unchanged for a very long time. The size of the shift in G is such as to reestablish, in the long run, an output level equal to that attained at time t_1. Illustrate by a phase diagram and by graphical time profiles what happens to R_t, Y_t, r_t and M_t for $t \geq t_2$. Comment.

Problem 3 *Short questions.*

a) Define the concept of superneutrality of money and list some cases within a neoclassical model framework where money is not superneutral.

b) What is the difference between "white noise fluctuations" and "business cycle fluctuations"?

Problem 4 *Briefly evaluate the following statements.*

a) "When leisure is a normal good, a decrease in early retirement compensation has a negative substitution effect and a negative income effect on leisure (retirement) and no wealth effect." True or false? Comment.

b) "In the Blanchard OLG model in steady state individual consumption grows faster than aggregate consumption per capita." True or false? Comment.

c) "When the long-run interest rate exceeds the long-run output growth rate, it is necessary and sufficient for sustainability of a given fiscal policy that it satisfies the No-Ponzi-Game condition." True or false? Comment.

d) "In a macro model with rational expectations and asynchronous wage setting in advance, changes in money supply must have real effects lasting longer than the contract period." True or false? Comment.

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