

**Advanced Macroeconomics**

Master's Course

January 24, 2007

(4-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The weighting of the problems is:

Problem 1: 30 %, Problem 2: 20 %, Problem 3: 30 %, Problem 4: 20 %.

**Problem 1** We consider a Blanchard OLG model for a closed economy. The dynamics of the model can be reduced to two differential equations:

$$\begin{aligned}\dot{\tilde{k}}_t &= f(\tilde{k}_t) - \tilde{c}_t - (\delta + g + n)\tilde{k}_t, \\ \dot{\tilde{c}}_t &= \left[ f'(\tilde{k}_t) - \delta - \rho - g \right] \tilde{c}_t - (n + p)(\rho + p)\tilde{k}_t,\end{aligned}$$

where  $\tilde{k}_t \equiv K_t/(T_t N_t)$  and  $\tilde{c}_t \equiv C_t/(T_t N_t) \equiv c_t/T_t$ . Here,  $K_t$  and  $C_t$  are aggregate capital and aggregate consumption, respectively, at time  $t$ ,  $N_t$  is population (= labour force),  $T_t$  is the technology level,  $T_t = T_0 e^{gt}$ , and  $f$  is an intensive production function, satisfying  $f(0) = 0$ ,  $f' > 0$ ,  $f'' < 0$  as well as the Inada conditions. The remaining symbols represent parameters which are all positive. Households satisfy their transversality condition.

- a) Briefly interpret the equations, including the parameters.
- b) Draw a phase diagram and illustrate the path the economy follows for  $t \geq 0$ , given a  $\tilde{k}_0$  above the long-run level. Comment.
- c) The model entails a simple theory of the real interest rate,  $r^*$ , in the long run. For example, the model implies that  $r^*$  must belong to an open interval, defined by the parameters  $\rho, g, \delta, n$  and  $p$ . Show this. (*Hint:* as to one of the end points of this interval you may use the fact that there exists a value of  $\tilde{k}$ ,  $\tilde{k}$ , which is less than the steady-state level and satisfies  $f'(\tilde{k}) - \delta = \rho + g + n + p$ .)

- d) Suppose we also want to know in what direction  $r^*$  varies with  $g$ . What is your answer to this question (a graphical argument is enough)? Give an intuitive interpretation.
- e) Suppose we are interested not only in the sign of this dependency, but also want a formula showing the elasticity of  $r^*$  w.r.t.  $g$ . Derive such a formula.

**Problem 2** Consider the government budget in a small open economy with perfect mobility of financial capital, but no mobility of labour. The real interest rate at the world capital market is a positive constant  $r$ . Time is continuous. Let

$$\begin{aligned} Y_t &= \text{GDP at time } t, \\ T_t &= \text{net tax revenue ( = gross tax revenue - transfer payments) at time } t, \\ G_t &= \text{government spending on goods and services at time } t, \\ B_t &= \text{government debt at time } t. \end{aligned}$$

All variables are in real terms (i.e., measured with the output good as numeraire). All government debt is short-term. Taxes and transfers are lump-sum. Assume there is no uncertainty and that the budget deficit is exclusively financed by debt issue (no money financing).

Suppose  $Y_t$  grows at a constant positive rate equal to  $g + n$ , where  $g$  is the rate of (Harrod-neutral) technical progress and  $n$  is the growth rate of the labour force (= employment). Assume  $T_t = \tau Y_t$  and  $G_t = \gamma Y_t$ , where  $\tau$  and  $\gamma$  are constant over time,  $0 < \gamma < 1$ . Let debt at time 0,  $B_0$ , be positive and suppose that  $r = g + n$  and, to begin with,  $\tau = \gamma$ .

- a) Is the current fiscal policy  $(\tau, \gamma)$  sustainable? Why or why not?

We now assume that the above conditions characterized the situation for  $t < 0$ . At time  $t = 0$  conditions unexpectedly change at the world capital market so that a new constant real interest rate  $r' > r$  is established.

- b) Is the current fiscal policy  $(\tau, \gamma)$  sustainable? Why or why not?

Let  $s$  denote the primary budget surplus as a share of GDP.

- c) Find the minimum  $s, \bar{s}$ , required for fiscal sustainability in the new situation. (*Hint*: one possible approach is to derive an expression for  $\dot{b}_t$ , where  $b_t \equiv B_t/Y_t$ ; another possible approach is based on the fact that  $\int_0^\infty e^{-at} dt = 1/a$  for a given constant  $a \neq 0$ .)
- d) How does  $\bar{s}$  depend on  $g, n$  and  $r'$ , respectively? Comment.

**Problem 3** Consider a small open economy satisfying the following assumptions:

1. Perfect mobility across borders of financial capital.
2. Domestic and foreign bonds are perfect substitutes and command the same expected rate of return.
3. Domestic and foreign output goods are imperfect substitutes.
4. Nominal prices (and wages) are fixed.
5. There is no uncertainty.

Suppose the short-term behaviour of the economy can be approximately described by the following model in continuous time:

$$\begin{aligned} \dot{Y}_t &= \lambda(D(Y_t, r_t, x_t) - Y_t), & \lambda > 0, 0 < D_Y < 1, D_r < 0, D_x > 0, \\ \frac{M}{P} &= L(Y_t, i_t), & L_Y > 0, L_i < 0, \\ i_t &= i^* + \frac{\dot{X}_t^e}{X_t}, \\ r_t &\equiv i_t - \pi_t^e, \\ x_t &\equiv \frac{X_t P^*}{P}. \end{aligned}$$

The endogenous variables are:  $Y_t$  = output,  $r_t$  = real interest rate,  $x_t$  = real exchange rate,  $i_t$  = nominal interest rate,  $X_t$  = nominal exchange rate,  $\pi_t$  = rate of inflation, all at time  $t$ ; the superscript  $e$  denotes expectation. The variables  $M, P, P^*$  and  $i^*$  are exogenous and constant; their interpretation is as follows:  $M$  = money supply,  $P$  = domestic price level,  $P^*$  = foreign price level and  $i^*$  = foreign nominal interest rate. The parameter  $\lambda$  is constant. The initial value,  $Y_0$ , of  $Y$  is given. Expectations are rational and speculative bubbles never occur.

- a) Briefly interpret the first three equations of the model.
- b) Derive two differential equations in  $Y$  and  $X$  that characterize the movement over time of the economy.
- c) Construct the corresponding phase diagram and illustrate the path that the economy follows for  $t \geq 0$ . Comment.

Suppose that the economy has been in steady state until time  $t_0$ .

- d) At time  $t_0$  an unanticipated upward shift in  $M$  occurs. After  $t_0$  everybody rightly expects the money supply to remain at the new higher level,  $M'$ , forever. Illustrate by a phase diagram and a separate figure with time profiles what happens to  $Y_t$ ,  $X_t$  and  $r_t$  for  $t \geq t_0$ . Explain in detail by words the economic intuition behind what happens.

- e) Assume instead that at time  $t_0$  everybody become aware that the monetary authority will at time  $t_1 > t_0$  carry into effect a shift in money supply to the level  $M' > M$ . Illustrate by a phase diagram and a separate figure with time profiles what happens to  $Y_t$ ,  $X_t$  and  $r_t$  for  $t \geq t_0$ . Comment.

**Problem 4**      *Short questions*

- a) “In the extended Slutsky equation for labour supply, where a ‘substitution effect’, an ‘income effect’ and a ‘wealth effect’ are distinguished, a wage increase tends to generate a positive substitution effect, a positive income effect, but a negative wealth effect.” True or false? Comment.
- b) Consider an economy described by Barro’s dynasty model without technical progress. Let  $1 + n$  be the average number of children per parent and let  $R > n$  denote the “pure” intergenerational rate of discount. “Then the economy has positive bequests in steady state if and only if it is dynamically efficient.” True or false? Comment.
- c) “The expectations theory of the term structure predicts that the long-term interest rate will always be higher than the short-term interest rate.” True or false? Comment.
- d) Give a brief account of the *minimum transaction rule*.

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