16.01.2006. Christian Groth

A suggested solution to the problem set at the exam in **Advanced Macroeconomics** January 5, 2006

Four hours. No auxiliary material[1]

1. Solution to Problem 1

For convenience we repeat the definitions:

$$
\begin{aligned}
Y_t &= GDP = C_t + I_t + \bar{G} + NX_t, && (1.1)\\
C_t &= \text{private consumption},\\
I_t &= \text{private gross investment},\\
\bar{G} &= \text{government consumption, an exogenous positive constant},\\
NX_t &= \text{net exports},\\
A_t^f &= \text{net foreign assets (financial claims on the rest of the world)},\\
\dot{A}_t^f &= NX_t + rA_t^f = \text{current account surplus}, && (1.2)\\
A_t &= K_t + B_t + A_t^f = \text{private financial wealth}, && (1.3)\\
K_t &= \text{capital stock},\\
B_t &= \text{government debt},\\
A_t^n &= A_t - B_t = K_t + A_t^f = \text{aggregate financial wealth ("national wealth")}, && (1.4)
\end{aligned}
$$

a) Maximizing profit $\Pi \equiv F(K,L) - (r+\delta)K - wL$ under perfect competition gives

$$
\begin{aligned}
\frac{\partial \Pi}{\partial K} &= F_K(K,L) - (r+\delta) = 0,\\
\frac{\partial \Pi}{\partial L} &= F_L(K,L) - w = 0.
\end{aligned}
$$

In equilibrium $L = \bar{L}$. Hence, the chosen K, K^*, satisfies $F_K\left(K^*, \bar{L}\right) = r+\delta$ and is unique (since $F_{KK} < 1$).

[1] The solution below contains more details than can be expected at a four hours exam.

We have $Y = GDP = F(K^*, \bar{L})$.

Since K^* is constant, $\dot{K} = 0$, so that $I = \delta K^* + \dot{K} = \delta K^*$.

Finally, $w = F_L(K^*, \bar{L})$.

b) For convenience we repeat the five equations:

$$\begin{aligned} \dot{A}_t &= rA_t + (1 - \tau_t)w\bar{L} - C_t, && (1.5) \\ \dot{C}_t &= (r - \rho)C_t - p(\rho + p)A_t, && (1.6) \\ \dot{B}_t &= rB_t + \bar{G} - T_t, \quad \text{where} \quad T_t = \tau_t w\bar{L}, && (1.7) \\ \lim_{t\to\infty} A_t e^{-(r+p)t} &= 0, && (1.8) \\ \lim_{t\to\infty} B_t e^{-rt} &= 0. && (1.9) \end{aligned}$$

The initial values A_0 and B_0 are historically given, and $B_0 > 0$. Interpretation:

The model is a Blanchard OLG model for an open economy with public debt and taxation of labour income. In this version there is a constant population, and technical progress is ignored. Individuals have finite, but uncertain remaining lifetime. The parameter p is the death rate, i.e., p is the expected number of deaths per time unit, say per year, relatively to the size of population. The model relies on the simplifying assumption that for a given individual the probability of having a remaining lifetime, X, longer than some arbitrary number x is $P[X > x] = e^{-px}$, the same for all (i.e., independent of age). It follows that for any person the probability of dying within one year from now is approximately equal to p. Since a constant population is assumed, the birth rate is equal to p. At the aggregate level the model appeals to the law of large numbers and considers the actual number of deaths (births) per year to be indistinguishable from the expected number.

Individuals can buy life annuity contracts from life insurance companies. These companies have negligible administrative costs so that in equilibrium with free entry (zero profits), the rate of return on these contracts is $r + p$ until death, where r is the safe (real) rate of interest, given from the world capital market. The actuarial premium p is financed through the wealth transfer to the insurance sector (the cancelling of the annuity liabilities of the insurance companies) that occurs when people die.

The equation (1.5) is just an aggregate book-keeping relation in that rA_t is private aggregate capital income and $(1 - \tau_t)w\bar{L}$ is after-tax labour income. These two sum to total private income. Subtracting consumption gives aggregate private saving, which is the same as the increase per time unit, $\dot{A}_t$, in private financial wealth.

As to the first term on the right-hand side of (1.6), notice that instantaneous utility in the Blanchard OLG model is logarithmic, so that the individual Keynes-Ramsey rule is simply $\dot{c}_t = [r + p - (\rho + p)]\, c_t = (r - \rho)c_t$, where ρ is the pure rate of time preference (impatience), and $\rho+p$ is the effective rate of discount of future utility (the addition of p to this discount rate reflects the probability of not being alive at the date in question). With $C_t = c_t\bar{L}$ this gives $\dot{C}_t = (r-\rho)C_t$. The second term on the right-hand side of (1.6) reflects the "generation replacement effect". The arrival of newborns is $\bar{L}p$ per time unit, and since they have no financial wealth, the inflow of these people lowers aggregate consumption by $p(\rho+p)A_t$ per time unit. Indeed, the average financial wealth in the population is $A_t/\bar{L}$, and the consumption effect of this is $(\rho + p)A_t/\bar{L}$, since the consumption function of the "average individual" is $c_t = (\rho + p)(A_t/\bar{L} + h_t)$, where h_t is individual human wealth. Thus, *ceteris paribus*, aggregate consumption is reduced by $\bar{L}p(\rho + p)A_t/\bar{L} = p(\rho + p)A_t$ per time unit.

Equation (1.7) is the dynamic government budget constraint, given that the budget deficit, $rB_t + \bar{G} - T_t$, is financed solely by debt creation. Equation (1.8) is the transversality condition of the "average individual". Finally, equation (1.9) is implied by the No-Ponzi-Game condition for the government combined with the presupposition that the government does not plan to procure more tax revenue than necessary.

c) For $t \geq 0$, $T_t = rB_0 + \bar{G} \equiv T^*$, so that $\dot{B}_t = 0$. The required tax rate satisfies $\tau w\bar{L} = T^*$, i.e.,

$$\tau = \frac{T^*}{w\bar{L}} = \frac{rB_0 + \bar{G}}{w\bar{L}}.$$

d) It is given that

$$p(\rho + p) > r(r - \rho) \quad \text{and} \quad r > \rho. \tag{1.10}$$

We have

$$\begin{aligned} \dot{A} &= 0 \text{ for } C = rA + (1-\tau)w\bar{L}, & (1.11)\\ \dot{C} &= 0 \text{ for } C = \frac{p(\rho+p)A}{r-\rho}. & (1.12) \end{aligned}$$

These two straight lines are shown in Fig. 1.1. The unique crossing point E is the steady state with coordinates A^* and C^*. The direction of movement of A and C in the different regions, as determined by (1.5) and (1.6), respectively, are indicated by arrows. These arrows show that E is a saddle point. There is one pre-determined variable, A_0, and one

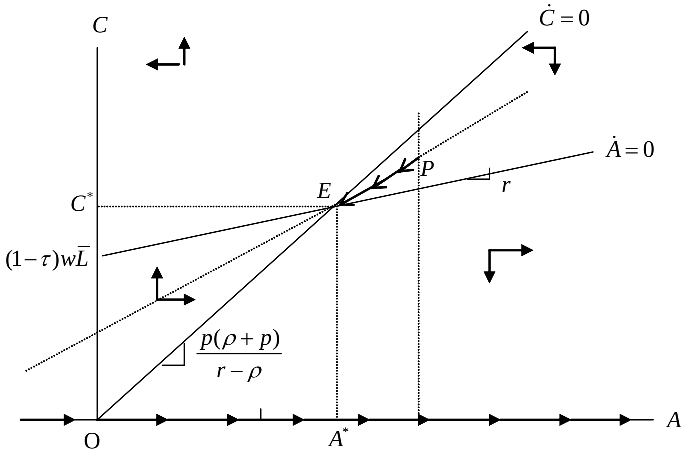

Figure 1.1:

jump variable, C_0. Given A_0, at $t = 0$ the economy must be at the point P (where the line $A = A_0$ crosses the saddle path) and then move along the saddle path and approach E for $t \to \infty$. All other paths starting at the line $A = A_0$ either violate the TVC condition of the average individual (paths starting below P) or the NPG condition of the average individual (paths starting above P).

e) We see from (1.11) and (1.12) that A^* is the solution in A to

$$\frac{p(\rho + p)A}{r - \rho} = rA + (1 - \tau)w\bar{L},$$

implying

$$\begin{aligned} A^* &= \frac{(r-\rho)(1-\tau)w\bar{L}}{p(\rho+p) - r(r-\rho)}, \qquad \text{and} \\ A^{n*} &= A^* - B_0. \end{aligned}$$

We find

$$\begin{aligned} \frac{\partial A^*}{\partial B_0} &= \frac{\partial A^*}{\partial \tau}\frac{\partial \tau}{\partial B_0} = \frac{-(r-\rho)w\bar{L}}{p(\rho+p) - r(r-\rho)}\frac{r}{w\bar{L}} = \frac{-(r-\rho)r}{p(\rho+p) - r(r-\rho)} < 0, \qquad \text{and} \\ \frac{\partial A^{n*}}{\partial B_0} &= \frac{\partial A^*}{\partial B_0} - 1 < -1. \end{aligned}$$

Comment: public debt has a crowding out effect on private financial wealth. Therefore there is more than 100 percent crowding out on national wealth. In a closed economy the effect would be smaller, because there would be an offsetting increase in the real rate of interest, implying an increase in aggregate private saving.

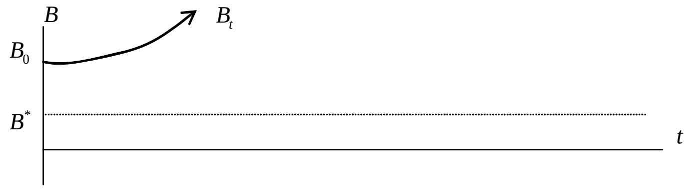

Figure 1.2:

f) In steady state $A^{f*} = A^* - K^* - B_0$, a constant, so that $\dot{A}_t^f = 0$, i.e.,

$$NX = -rA^{f*} = -r(A^* - K^* - B_0) \equiv NX^*.$$

Alternative approach 1:

$$\begin{aligned} NX &= Y - C - I - \bar{G} = F(K^*, \bar{L}) - C^* - \delta K^* - \bar{G} \qquad (1.13) \\ &= F_K(K^*, \bar{L})K^* + F_L(K^*, \bar{L})\bar{L} - C^* - \delta K^* - \bar{G} \quad \text{(by Euler's theorem)} \\ &= (r+\delta)K^* + w\bar{L} - (rA^* + (1-\tau)w\bar{L}) - \delta K^* - \bar{G} \\ &= r(K^* - A^*) + \tau w\bar{L} - (\tau w\bar{L} - rB_0) \qquad \text{(balanced budget)} \\ &= r(K^* + B_0 - A^*) = -r(A^* - K^* - B_0) = -rA^{f*}. \end{aligned}$$

Alternative approach 2: Inserting $C^* = rA^* + (1-\tau)w\bar{L} = \frac{p(\rho+p)(1-\tau)w\bar{L}}{p(\rho+p)-r(r-\rho)}$ into (1.13) is another way of expressing the result.

g) $G' > \bar{G}$ for $t \geq t_0$. With unchanged tax rate, the tax revenue is still T^*. We have

$$\int_{t_0}^{\infty} (T^* - G')e^{-r(t-t_0)}dt = \frac{T^* - G'}{r} < \frac{T^* - \bar{G}}{r} = B_0,$$

where the last equality follows from $T^* = rB_0 + \bar{G}$. Hence, the fiscal policy (G', τ) is not sustainable. Two other approaches to this question are described in the next section.

h) For $t \geq t_0$, $\dot{B}_t = rB_t + G' - T^*$. This linear differential equation has the solution

$$B_t = (B_0 - B^*)e^{rt} + B^*, \text{ where } B^* = \frac{T^* - G'}{r} < B_0. \qquad (1.14)$$

The time path of B is shown in Fig. 1.2. Since $r > 0$, the debt B_t goes towards infinity for $t \to \infty$, and so does the debt-income ratio, since Y is constant. Hence, the fiscal policy (G', τ) is not sustainable.

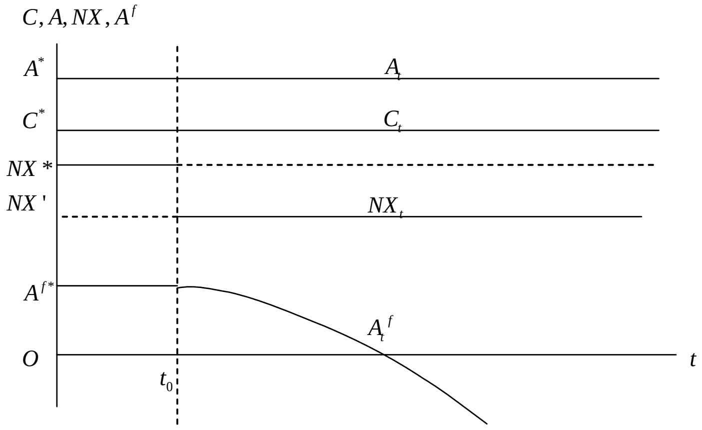

Figure 1.3:

Yet another way to show unsustainability is the following. Multiplying on both sides in (1.14) gives

$$B_t e^{-rt} = B_0 - B^* + B^* e^{-rt} \to B_0 - B^* > 0 \quad \text{for } t \to \infty.$$

Hence, the NPG condition for the government is violated. In view of $r > 0 = \dot{Y}/Y$, this implies that the fiscal policy (G', τ) is not sustainable.

i) Since the tax rate is unchanged and concealment is successful at least beyond time $t_1 > t_0$, the private sector does not change behaviour in the time interval $[0, t_1)$. Hence, for $t \in [t_0, t_1)$, $C_t = C^*$ and $A_t = A^*$, but

$$\begin{aligned} NX_t &= Y - C_t - I - G' = F(K^*, \bar{L}) - C^* - \delta K^* - G' \\ &= F(K^*, \bar{L}) - C^* - \delta K^* - \bar{G} - (G' - \bar{G}) \\ &= NX^* - (G' - \bar{G}) \equiv NX' < NX^*, \end{aligned} \tag{1.15}$$

and

$$A_t^f = A_t - K_t - B_t = A^* - K^* - (B_0 - B^*)e^{rt} - B^* \to -\infty \quad \text{for } t \to \infty.$$

These results are illustrated in Fig. 1.3.

Let $GBS_t \equiv$ government budget surplus $= T_t - rB_t - G$, and let $CA_t \equiv$ current account

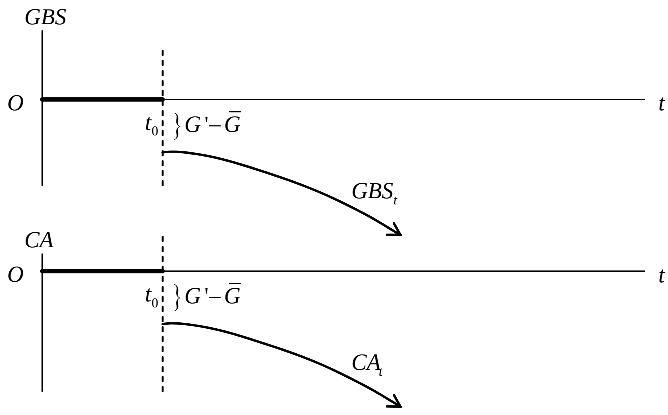

Figure 1.4:

surplus $= NX_t + rA_t^f$. Then, for $t \in [t_0, t_1)$

$$\begin{aligned}
GBS_t &= T^* - rB_t - G' = T^* - rB_0 - \bar{G} - r(B_t - B_0) - (G' - \bar{G}) \\
&= 0 - r(B_t - B_0) - (G' - \bar{G}), \\
CA_t &= NX' + rA_t^f = NX' - NX^* + NX^* + rA^{f*} + r(A_t^f - A^{f*}) \\
&= -(G' - \bar{G}) + 0 + r\left[A^* - K^* - B_t - (A^* - K^* - B_0)\right] \quad \text{(by (1.15) and (1.3))} \\
&= -(G' - \bar{G}) - r(B_t - B_0) = GBS_t.
\end{aligned}$$

These developments are illustrated in Fig. 1.4. Comment: at time t_0, the shift to $G' > \bar{G}$ and the absence of adjustment of the tax rate, causes the government budget surplus to be immediately reduced from zero to a negative number. That is, a government budget deficit is created. This is reflected in an instantaneous reduction of the current account surplus from zero to a negative number, since private behaviour is not changed. That is, a current account deficit is created. The government budget deficit causes government debt to increase, and the current account deficit causes net foreign assets to decline. These two developments causes tha government budget deficit and the current account deficit to increase further, and so on. This explains the divergent paths of A_t^f, GBS_t and CA_t.

j) For $t \geq t_1$, $G = \bar{G}$, again, and $\dot{B}_t = 0$, so that $T = rB_{t_1} + \bar{G} \equiv T'$, where $B_{t_1} = (B_0 - \frac{T^* - G'}{r})e^{rt_1} + \frac{T^* - G'}{r} > B_0$. The required tax rate, τ', satisfies $\tau' w\bar{L} = T'$, so that

$$\tau' = \frac{rB_{t_1} + \bar{G}}{w\bar{L}} > \tau,$$

since $B_{t_1} > B_0$. We assume t_1 is small enough such that $\tau' < 1$.

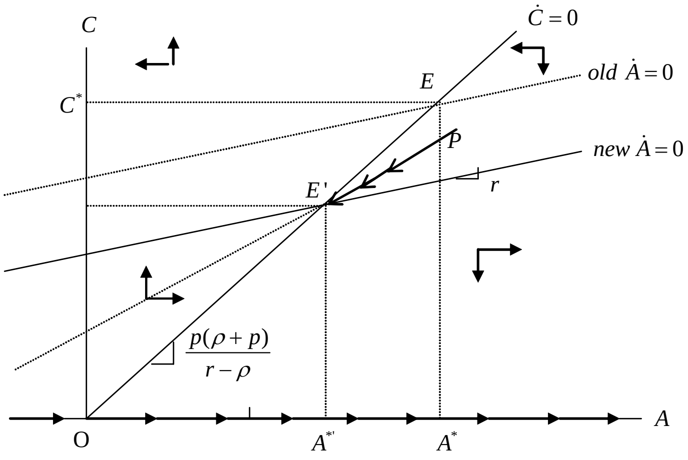

Figure 1.5:

k) For $t \geq t_1$, the dynamics are governed by:

$$\dot{A}_t = rA_t + (1-\tau')w\bar{L} - C_t, \tag{1.16}$$

$$\dot{C}_t = (r-\rho)C_t - p(\rho+p)A_t. \tag{1.17}$$

The only change is that τ has been replaced by τ'. The new phase diagram is shown on Fig. 1.5. The $\dot{C} = 0$ line is unchanged, but the $\dot{A} = 0$ line is shifted downwards. At time t_1, the economy jumps from the point E to the point P on the new saddle path. This is because the tax increase decreases after-tax human wealth, so that there is an immediate drop in consumption. Yet this drop is smaller than the tax increase, so that private saving becomes negative and private financial wealth gradually falls to the new long-run level, $A^{*\prime}$ ($< A^*$). To this corresponds the new long-run level of consumption, $C^{*\prime} < C^*$.

Net foreign assets are $A_t^f = A_t - K_t - B_t = A_t - K^* - B_{t_1} \to A^{*\prime} - K^* - B_{t_1} \equiv A^{f*\prime} < A^{f*}$ for $t \to \infty$.

ℓ) The time profiles of C_t, A_t, A_t^f and NX_t for $t \geq 0$ are shown in Fig. 1.6 and Fig. 1.7. Notice that for $t \to \infty$

$$\begin{aligned} NX_t &= F(K^*,\bar{L}) - C_t - \delta K^* - \bar{G} \to F(K^*,\bar{L}) - C^{*\prime} - \delta K^* - \bar{G} \\ &= F(K^*,\bar{L}) - C^* - \delta K^* - \bar{G} + (C^* - C^{*\prime}) = NX^* + C^* - C^{*\prime} > NX^*. \end{aligned}$$

In the new steady state, $\dot{A}_t^f = 0$, so that $NX^{*\prime} = -rA^{f*\prime} > -rA^{f*}$, since $A^{f*\prime} < A^{f*}$. Indeed, as Fig. 1.6 happens to be drawn, $A^{f*\prime} < 0 < A^{f*}$, so that positive net exports are

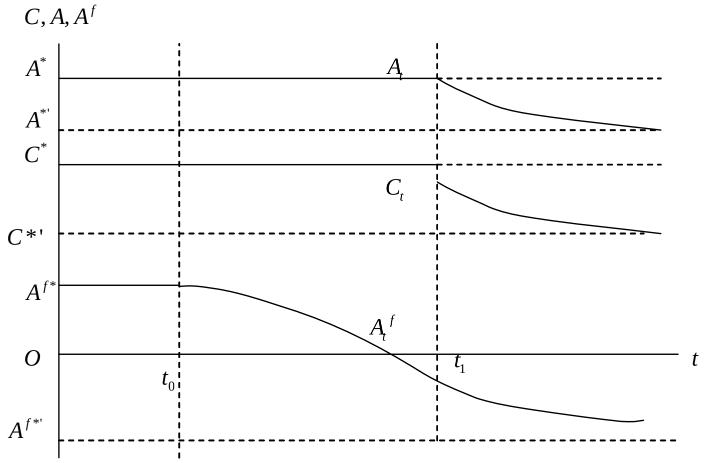

Figure 1.6:

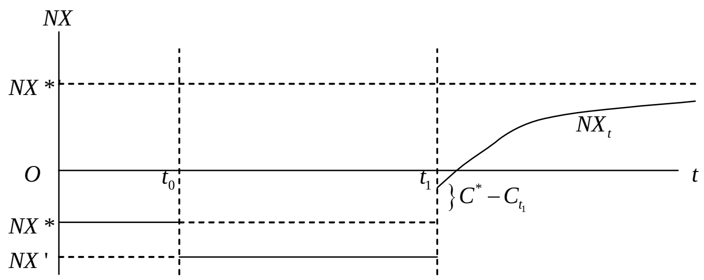

Figure 1.7:

needed in the long run to cover the interest payments on the positive net foreign debt, -$A^{f*\prime}$; i.e., $NX^{*\prime} > 0 > NX^*$.

These allocation effects of the temporary government deficit and the debt build-up are due to both the concealment and the finite time horizon of the individuals in the Blanchard OLG framework. In a representative agent framework (for example the Barro model with an operative bequest motive) the Keynes-Ramsey rule, $\dot{C} = (r - \rho)C$, would hold at the aggregate level and replace (1.17). Consequently the model would, as a model of a small open economy, be meaningful only if $\rho = r$ (since otherwise the economy would either cease being small, namely if $\rho < r$, or it would entail de-cumulation forever and starvation in the long run). In case $\rho = r$ the adjustment of C to its new steady state level after t_2 would be immediate. In the Blanchard OLG framework, however, the adjustment is only gradual, because the Keynes-Ramsey rule does *not* hold at the aggregate level.

2. Solution to Problem 2

For convenience, the model is repeated here. Given the function $D(Y_t, r_t, x_t, T)$, where $0 < D_Y < 1, D_r < 0, D_x > 0$ and $D_T < 0$, the model is:

$$
\begin{aligned}
Y_t^d &= D(Y_t, r_t, x_t, T) + G, && && (2.1)\\
\dot{Y}_t &= \lambda(Y_t^d - Y_t), && \lambda > 0, && (2.2)\\
\frac{M}{P} &= L(Y_t, i_t), && L_Y > 0,\ L_i < 0, && (2.3)\\
i_t &= i^* + \frac{\dot{X}_t^e}{X_t}, && && (2.4)\\
r_t &\equiv i_t - \pi_t^e, && && (2.5)\\
x_t &\equiv \frac{X_t P^*}{P}. && && (2.6)
\end{aligned}
$$

a) Evidently, the model is the Blanchard-Fischer version of Dornbusch's "overshooting" model, i.e., a dynamic IS-LM model for a SOE with a floating exchange rate. It is a model of short-run mechanisms, since the price level P is an exogenous constant. Equation (2.1) defines aggregate output demand as the sum of government spending, G, and private demand, $D(Y_t, r_t, x_t, T)$, including net exports. The signs of the derivatives of D have the following interpretation: $D_Y > 0$, because private consumption and perhaps also investment depends positively on aggregate production and income (which affect aggregate wealth); a further possible source to the positive dependence of Y^d on current income is that liquidity constraints may be operative; $D_Y < 1$ because the marginal propensity to spend can, realistically, be assumed less than one (that net exports tend to depend negatively on Y helps in this direction); $D_r < 0$, because consumption and investment depend negatively on the real rate of interest; $D_x > 0$, because net exports depend positively on the real exchange rate $x \equiv XP^*/P$, cf. (2.6); indeed, x is an indicator of competitiveness; and $D_T < 0$, because higher taxation implies smaller after-tax income (everything else equal), hence smaller consumption.

Equation (2.2) says that the adjustment of output to demand takes time; the parameter λ is the speed of adjustment. Equation (2.3) expresses equilibrium at the money market. Naturally, real money demand depends positively on Y through the "transaction motive" and negatively on the (short-term) nominal rate of interest, the opportunity cost of holding money. Equation (2.4) is the UIP assumption, saying that domestic and foreign financial assets pay the same rate of return (measured in the same currency). Indeed, on the left-hand side appears the interest rate on a bond denominated in domestic currency (henceforth a "domestic bond"). On the right-hand side appears the expected rate of return on investing in a bond denominated in foreign currency (henceforth a "foreign bond") plus the expected rate of depreciation of the domestic currency; this sum is in fact the expected rate of return on the foreign asset when measured in the domestic currency. Equation (2.5) defines the domestic real rate of interest as the domestic nominal rate of interest minus the expected rate of inflation. Finally, (2.6) defines the real exchange rate.

Expectations are rational, there is no uncertainty and no speculative bubbles.

b) The assumption of rational expectations (here perfect foresight) implies $\pi_t^e = E_t\pi_t = \pi_t = 0$ for all t, since the price level P is constant in the model. Therefore, equation (2.5) reduces to $r_t = i_t$. In view of (2.3) we can write i_t as a function of Y_t and M/P, that is, $i_t = i(Y_t, M/P)$, where $i_Y = -L_Y/L_i > 0$ and $i_{M/P} = 1/L_i < 0$. Inserting into (2.2) gives

$$\dot{Y}_t = \lambda(D(Y_t, i(Y_t, M/P), X_tP^*/P, T) + G - Y_t). \tag{2.7}$$

Similarly, $\dot{X}_t^e = E_t\dot{X}_t = \dot{X}_t$, so that (2.4) can be written

$$\dot{X}_t = (i(Y_t, M/P) - i^*)X_t. \tag{2.8}$$

In this way the model has been reduced to two coupled differential equations in Y_t and X_t, where Y_t is pre-determined and X_t is a jump variable.

c) To draw the corresponding phase diagram, note that (2.7) implies that $\dot{Y} = 0$ for

$$D(Y, i(Y, M/P), XP^*/P, T) + G = Y. \tag{2.9}$$

Take the total differential on both sides w.r.t. Y, X and M (for later use):

$$\begin{aligned} D_Y dY + D_r(i_Y dY + i_{M/P}\frac{1}{P}dM) + D_x\frac{P^*}{P}dX &= dY \Rightarrow \\ (1 - D_Y - D_r i_Y)dY &= D_r i_{M/P}\frac{1}{P}dM + D_x\frac{P^*}{P}dX. \end{aligned} \tag{2.10}$$

Setting $dM = 0$, we find

$$\frac{dX}{dY}\Big|_{\dot{Y}=0} = \frac{1 - D_Y - D_r i_Y}{D_x P^*/P} > 0. \tag{2.11}$$

It follows that the $\dot{Y} = 0$ locus (the "IS curve") is upward-sloping as shown in Fig. 2.1.

Equation (2.8) implies that $\dot{X} = 0$ for $i(Y, M/P) = i^*$. The value of Y satisfying this equation is unique (because $i_Y \neq 0$) and is called $\bar{Y}$. That is, $\dot{X} = 0$ for $Y = \bar{Y}$, which

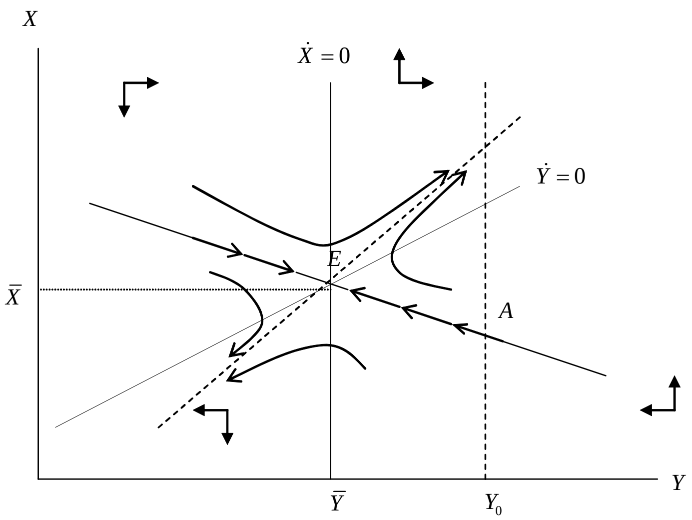

Figure 2.1:

says that the $\dot{X} = 0$ locus (the "LM curve") is vertical, cf. Fig. 2.1. The figure also indicates the direction of movement in the different regions, as determined by (2.7) and (2.8). We see that the steady state point, E, is a saddle point. This implies that two and only two solution paths - one from each side - converge towards E.

At time $t = 0$, the economy must be somewhere on the vertical line $Y = Y_0$. In view of the absence of speculative bubbles, the explosive or implosive paths of X in Fig. 2.1 cannot arise. Hence, we are left with the saddle path, the path AE in Fig. 2.1, as the unique solution to the model. Gradually over time the economy moves along the saddle path towards the steady state E.

d) In steady state

$$\frac{M}{P} = L(\bar{Y}, i^*). \tag{2.12}$$

To see how $\bar{Y}$ is affected by the decrease in M, we take the total differential on both sides:

$$\frac{1}{P}dM = L_Y d\bar{Y} + L_i di^*.$$

With $di^* = 0$, this gives $\partial\bar{Y}/\partial M = 1/(PL_Y) > 0$, implying

$$d\bar{Y} = (\partial\bar{Y}/\partial M)dM = dM/(PL_Y) < 0, \tag{2.13}$$

since $dM < 0$. Hence, the $\dot{X} = 0$ line is shifted to the left, cf. Fig. 2.2.

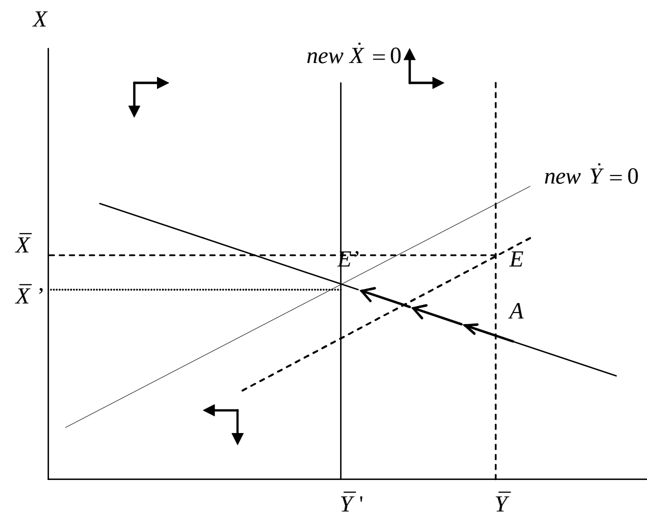

Figure 2.2:

As to the shift of the $\dot{Y}=0$ locus, (2.10) gives, for given X, i.e., $dX=0$,

$$\frac{\partial Y}{\partial M}_{|\dot{Y}=0,X=X_0} = \frac{D_r i_{M/P} 1/P}{1-D_Y-D_r i_Y} = \frac{D_r/(PL_i)}{1-D_Y-D_r L_Y/L_i} > 0.$$

Hence, the decrease in M shifts the $\dot{Y}=0$ locus leftward, cf. Fig. 2.2. Another way of understanding the shift of the $\dot{Y}=0$ locus is to consider Y as fixed, i.e., $dY=0$. Then (2.10) gives

$$\frac{\partial X}{\partial M}_{|\dot{Y}=0,Y=Y_0} = -\frac{D_r i_{M/P} 1/P}{D_x P^*/P} = -\frac{D_r/L_i}{D_x P^*} < 0.$$

Hence, we can also say that the decrease in M shifts the $\dot{Y}=0$ locus upward.

Since both the $\dot{X}=0$ locus and the $\dot{Y}=0$ locus shift left-ward, it might seem ambiguous in what direction $\bar{X}$ moves. However, we know that $\bar{Y}' < \bar{Y}$. Given $r=i^*$, this requires lower competitiveness in the new steady state, i.e., $\bar{X}' < \bar{X}$.

[More formally, the two equations describing steady state are (2.12) and

$$D(\bar{Y}, i^*, \bar{X}P^*/P, T) + G = \bar{Y}. \tag{2.14}$$

First, $d\bar{Y}$ is given by (2.13), i.e., determined from (2.12), independently of (2.14). Then, by (2.14) we see that $D_Y d\bar{Y} + D_x \frac{P^*}{P} d\bar{X} = d\bar{Y}$, from which follows

$$\begin{aligned} d\bar{X} &= \frac{1-D_Y}{D_x P^*/P} d\bar{Y} \\ &= \frac{1-D_Y}{D_x P^* L_Y} dM < 0, \quad \text{(by (2.13))} \end{aligned}$$

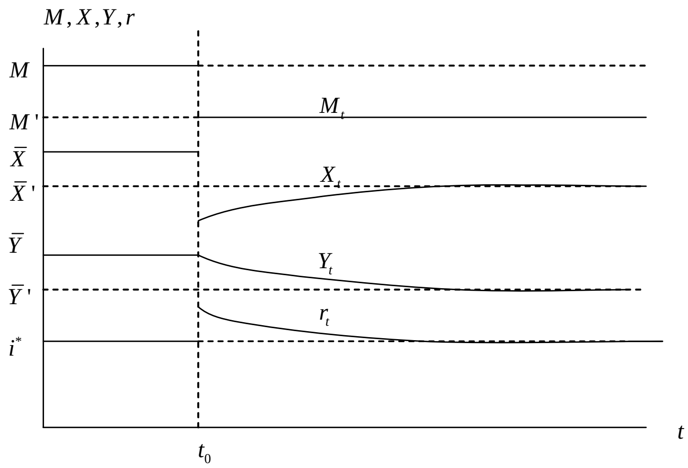

Figure 2.3:

where the inequality is due to $dM < 0$.]

The time profiles of Y_t, X_t and r_t $(= i_t)$ are shown in Fig. 2.3.

The long-run effect of the fall in M is a fall in X, that is, *appreciation* (rise in the value of the domestic currency vis-a-vis the foreign currency). The impact effect is, however, a *larger* appreciation ("overshooting"). The reason is that the fall in money supply immediately leads to a rise in i, since output is given in the short run. This leads to capital inflow, hence appreciation. The capital inflow and appreciation occur instantly. If we for a moment imagine they occurred gradually over time, there would be expected appreciation of the domestic currency, implying that domestic bonds became even more attractive relative to foreign bonds, thereby reinforcing the capital inflow and the fall in X. Hence, the capital inflow and appreciation occur in a jump, that is, X drops from its previous steady state level to a level low enough so that the concomitant *expected gradual depreciation* is at the level needed to make domestic bonds not more attractive than foreign bonds. This happens where the vertical line $Y = \bar{Y}$ crosses the new saddle path, i.e., at the point A in Fig. 2.2.

For $t > t_0$ the economy moves along the new saddle path: X gradually rises, and Y gradually falls in response to the low output demand generated by the high interest rate $r_t = i_t$ and the low exchange rate X_t (low competitiveness). The time profiles of the variables are shown in Fig. 2.3.

e) Fig. 2.4 illustrates what happens from time t_0 when people become aware that a

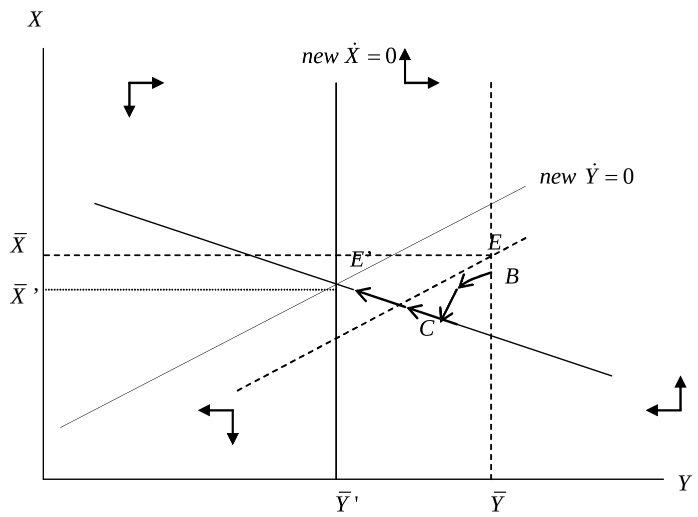

Figure 2.4:

reduction in the money supply is going to take place at time t_1. As soon as the future tight monetary policy becomes anticipated, there is an immediate effect on X in the same direction as the long-run effect, i.e., the economy drops to some point B as in Fig. 2.4. This is because agents are anticipating that from time t_1 the tight monetary policy will cause the rate of interest to be higher than i^*, thereby engendering gradual depreciation (rise in X) along the new saddle path. And since the exchange rate is an asset price, an anticipated discrete jump in X at time t_1 is ruled out. Instead X adjusts continuously after time t_0.

In the time interval (t_0, t_1) the movement of (Y, X) is governed by the "old" phase diagram. That is, in the time interval (t_0, t_1) the economy must follow that trajectory (BC in Fig. 2.4), which takes exactly $t_1 - t_0$ time units to reach the new saddle path. At time t_0, therefore, X drops precisely to the level X_B in Fig. 2.4. This appreciation implies lower competitiveness, hence lower output demand, so that output begins its downward adjustment already before monetary policy has been tightened. Given that M has not changed yet, the rate of interest begins to fall (lower transaction demand for money), causing a state where $i_t < i^*$. This leads to expected and actual gradual appreciation ($\dot{X} < 0$), further pulling output demand and actual output downwards. Hence, the interest rate falls further, and the process continues until the tight monetary policy is *actually* implemented. This occurs at time t_1. Exactly at this time the economy's trajectory, governed by the old dynamic regime, crosses the new saddle path (cf. the point C in Fig. 2.4). The actual drop in M at time t_1 triggers an upward jump in the rate of

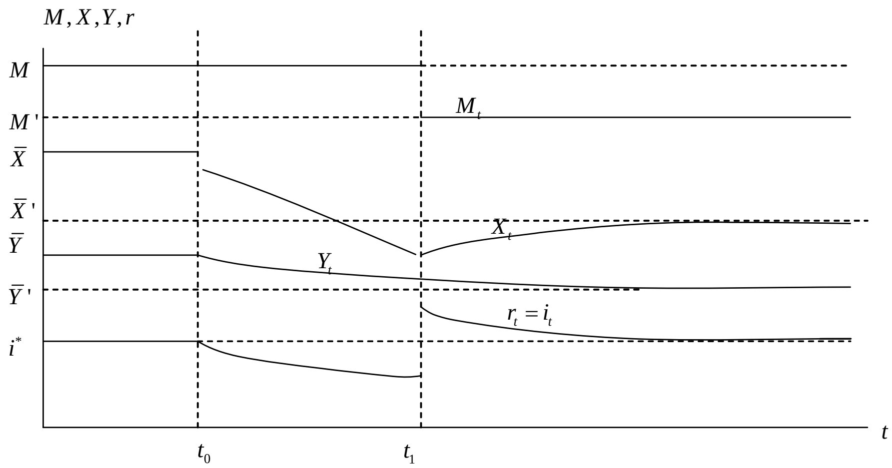

Figure 2.5:

interest, so that for $t > t_1$ we have $i > i^*$, and the new dynamics with gradual depreciation ($\dot{X} > 0$) set in.

The resulting time profiles of Y_t, X_t and r_t $(= i_t)$ are shown in Fig. 2.5. If the length of the time interval (t_0, t_1) is small enough, X may already immediately after time t_0 be below its new long-run level. However, Fig. 2.4 and Fig. 2.5 depict the opposite case, where the time interval (t_0, t_1) is somewhat larger.

f) A strength of this Blanchard-Fischer version of the Dornbusch "overshooting" model is, for example, that output is not fixed (as in Dornbusch's own version), but adjusts gradually to demand. Among the weaknesses are that the price level is completely fixed, instead of governed by some kind of a Phillips curve, and that the long-run level of the real exchange rate is not anchored by some kind of purchasing power parity relation. This implies that monetary shocks have *permanent* real effects in the model, contrary to what the data seem to indicate.

An interesting feature of the model is its capability of reconciling rational expectations with large volatility in exchange rates. Large volatility in floating exchange rates is in fact what the data show. It is, however, a weakness of the model that, in fact, it seems to *exaggerate* the exchange rate fluctuations.

3. Solution to Problem 3

Fischer's and Taylor's AD-AS models with asynchronous wage setting are discrete-time stochastic aggregate demand-aggregate supply models with some kind of imperfect competition at the labour market. Among other things the models are intended to throw light on the question:

> Do rational expectations rule out persistent real effects of changes in money supply on output?

Both models consider an economy with two "local" labour markets. Wages at these markets are set in advance for *two* periods in a *staggered* pattern (asynchronous wage setting). The Fischer model focuses on the case with possibly different wage levels in the two periods at the local labour market, whereas the Taylor model studies the case where the wage level is constant during the two periods (the contract period).

In both models workers set or negotiate wages so as to achieve, in expected value, a certain target real wage. In the Fischer model this target real wage is a given constant, whereas in the Taylor model it depends positively on expected output (or expected employment).

Main results from the Fischer model are:

(a) Only unanticipated changes in money supply affect output.

(b) Real effects of monetary shocks last only two periods (the length of the contract period). This amounts to (almost) absence of persistence.

(c) If policy makers can act every period, there is scope for stabilization policy, since wages are preset for two periods in advance.

In contrast the main results from the Taylor model are:

(a) Also anticipated changes in money supply can affect output.

(b) Real effects of monetary shocks last more than two periods (i.e., longer than the length of the contract period). Thus there is persistence.

(c) There is scope for stabilization policy (and this is so even if policy makers can act no more often than private agents).

The basic reason for the difference in results is that in the Taylor model output has always a *backward* link. This is because output in period t depends on demand, which depends negatively on the general price level, which depends positively on the current general wage level. But this wage level is formed as an average of the wage set at one local labour market at the end of period $t-1$ *and* the wage set at the other local labour market at the end of period $t-2$. Each of these two local wage levels were set with a view of how the wage at the other local labour market were set a period ago. That is, not only do the expected circumstances in period t matter as seen from the previous period, but also (in contrast to the Fischer model) the *expected circumstances in period* $t-1$ as seen from the end of period $t-2$. And so on backward in time. The system never gets completely free from its previous history.

Therefore, the effects of changes in the money supply last much longer than the time during which each nominal wage is fixed. The intuition behind the result is perhaps most clearly seen, when it is realized that in the Taylor model workers act, effectively, as if they care about *relative* wages.

4. Solution to Problem 4

a) False. The NPG condition for the government is only a necessary condition for fiscal sustainability. It is not sufficient since, if GNP growth is absent, then the debt-income ratio can explode even if the NPG condition is satisfied.

b) The claim is economic nonsense. According to the extended Slutsky equation the total effect of a tax cut on labour supply can be decomposed into three effects:

$$\text{total effect} = \text{substitution effect} + \text{income effect} + \text{wealth effect}.$$

Evidently, the tax cut increases the effective price on leisure and has therefore a *positive substitution effect* on labour supply. At the same time, by making leisure more expensive, the tax cut implies that a given budget can buy less consumption of goods *and leisure*. This amounts to a *positive income effect* of the tax cut on labour supply. Finally, the tax cut increases the present discounted value of future after-tax income, and this makes it possible to buy *more* consumption of goods *and leisure*. This amounts to a *negative wealth effect* of the tax cut on labour supply. This negative wealth effect tends to offset the other two effects.

Since the decrease in taxation of labour income proposed by the Welfare Commission is *not* accompanied by simultaneous increases in other taxes, the negative wealth effect on labour supply *is* present. This tends to make the effect on labour supply *smaller* (not larger as the journalist claims) than it would be in case of a fully financed decrease in the marginal tax on labour income.

c) In short, the term *effective demand* can be understood as actual (or operative) demand, rather than notional or Walrasian demand. It refers to an important concept in Keynesian macroeconomics.

The *minimum transaction rule* is a key element in understanding the concept of effective demand. Prices are considered as preset by price-setting firms that face a downward-sloping demand curve (usually monopolistic competition). Hence, the preset price $\bar{P}_i$ of firm i is above (expected) marginal cost, and output of the firm becomes

$$y_i = \min\left[y^d(\bar{P}_i/P, Y), y^c(\bar{P}_i, W)\right],$$

where P is the general price level (of competing firms supplying more or less differentiated goods), Y is a measure of the aggregate level of demand, and W is the wage rate. That is, production is determined as the minimum of demand, $y^d(\bar{P}_i/P, Y)$, and the "classical supply", $y^c(\bar{P}_i, W)$, at the preset price $\bar{P}_i$ (by "classical supply" is meant the output level, where marginal cost equals $\bar{P}_i$). This principle is called *the minimum transaction rule*. It follows that as long as demand $y^d(\bar{P}_i/P, Y)$ defines the most narrow limit (i.e., as long as $y^d(\bar{P}_i/P, Y) < y^c(\bar{P}_i, W)$), production will be determined by demand.

As an implication, the demand for labour by the firms depends not only on the "price signals" $\bar{P}_i$, P and W, but also on "quantity signals", like $y^d(\bar{P}_i/P, Y)$, which in turn depend on the quantity signal Y. In the next instance, there is a feedback from labourers, whose consumption demand depends *not* so much on how much labour they would *prefer* to sell at the going wage rate, but on how much they are *able* to sell. Summing over all firms and households we see that actual aggregate demand is determined not by fully adjusted equilibrium prices and wages (as is Walrasian demand), but by the going, somewhat sticky, prices and wages *and by quantity signals*. The quantity signals come from the constraints on how much the different agents can sell at the going prices and wages. The Keynesian concept of *effective demand* is defined as demand based not only on price signals, but also on such quantity signals.

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