Written exam for the M. Sc. in Economics Winter 2005/2006

Advanced Macroeconomics

January 5, 2006

Four hours. No auxiliary material. To be answered in Danish or English[1]

The weighting of the problems is:

Problem 1: 50 %, Problem 2: 30 %, Problem 3: 10 %, Problem 4: 10 %.

Problem 1. We consider the Blanchard OLG model for a small open economy (henceforth SOE) with government and taxation of labour income. For simplicity, population $\bar{L}$ is a given constant, each person supplies one unit of labour (per time unit) inelastically, people work until death (no retirement) and there are no transfers. There is perfect competition on all markets, perfect mobility of goods and financial capital across borders, but no mobility of labour. The SOE faces a given and constant real rate of interest $r > 0$ from the world capital market. There are no capital adjustment costs. Time is continuous. Let

$$
\begin{aligned}
Y_t &= GDP = C_t + I_t + \bar{G} + NX_t, \\
C_t &= \text{private consumption,} \\
I_t &= \text{private gross investment,} \\
\bar{G} &= \text{government consumption, an exogenous positive constant,} \\
NX_t &= \text{net exports,} \\
A_t^f &= \text{net foreign assets (financial claims on the rest of the world),} \\
\dot{A}_t^f &= NX_t + rA_t^f = \text{current account surplus,} \\
A_t &= K_t + B_t + A_t^f = \text{private financial wealth,} \\
K_t &= \text{capital stock,} \\
B_t &= \text{government debt,} \\
A_t^n &= A_t - B_t = K_t + A_t^f = \text{aggregate financial wealth ("national wealth"),}
\end{aligned}
$$

all at time t and all real. The technology of the representative firm is given by

$$Y_t = F(K_t, L_t),$$

where L_t is employment at time t, and F is a neoclassical production function which has constant returns to scale and satisfies the Inada conditions. For simplicity, technical progress is ignored. The rate of physical capital depreciation is a constant $\delta > 0$, so that $\dot{K}_t = I_t - \delta K_t$.

  a) Show how the capital stock K chosen by the profit maximizing representative firm in equilibrium is determined. Given this K, determine GDP, the gross investment level I and the equilibrium real wage w.

The dynamics of household variables are given by

$$\dot{A}_t = rA_t + (1 - \tau_t)w\bar{L} - C_t, \qquad A_0 \text{ given,}$$
$$\dot{C}_t = (r - \rho)C_t - p(\rho + p)A_t,$$

where ρ and p are parameters, $\rho \geq 0, p > 0$, and τ_t is a tax rate. The dynamic government budget constraint is

$$\dot{B}_t = rB_t + \bar{G} - T_t, \qquad B_0 > 0 \text{ given,}$$

where $T_t = \tau_t w\bar{L}$. Further, we have

$$\lim_{t \to \infty} A_t e^{-(r+p)t} = 0,$$
$$\lim_{t \to \infty} B_t e^{-rt} = 0.$$

b) Give a brief interpretation of these five equations, including the parameters.

Suppose that from time 0 the government budget is balanced.

c) Find the required tax rate τ. Assume (as is natural) that $\bar{G}$ and B_0 are small enough for τ to satisfy $\tau < 1$.

Assume that

$$p(\rho + p) > r(r - \rho) \quad \text{and} \quad r > \rho.$$

d) Draw a phase diagram in (A, C) space and illustrate the movement of the economy by arrows. Comment.

e) How does the steady state levels of private financial wealth and national wealth, respectively, depend on B_0? Comment.

f) Find net exports in steady state. *Hint:* you may use either that $A^f = A - K - B_0$ is constant in steady state or that, by the Euler theorem, $F(K, L) = F_K(K, L)K + F_L(K, L)L$, when F is homogeneous of degree one.

Suppose that the economy has been in steady state until time $t_0 > 0$. Then the government increases public consumption to the new constant level $G' > \bar{G}$, leaving the tax rate unchanged.

g) Is the fiscal policy (G', τ) sustainable? Comment. *Hint:* depending on your approach to this question, you may find the hint to the next question useful also here.

h) Find the induced time path for B_t and illustrate in (t, B) space. Comment. *Hint:* the linear differential equation $\dot{x}_t = ax_t + c$, where a and c are constants, $\alpha \neq 0$, has the solution $x_t = (x_0 - \bar{x})e^{at} + \bar{x}$, where $\bar{x} = -c/a$.

Suppose the change in fiscal policy at time t_0 is kept secret from the public and that the emerging budget deficit is for some time secretly financed by government borrowing at the international capital market.

i) Assuming the concealment is successful at least beyond time t_1, where t_1 is somewhat larger than t_0, illustrate graphically the time profiles of C, A, NX and A^f for $t \in [0, t_1)$. Illustrate also the time profiles of the government budget surplus and the current account surplus for $t \in [0, t_1)$. Comment.

Suppose that at time $t_1 > t_0$, a new government comes into power, discloses the concealment, reduces public consumption to the old level $\bar{G}$, shifts to a balanced budget fiscal policy, immediately increases the tax rate correspondingly and announces (credibly) that it will pursue this fiscal policy as long as in power.

j) Find the required tax rate, and assume it is less than one (i.e., assume t_1 is small enough for this to be the case).

k) Presupposing the public believes that also future governments will pursue this fiscal policy, draw a phase diagram in (A, C) space to illustrate the movement of the economy for $t \geq t_1$. Compare to the situation before t_0.

ℓ) Illustrate the time profiles of C_t, A_t, NX_t and A_t^f for $t \geq 0$. Comment.

Problem 2. Consider a small open economy satisfying the following assumptions:

1. Perfect mobility across borders of financial capital.

2. Domestic and foreign bonds are perfect substitutes and command the same expected rate of return.

3. Domestic and foreign output goods are imperfect substitutes.

4. Nominal wages and prices are fixed.

5. There is no uncertainty.

Suppose the short-term behaviour of the economy can be approximately described by the following model in continuous time. Given the function $D(Y_t, r_t, x_t, T)$, where $0 < D_Y < 1, D_r < 0, D_x > 0$ and $D_T < 0$, the model is:

$$\begin{aligned}
Y_t^d &= D(Y_t, r_t, x_t, T) + G, \\
\dot{Y}_t &= \lambda(Y_t^d - Y_t), & \lambda > 0, \\
\frac{M}{P} &= L(Y_t, i_t), & L_Y > 0, \; L_i < 0, \\
i_t &= i^* + \frac{\dot{X}_t^e}{X_t}, \\
r_t &\equiv i_t - \pi_t^e, \\
x_t &\equiv \frac{X_t P^*}{P}.
\end{aligned}$$

The endogenous variables are: Y_t^d = output demand, Y_t = output, r_t = real interest rate, x_t = real exchange rate, i_t = nominal interest rate, X_t = nominal exchange rate, π_t = rate of inflation, all at time t; the superscript e denotes expectation. The variables T, G, M, P, P^* and i^* are exogenous and constant; their interpretation is as follows: T = net tax revenue, G = government spending on goods and services, M = money supply, P = domestic price level, P^* = foreign price level and i^* = foreign nominal interest rate. The parameter λ is constant. The initial value, Y_0, of Y is given. Expectations are rational and speculative bubbles never occur.

a) Briefly interpret the model.

b) To characterize the movement over time of the economy, derive two differential equations in Y and X.

c) Construct the corresponding phase diagram and illustrate the path that the economy follows. Comment.

Suppose that the economy has been in steady state until time t_0.

d) At time t_0 an unanticipated tightening of monetary policy (downward shift in M) occurs. After t_0 everybody rightly expects the money supply to remain at the new lower level, M', forever. Illustrate by a phase diagram and a separate figure with time profiles what happens to Y_t, X_t and r_t for $t \geq t_0$. Comment.

e) Assume instead that at time t_0, due to foreseeable overheating problems everybody become aware that the monetary authority will at time $t_1 > t_0$ carry into effect a shift in money supply to the level $M' < M$. Illustrate by a phase diagram and a separate figure with time profiles what happens to Y_t, X_t and r_t for $t \geq t_0$. Comment.

f) Briefly discuss the model.

Problem 3. Briefly compare the main results in Fischer's and Taylor's AD-AS models with asynchronous wage setting.

Problem 4. *Short questions*

a) "When the real interest rate remains above the *GDP* growth rate of the economy, then the NPG condition for the government is a necessary and sufficient condition for fiscal sustainability." True or false? Comment.

b) Commenting on the fact that the Danish Welfare Commission has proposed a decrease in taxation on labour income without a corresponding simultaneous increase in other taxes, a journalist made the claim: "Because the proposed decrease in taxation on labour income is not accompanied by simultaneous increases in other taxes, the positive effect on labour supply is likely to be considerable." Give your evaluation of the claim in terms of the different relevant "effects" (substitution effect etc.).

c) Give a brief account of the concept of *effective demand*.

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