

**Advanced Macroeconomics**

January 19, 2004

Four hours. No auxiliary material

To be answered in Danish or English<sup>1</sup>

The weighting of the problems is:

Problem 1: 30 %, Problem 2: 30 %, Problem 3: 30 %, Problem 4: 10 %.

**Problem 1.** Consider a closed economy with public sector, public debt, and lump-sum taxation. The dynamics of the economy are described by the differential equations

$$\dot{C}_t = (F_K(K_t, L) - \delta - \rho)C_t - p(\rho + p)(K_t + B_t), \quad (1)$$

$$\dot{K}_t = F(K_t, L) - \delta K_t - C_t - G, \quad (2)$$

$$\dot{B}_t = [F_K(K_t, L) - \delta] B_t + G - T_t, \quad (3)$$

the condition

$$\lim_{t \rightarrow \infty} (K_t + B_t) e^{-\int_0^t [F_K(K_s, L) - \delta + p] ds} = 0, \quad (4)$$

and a requirement that the government remains solvent. Here,  $C_t$  is aggregate private consumption,  $K_t$  is physical capital,  $L$  is population = labour supply,  $B_t$  is public debt,  $G$  is government spending on goods and services,  $T_t$  is net tax revenue (= gross tax revenue – transfer payments), and  $F$  is an aggregate neoclassical production function with constant returns to scale and satisfying the Inada conditions. The other symbols stand for parameters, and all these are positive;  $L$  and  $G$  are positive constants. A dot over a variable denotes the derivative w.r.t. time  $t$ .

- a) Give a brief interpretation of the model, including the parameters.
- b) Assuming  $B_0 > 0$  and a balanced budget for all  $t \geq 0$ , draw a phase diagram and illustrate the path that the economy follows, for a given  $K_0$ . It is understood that  $G$  and  $B_0$  are "modest" relative to the production possibilities of the economy, given this  $K_0$ . Comment on the phase diagram.
- c) Suppose two countries, country I and country II, are well described by the model. The countries are similar at time  $t = 0$ , except that they differ w.r.t.  $B_0$  (but they have the same  $K_0$ ). Comment on the implied long-run differences between the countries.
- d) Suppose country I has been in its steady state until time  $t_0$ . Then, suddenly fiscal policy shifts such that, at least for a while,  $T_t = \bar{T}$  where  $\bar{T}$  is a constant which is smaller than the tax revenue in the old steady state. Is the fiscal policy  $(G, \bar{T})$  sustainable? Why or why not?

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<sup>1</sup>You are also allowed to write in Swedish or Norwegian.

- e) Suppose there were age-dependent labour supply (or age-dependent labour productivity) as in the extended Blanchard OLG model. Answer in this situation again question d). An informal answer is enough.

**Problem 2.** Consider the following dynamic model in continuous time:

$$\begin{aligned} \dot{Y}_t &= \lambda(D(R_t, Y_t) - Y_t), & D_R < 0, 0 < D_Y < 1, \\ \frac{\dot{M}_t}{P} &= L(Y_t, i_t), & L_Y > 0, L_i < 0. \\ R_t &= 1/Q_t, \\ \frac{1 + \dot{Q}_t^e}{Q_t} &= r_t, \\ r_t &\equiv i_t - \pi_t^e, \end{aligned}$$

where a dot over a variable denotes the derivative w.r.t. time  $t$ , and  $Y_t =$  output,  $R_t =$  real rate of interest on a consol,  $M_t =$  money supply,  $P =$  output price,  $i_t =$  nominal short-term interest rate,  $\pi_t =$  rate of inflation,  $Q_t =$  real price of the consol. Further, the superscript  $e$  denotes the "subjective" expected value,  $\lambda$  and  $P$  are exogenous positive constants, and  $i_t$  is the instrument of the monetary authority. The initial value of  $Y$ ,  $Y_0$ , is given.

- a) Briefly, interpret the model.

Assume rational expectations, no uncertainty, and no speculative bubbles. Until further notice, let  $i_t = \bar{i}$  for all  $t$ , where  $\bar{i}$  is an exogenous positive constant.

- b) To characterize the movement over time of the economy, derive two differential equations. Draw the corresponding phase diagram and illustrate the path that the economy follows. Comment.
- c) Determine the short-term and long-term real interest rates in steady state and find an implicit solution for  $Y$  in steady state. How does  $Y$  in steady state depend on  $\bar{i}$ ?

Now, suppose that the economy has been in its steady state until time  $t_0$ .

- d) Then an unanticipated upward shift in the instrument variable occurs, but apart from this shift, everybody expects  $i$  to remain unchanged forever. Illustrate by a phase diagram and by graphical time profiles what happens to  $Y_t$ ,  $R_t$ ,  $r_t$ ,  $Q_t$ , and  $M_t$  for  $t \geq t_0$ . Comment.
- e) Assume instead that at time  $t_0$ , the monetary authority credibly announces an upward shift in the instrument variable to take place at time  $t_1 > t_0$ . Illustrate by a phase diagram and by graphical time profiles what happens to  $Y_t$ ,  $R_t$ ,  $r_t$ ,  $Q_t$ , and  $M_t$  for  $t \geq t_0$ . Comment.

*Hint:* The following formula may be helpful:

$$R_t = 1/Q_t = 1/\left(\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds\right).$$

**Problem 3.** Consider a decision problem in discrete time for a given household facing uncertainty. As seen from period 0, the problem is:

$$\begin{aligned} \max E_0(U_0) &= E_0\left[\sum_{t=0}^{T-1} (\log c_t - \gamma \frac{\sigma}{1+\sigma} \ell_t^{(1+\sigma)/\sigma})(1+\rho)^{-t}\right] && \text{s.t.} \\ c_t &\geq 0, 0 \leq \ell_t \leq \bar{\ell}, \\ a_{t+1} &= (1+r_t)a_t + w_t \ell_t - c_t, && a_0 \text{ given,} \\ a_T &\geq 0, \end{aligned}$$

where  $c$  = consumption,  $\ell$  = labour supply,  $a$  = financial wealth,  $r$  = real rate of return on financial wealth, and  $w$  = real wage. The parameters  $\gamma$ ,  $\sigma$ ,  $\rho$ , and  $\bar{\ell}$  are all positive. The upper limit,  $\bar{\ell}$ , to labour supply is large enough so as to never be binding. The symbol  $E_0$  (generally  $E_t$ ) denotes the "true" expected value, conditional on the information available in period 0. This information includes knowledge of all realizations of the variables up to period 0 (generally period  $t$ ), including that period. There is uncertainty about future values of  $r_t$  and  $w_t$ , but the household knows the stochastic processes that these variables follow.

- a) Derive two first order conditions, the first of which (call it (\*)) describes the trade-off between consumption and labour supply in, say, period  $t$ , and the second of which (call it (\*\*)) describes the trade-off between consumption in period  $t$  and consumption in period  $t+1$ , both conditions as seen from period  $t$  ( $t = 0, 1, \dots$ ). *Hint:* You may use whatever method you wish, for example the substitution method.
- b) Give an interpretation of the two conditions.

Among the "stylized facts" of business cycle fluctuations (based on time series data after detrending) are the following: (i) Employment (aggregate labour hours) is procyclical and fluctuates almost as much as GDP. (ii) Consumption and employment are positively correlated. (iii) Real wages are weakly procyclical and do not fluctuate much.

- c) In the light of these facts and the condition (\*), briefly give an informal evaluation of the Real Business Cycle (RBC) theory.

In order to simplify, suppose for a moment there is no uncertainty.

- d) On the basis of this, find an expression for how  $\ell_t/\ell_{t+1}$  depends on  $w_t/w_{t+1}$ . From this expression, give an interpretation of the parameter  $\sigma$ . Relate this, together with your empirical knowledge, to the discussion under c).

- e) Within the market-clearing framework of the RBC approach, if fluctuations in the real wage are negligible, is it then likely that fluctuations in  $r_{t+1}$  could be a driving force behind fluctuations in employment? Relate your answer to both your result under d), the condition (\*) and the stylized facts above.

Now go back to the original formulation of the household's decision problem *with* uncertainty. Suppose there is also uncertainty as to the prospect of employment in the future, and add the constraint  $\ell_t \leq x_t$  for all  $t \geq 0$ , where  $x_t \geq 0$  is the exogenous maximum employment offered the household in period  $t$ .

- f) It can be shown that when the employment constraint  $\ell_t \leq x_t$  is binding, the equality sign in (\*) is replaced by an inequality sign. Write down the new (\*) and give an interpretation.
- g) Is it possible to reconcile theory with the stylized facts within this extended framework? Why or why not?
- h) Suppose an increase in uncertainty occurs. Discuss how this is likely to affect the current consumption of the household, given its specific period utility function.

**Problem 4.**      *Short questions*

- a) "If leisure is a 'normal good', then the income effect as well as the wealth effect on leisure of an increase in the wage rate are capable at offsetting the substitution effect." True or not true? Comment.
- b) "Ignoring uncertainty, the difference between the short-term and the long-term interest rate is determined only by expectations." True or not true? Comment.
- c) "In the Blanchard-Kiyotaki model with monopolistic competition and menu costs, there may be more than one equilibrium." True or not true? Comment.

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