Advanced Macroeconomics

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A suggested solution to the problem set at the exam January 7, 2005

Four hours. No auxiliary material

1. Solution to Problem 1

For convenience we repeat the basic equations. The technology of the representative firm is given by

$$Y_t = F(K_t, E_t L) \equiv E_t L f(\hat{k}_t), \tag{1.1}$$

where F is a neoclassical production function with CRS, Y_t and K_t are output and capital input, respectively, and $\hat{k}_t \equiv K_t/(E_t L)$. The whole labour force is employed. Efficiency of labour, E_t, grows according to $E_t = e^{gt}$. The rate of physical capital depreciation is $\delta \geq 0$.

Government spending on goods and services is

$$G_t = \gamma(N - L)w_t, \qquad \gamma > 0, \tag{1.2}$$

Transfer payments including pensions are given by

$$O_t = \alpha w_t(N - L), \qquad 0 < \alpha < 1, \tag{1.3}$$

Gross tax revenue is

$$\tilde{T}_t = \tau(w_t L + O_t), \qquad 0 < \tau < 1. \tag{1.4}$$

where the tax rate τ is constant (capital income taxation, consumption taxes etc. are ignored). Finally, B_t is real public debt, and we have $B_0 > 0$.

a) Maximizing profit $\Pi \equiv F(K, EL) - (r + \delta)K - wL$ under perfect competition gives

$$F_1(K, EL) = r + \delta, \text{ that is, } \frac{\partial\left[ELf\left(\hat{k}\right)\right]}{\partial K} = f'\left(\hat{k}\right) = r + \delta, \tag{1.5}$$

$$F_2(K, EL)\,E = w, \text{ that is, } \frac{\partial\left[ELf\left(\hat{k}\right)\right]}{\partial L} = \left[f\left(\hat{k}\right) - \hat{k}f'\left(\hat{k}\right)\right]E = w. \tag{1.6}$$

In view of $f'' < 0$, a $\hat{k}$ satisfying (1.5) is unique. Call it $\hat{k}^*$, i.e, $f'(\hat{k}^*) = r + \delta$.

Comment: Since the exogenous rate of interest, r, faced by our SOE is assumed constant, $\hat{k}^*$ is constant over time.

b) With $\hat{k} = \hat{k}^*$ (1.6) gives

$$w_t = \left[f\left(\hat{k}^*\right) - \hat{k}^* f'\left(\hat{k}^*\right) \right] E_t = \left[f\left(\hat{k}^*\right) - \hat{k}^* f'\left(\hat{k}^*\right) \right] e^{gt}. \tag{1.7}$$

Hence, w_t grows at the same constant rate, g, as technology. We see that $w_0 = f\left(\hat{k}^*\right) - \hat{k}^* f'\left(\hat{k}^*\right)$, allowing us to write

$$w_t = w_0 e^{gt}. \tag{1.8}$$

c) The budget deficit is total expenditure (the sum of interest payments, transfers and expenditure on goods and services) minus gross taxes, i.e., $rB_t + O_t + G_t - \tilde{T}_t$. Since there is no financing by money creation,

$$\dot{B}_t = rB_t + O_t + G_t - \tilde{T}_t. \tag{1.9}$$

d) The primary surplus is

$$\begin{aligned} S_t &= \tilde{T}_t - (G_t + O_t) \\ &= \tau(w_t L + \alpha w_t (N - L)) - \gamma(N - L) w_t - \alpha w_t (N - L) \\ &= \left[\tau L + (\tau\alpha - \gamma - \alpha)(N - L)\right] w_t \\ &= \left\{\left[(1 - \alpha)\tau + \gamma + \alpha\right] L - (\alpha(1 - \tau) + \gamma) N\right\} w_0 e^{gt} \\ &= S_0 e^{gt}. \end{aligned} \tag{1.10}$$

Therefore, the growth rate of the primary surplus is

$$\frac{\dot{S}_t}{S_t} = g. \tag{1.11}$$

We see that

$$\frac{\partial S_t}{\partial L} = \left[(1 - \alpha)\tau + \gamma + \alpha\right] w_0 e^{gt} > 0. \tag{1.12}$$

Given N, a larger labour force, L, reflects a smaller number of retired people. This results in larger tax revenue and less need for transfers and elder care. Hence, the primary surplus ends up larger.

e) In view of (1.1), $Y_t = e^{gt}Lf(\hat{k}^*)$ so that

$$Y_t = Y_0 e^{gt}. \tag{1.13}$$

This shows that the rate of growth of GDP is g. Hence, the assumption $r > g$ tells us that the rate of interest is for ever larger than the GDP growth rate. We know that in this case, the current fiscal policy is sustainable (the government remains solvent) only if the government NPG is fulfilled, i.e.,

$$\lim_{t\to\infty} B_t e^{-rt} \leq 0. \tag{NPG}$$

And this is fulfilled if and only if the intertemporal government budget constraint,

$$\int_0^\infty (G_t + O_t)e^{-rt}dt \leq \int_0^\infty \tilde{T}e^{-rt}dt - B_0, \tag{IGBC}$$

is satisfied. This inequality is equivalent to

$$\int_0^\infty S_t e^{-rt}dt \geq B_0.$$

Inserting (1.10) gives

$$\int_0^\infty S_0 e^{-(r-g)t}dt \geq B_0. \tag{IGBC'}$$

The minimum size of the initial primary surplus consistent with fiscal sustainability is that S_0 which satisfies (IGBC') with strict equality:

$$\int_0^\infty \bar{S}_0 e^{-(r-g)t}dt = B_0 \qquad \Rightarrow$$

$$\frac{\bar{S}_0}{r-g} = B_0, \qquad \text{or}$$

$$\bar{S}_0 = (r-g)B_0 > 0. \tag{1.14}$$

Comment: Since $B_0 > 0$ and $r > g$, sustainability requires a positive primary surplus.

An alternative way of proving (1.14) is shown in the appendix.

f) From now it is assumed that $S_0 = \bar{S}_0$. Therefore, in view of (1.10) and (1.14) we have $S_0 =$

$$\{[(1-\alpha)\tau + \gamma + \alpha]L - (\alpha(1-\tau) + \gamma)N\}w_0 = (r-g)B_0 \qquad \Rightarrow$$

$$[L + \alpha(N-L)]\tau - (\alpha+\gamma)(N-L) = \frac{(r-g)B_0}{w_0} \qquad \Rightarrow$$

$$\tau = \frac{\frac{(r-g)B_0}{w_0} + (\alpha+\gamma)(N-L)}{L + \alpha(N-L)}, \tag{1.15}$$

a number in the interval (0,1).

Comment: It is seen that a higher initial debt implies a higher required tax rate (when $r > g$) in order to keep government net wealth (the right-hand side of (IGBC)) unchanged. On the other hand, given N, a larger labour force implies a lower required tax rate. This is because a larger labour force reflects a smaller number of retired people - a lower dependency ratio.

g) The debt-income ratio is $b_t \equiv B_t/Y_t$. Log-differentiation w.r.t. t gives

$$\frac{\dot{b}_t}{b_t} = \frac{\dot{B}_t}{B_t} - \frac{\dot{Y}_t}{Y_t} = \frac{rB_t - S_t}{B_t} - g = r - g - \frac{S_t}{B_t}, \quad \text{so that}$$

$$\dot{b}_t = (r-g)b_t - \frac{S_t}{Y_t}, \quad \text{or}$$

$$\dot{b}_t - (r-g)b_t = -s, \qquad \text{for } t \geq 0, \tag{1.16}$$

where

$$s \equiv \frac{S_t}{Y_t} = \frac{S_0 e^{gt}}{Y_0 e^{gt}} = \frac{S_0}{Y_0} = \frac{(r-g)B_0}{Y_0} = (r-g)b_0, \tag{1.17}$$

in view of (1.14) and $S_0 = \bar{S}_0$. The linear differential equation (1.16) has the solution

$$b_t = (b_0 - b^*)e^{(r-g)t} + b^*, \tag{1.18}$$

where

$$b^* = \frac{s}{r-g} = b_0.$$

Hence,

$$b_t = b_0 \text{ for all } t \geq 0, \tag{1.19}$$

as illustrated in Fig. 1.1.

Comment: We see that the debt-income ratio stays constant for ever. Hence, the fiscal policy is sustainable. The general solution (1.18) shows that because $r > g$, the debt-income ratio explodes unless $b^* \geq b_0$. Since we consider the case where the primary surplus is not larger than necessary for sustainability, $b^* > b_0$ is excluded. Hence,we have strict equality between b^* and b_0.

h) For $t < t_1$,

$$L = \frac{p}{\omega + p}N,$$

whereas for $t \geq t_1$, the labour force is

$$L' = \frac{p'}{\omega + p'}N < L,$$

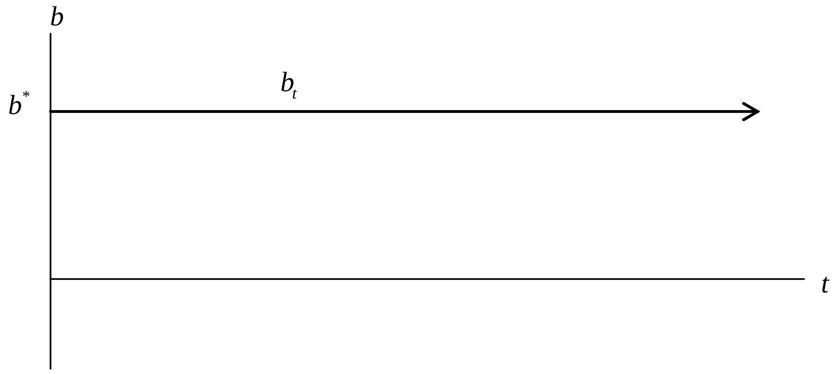


Figure 1.1:

where $p' < p$. That is, for $t \geq t_1$, life expectancy for a young person just entering the labour force will be $1/p'$, which is larger than life expectancy before the demographic change, $1/p$. Since N is unchanged, the birth rate must follow the death rate, i.e., for $t \geq t_1$, the birth rate is p'. With unchanged fiscal policy the reduction in the labour force at time t_1 results is a lower primary surplus S than otherwise (this follows from (1.12)). Fig. 1.2 illustrates. Hence

$$S_{t_1} < \bar{S}_{t1} \tag{1.20}$$

where $\bar{S}_{t_1}$ is the minimum size of the primary surplus at time t_1, determined according to the rule (1.11) as $\bar{S}_{t_1} = \bar{S}_0 e^{gt_1} = (r-g)B_0 e^{gt_1} = (r-g)B_{t_1}$.

From this follows intuitively that the current fiscal policy is not sustainable. A more formal approach to the derivation of this conclusion is the following. Application of the formula (1.10) with t_1 as initial time gives

$$S_t = S_{t_1} e^{g(t-t_1)} \text{ for } t \geq t_1.$$

Hence,

$$\int_{t_1}^{\infty} S_t e^{-r(t-t_1)} dt = \int_{t_1}^{\infty} S_{t_1} e^{g(t-t_1)} e^{-r(t-t_1)} dt = \frac{S_{t_1}}{r-g} < \frac{\bar{S}_{t_1}}{r-g}, \tag{1.21}$$

by (1.20). We see by (1.21) that the current fiscal policy implies that the present discounted value of future primary surpluses as seen from time t_1 is smaller than what is required for fiscal sustainability. Hence, the current fiscal policy is not sustainable.

Another approach to this question could be to consider the time path of the debt-income ratio for $t \geq t_1$. Application of the formula (1.16) with t_1 as initial time gives

$$\dot{b}_t - (r-g)b_t = -s', \quad \text{for } t \geq t_1, \tag{1.22}$$

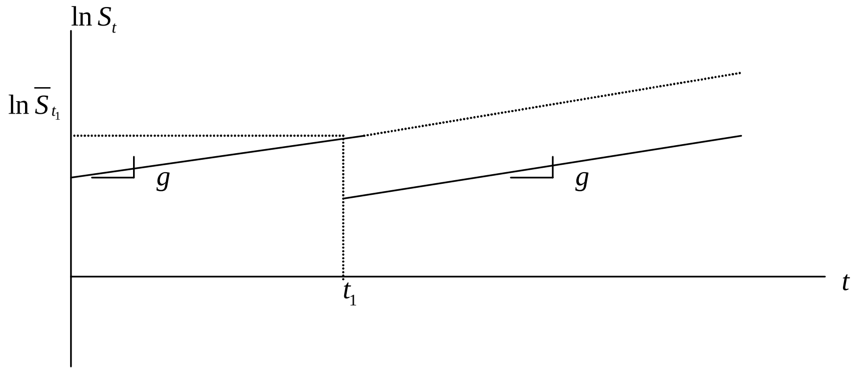


Figure 1.2:

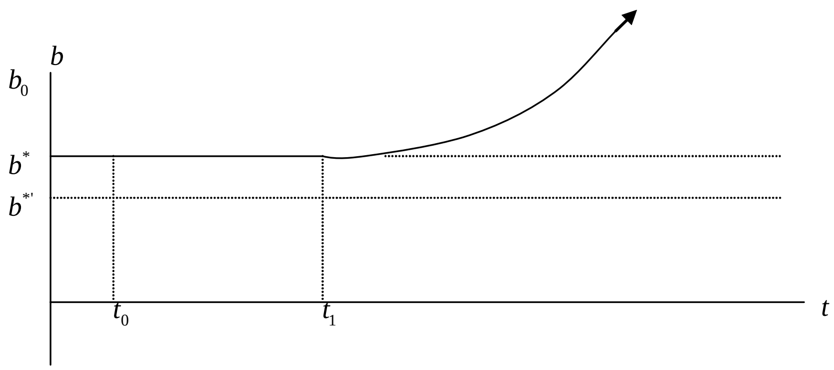


Figure 1.3:

where, in analogy with (1.17),

$$s' \equiv \frac{S_{t_1} e^{g(t-t_1)}}{Y_{t1} e^{g(t-t_1)}} = \frac{S_{t_1}}{Y_{t1}} < \frac{\bar{S}_{t_1}}{Y_{t1}} = s.$$

The solution of (1.22) is

$$b_t = (b_{t_1} - b^{*\prime}) e^{(r-g)(t-t_1)} + b^{*\prime}, \tag{1.23}$$

where

$$b^{*\prime} = \frac{s'}{r-g} < \frac{s}{r-g} = b^* = b_{t_1}.$$

In view of this inequality and the fact that $r > g$, (1.23) shows that $b_t \to \infty$ for $t \to \infty$; Fig. 1.3 illustrates. Hence, this approach ends up with the same conclusion as the previous approach: the current fiscal policy is not sustainable.

i) Let τ' denote the minimum size of the (constant) tax rate required for fiscal sustainability from time t_1, assuming γ and α to be unchanged for ever and no change in taxation before time t_1 (Policy I). Using the formula (1.15) (with initial time equal to t_1 instead of 0) gives

$$\tau' = \frac{\frac{(r-g)B_{t_1}}{w_{t_1}} + (\alpha + \gamma)(N - L')}{L' + \alpha(N - L')} > \tau, \tag{1.24}$$

where the inequality is a consequence of $L' < L$. Comment: With unchanged "welfare arrangements" (γ and α), the higher dependency ratio caused by lower L implies a higher required tax rate.

j) Given $\dot{b}_t = -c$, the path of b_t for $t \geq t_0$ is

$$b_t = b_{t_0} + \int_{t_0}^{t} \dot{b}_\tau d\tau = b_{t_0} - c(t - t_0),$$

If we require $b_{t_1} = 0$, the needed value of c satisfies

$$0 = b_{t_1} = b_{t_0} - c(t_1 - t_0), \quad \text{or}$$
$$c = \frac{b_{t_0}}{t_1 - t_0} = \frac{b_{t_0}}{35} = \frac{b_0}{35}.$$

k) For $t \geq t_1$, we let γ and α be back at their pre t_0 level and let the tax rate take the minimum value, τ'', now needed to obtain fiscal sustainability from time t_1 (Policy II). Then, with $B_{t_1} = 0$ in the formula (1.24) we get

$$\tau'' = \frac{(\alpha + \gamma)(N - L')}{L' + \alpha(N - L')} < \tau'. \tag{1.25}$$

Comment: the inequality is due to the fact that when $r > g$, the required tax rate is an increasing function of the initial level of government debt.

l) Policy III is the following alternative policy: let τ, γ and α stay at their pre t_0 level for ever and at time t_1 the retirement rate ω is adjusted such that fiscal sustainability is obtained. The primary surplus, S, at time t_1 is a function of ω. Hence, the required ω must be such that $S_{t_1} = \bar{S}_{t_1}$. Calling the required labour force L'', the equation $S_{t_1} = \bar{S}_{t_1}$ is satisfied if and only if $L'' = L$, i.e.,

$$
\begin{aligned}
L'' &= \frac{p'}{\omega' + p'} N = L = \frac{p}{\omega + p} N, \quad \text{or} \\
\omega' &= \frac{N - L}{L} p' < \frac{N - L}{L} p = \omega, \quad \text{or} \\
\omega' &= \frac{p'}{p} \omega.
\end{aligned}
$$

m) We now compare Policy II and Policy III w.r.t. the implied intergenerational "burden" and "benefit" distributions under the assumption $\tau'' > \tau$.

Policy II. Currently young generations will bear part of the costs of the adjustment since when becoming older they face higher taxation. The currently old bear another part of the cost of the adjustment, since they get lower transfers and less welfare and health services. Future generations bear the remaining part of the costs since they face a higher tax rate. But in contrast to current generations they also get the benefit of higher life expectancy and a longer period as retired. In this sense Policy II is favorable to future generations.

Policy III. Current generations bear no costs and get no benefits, future generations bear the cost in the form of later retirement, and they get the benefit of higher life expectancy. One might argue that this seems a more fair policy.

2. Solution to Problem 2

For convenience, the model is repeated here. Given the function $D(Y_t, R_t, \frac{eP^*}{P}, G, F)$, where $0 < D_Y < 1, D_R < 0, D_{\frac{eP^*}{P}} > 0, 0 < D_G < 1$ and $0 < D_F < 1$, we have

$$Y_t^d = D(Y_t, R_t, \frac{eP^*}{P}, G, F), \tag{2.1}$$

$$\dot{Y}_t = \lambda(Y_t^d - Y_t), \qquad \lambda > 0, \tag{2.2}$$

$$i_t = i^*, \tag{2.3}$$

$$\frac{M_t}{P} = L(Y_t, i_t), \qquad L_Y > 0,\ L_i < 0, \tag{2.4}$$

$$R_t = 1/Q_t, \tag{2.5}$$

$$\frac{1 + E_t\dot{Q}_t}{Q_t} = r_t, \tag{2.6}$$

$$r_t \equiv i_t - E_t\pi_t. \tag{2.7}$$

a) Evidently, the model is a dynamic IS/LM model (Blanchard's) extended to a SOE with a fixed exchange rate. It is a short-run model, since the price level P is an exogenous constant. Equation (2.1) gives aggregate output demand, which naturally depends negatively on the long-term interest rate R (high R means high costs of investment) and positively on the real exchange rate eP^*/P (an indicator of competitiveness). Equation (2.2) says that the adjustment of output to demand takes time; the parameter λ is the speed of adjustment. Equation (2.3) says that the short-term nominal interest rate, i, equals the foreign short-term nominal interest rate. This is a no-arbitrage condition, given the fixed exchange rate and the perfect capital mobility.

Equation (2.4) expresses equilibrium at the "money market". Naturally, real money demand depends positively on Y (the "transaction motive", Y is a proxy for the number of transactions per time unit) and negatively on the short-term nominal rate of interest, the opportunity cost of holding money.

The inverse relation between the long-term interest rate and the market value of a long-term bond in equation (2.5) comes from the definition of the long-term rate as the internal rate of return on a consol paying one unit of account (the output good) per time unit for ever. This internal rate of return is the solution in R_t to

$$Q_t = \int_t^\infty e^{-R_t(s-t)} ds.$$

Since this integral is $1/R_t$, we get (2.5). Equation (2.6) is a no-arbitrage condition saying that, absent uncertainty, the rate of return on the long-term bond is at any time equal to the rate of return on the short-term bond. Finally, equation (2.7) defines r_t as the short-term nominal rate of interest minus the expected rate of inflation.

Expectations are rational, there is no uncertainty and no speculative bubbles.

b) The assumption of rational expectations (here perfect foresight) implies $E_t\dot{Q}_t = \dot{Q}_t$. Since the price level P is an exogenous constant in the model, we have $E_t\pi_t = \pi_t = 0$ for all t. Therefore, equation (2.7) reduces to $r_t = i_t = i^* > 0$, in view of (2.3). Further, (2.5) gives $Q_t = R_t^{-1}$ so that $\dot{Q}_t = d(R_t^{-1})/dt = -R_t^{-2}\dot{R}_t$. Together with (2.6) this entails

$$R_t - \dot{R}_t/R_t = r_t = i^*.$$

Ordering gives

$$\dot{R}_t = (R_t - i^*)R_t. \tag{2.8}$$

The other differential equation is immediately obtained from (2.2), which can be written

$$\dot{Y} = \lambda(D(Y, R; \frac{eP^*}{P}, G, F) - Y) \tag{2.9}$$

The differential equations (2.8) and (2.9) in R and Y constitute the dynamic system of the model.

c)

To draw the corresponding phase diagram, note that (2.8) implies

$$\dot{R} = 0 \quad \text{for} \quad R = i^*.$$

Hence, the $\dot{R} = 0$ locus (the "LM curve") is horizontal, cf. Fig. 2.1. Similarly, (2.9) implies

$$\dot{Y} = 0 \quad \text{for} \quad D(Y, R; \frac{eP^*}{P}, G, F) = Y. \tag{2.10}$$

Totally differentiating this gives $D_Y dY + D_R dR = dY$, implying

$$\frac{dR}{dY}\Big|_{\dot{Y}=0} = \frac{1 - D_Y}{D_R} < 0. \tag{2.11}$$

It follows that the $\dot{Y} = 0$ locus (the "IS curve") is downward-sloping as shown in Fig. 2.1. The figure also shows the direction of movement in the different regions, as determined by (2.8) and (2.9). We see that the steady state point, E, is a saddle point.[1] This implies that

[1] More formally, the determinant of the Jacobian matrix for the right hand sides of the two differential equations, evaluated in the steady state point $(\bar{Y}, \bar{R})$, is $\bar{R}\lambda(D_Y - 1) < 0$.

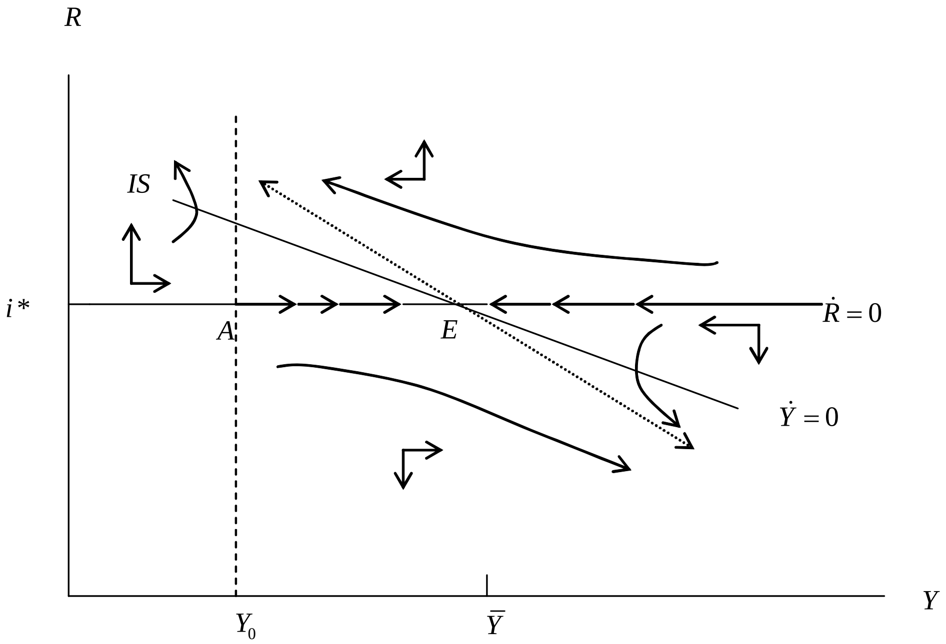


Figure 2.1:

two and only two solution paths – one from each side – converges towards E. These two saddle paths coincide with the $\dot{R} = 0$ locus. Since Y is (in this model) a predetermined variable, and R is a jump variable, the steady state is saddle-point stable.

At time $t = 0$, the economy must be somewhere on the vertical line $Y = Y_0$. In view of the absence of speculative bubbles, the explosive or implosive paths of Q in Fig. 2.1 cannot arise. Hence, we are left with the saddle path, the path AE in Fig. 2.1, as the unique solution to the model.

d) In steady state

$$R_t = \bar{R} = i^*, \tag{2.12}$$

and $Y = \bar{Y}$, where $(\bar{Y}, i^*)$ satisfies (2.10). Hence, inserting $(\bar{Y}, i^*)$ into (2.10), this equation defines $\bar{Y}$ as an implicit function of i^* and the other exogenous variables, $\bar{Y}$ $= \phi(i^*, \frac{eP^*}{P}, G, F)$. To find the partial derivative of $\bar{Y}$ w.r.t. G and F, respectively, we totally differentiate the equation (2.10) to get

$$D_Y dY + D_G dG + D_F dF = dY.$$

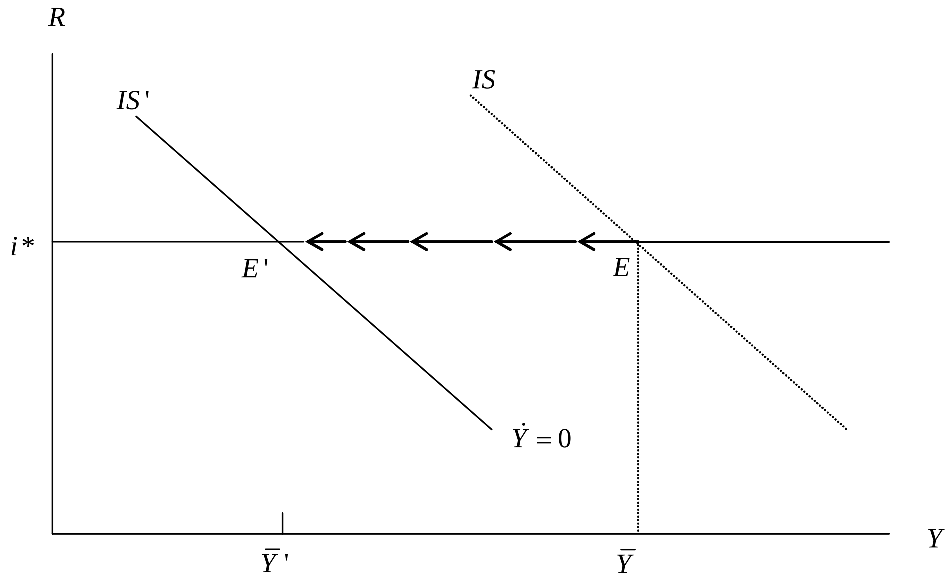


Figure 2.2:

This gives

$$\frac{\partial \bar{Y}}{\partial G} = \frac{D_G}{1 - D_Y} > 0, \quad \text{and}$$
$$\frac{\partial \bar{Y}}{\partial F} = \frac{D_F}{1 - D_Y} > 0. \tag{2.13}$$

e) In view of (2.13), the downward shift in the government budget deficit, F, at time t_0 moves the IS curve (the $\dot{Y} = 0$ locus) to the left. The lower output demand results in a gradual decline in production, which further lowers demand and so on. The system approaches the new steady state at E' in Fig. 2.2.

Figures 2.3 - 2.5 show the time profiles of F, Y, M, R and r. After t_1 money supply gradually adjusts downward along with the decreasing output. The fall in output implies lower money demand because the amount of transactions is lower. The mechanism is that the lower money demand generates an incipient tendency for the short-term nominal interest rate, i, to decrease. But this tendency is immediately counteracted by the decline in M due to citizens converting home currency into foreign currency in order to take advantage of a higher foreign interest rate. Fig. 2.5 illustrates that r and R remain unaffected by the decrease in output and money demand. This is due to the no-arbitrage condition (2.3) and the fact that the foreign short-term nominal interest rate i^* remains an unchanged exogenous constant.

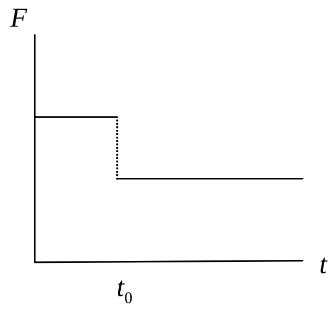
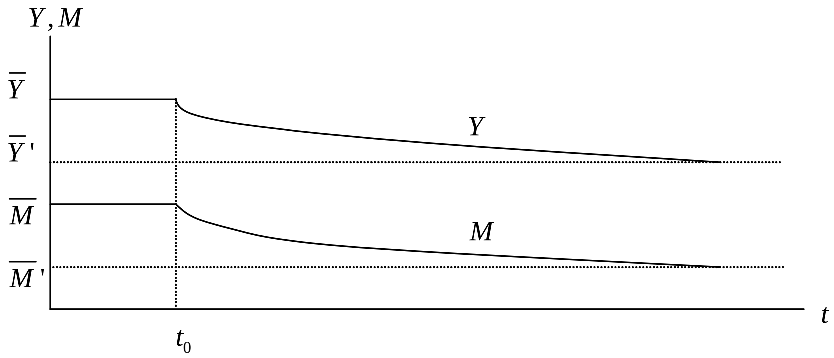


Figure 2.3:



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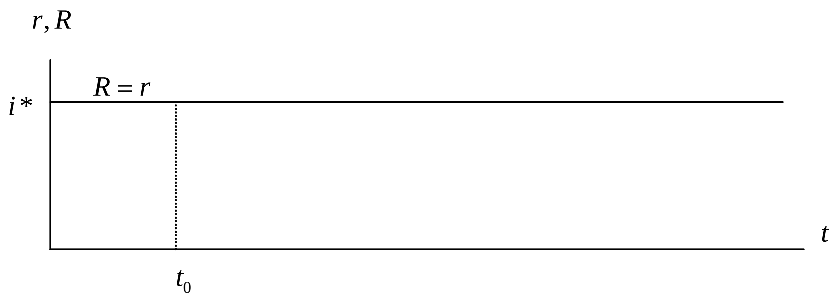

Figure 2.5:

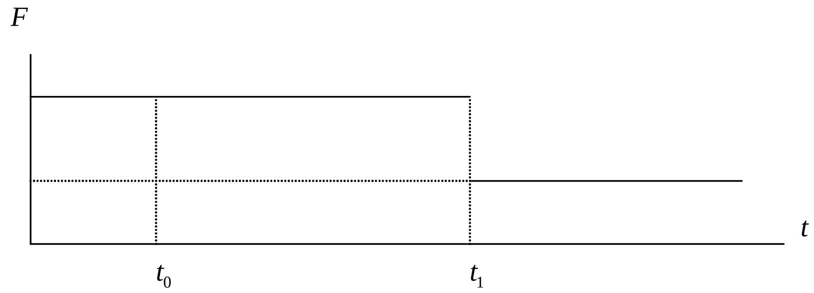

Figure 2.6:

f) Fig. 2.6 illustrates that at time t_0 people anticipate a reduction in the government budget deficit to take place at time t_1. But nothing happens until the fall in F actually takes place. This is because the anticipation itself does not affect the forward-looking variable, R, which remains unchanged. The explanation for this is that the foreign short-term nominal interest rate i^* remains unchanged, hence implying unchanged domestic short-term nominal interest rate i. Since there is no anticipating response in the time interval (t_0, t_1), the phase diagram in Fig. 2.2 still describes the (Y, R) dynamics. Only the interpretation is now that the diagram depicts what happens for $t \geq t_1$. Similarly, the time profiles of Y, M, r and R in Fig. 2.4 and Fig. 2.5 are still valid if we interpret t_0 in the figures as t_1.

3. Solution to Problem 3

The menu cost theory was introduced in the 1980's as an attempt at providing some micro foundation for the Keynesian presumption that nominal prices and wages are rigid in the short run when demand changes. For simplicity, here we shall talk mostly on prices and price-setting firms. The idea is that there are fixed costs ("menu costs") associated with changing prices. Hence, firms change prices less often than otherwise.

The important theoretical insight in the menu cost theory is that *small* menu costs can be enough to prevent firms from changing their price in response to a change in demand. This is because the gross loss by not changing price is only of second order, i.e., "small"; this is due to the envelope theorem. But, due to monopolistic competition, the effect on output, employment and welfare of not changing price is of first order, i.e., "large".

To be more specific, consider the Blanchard-Kiyotaki model for a closed economy with monopolistic competition. The profit function of firm i is written

$$V_i = V_i(P_i, P, W, M),$$

where P_i = output price of firm i, P = general price level, W = general wage level and M = money supply. Facing a downward sloping demand curve, firm i chooses P_i with a view to the maximization of profit. Suppose that initially, $P_i = P_i^*$, where P_i^* is the price that maximizes V_i.

Let money supply shift to the new level $M' > M$. Initially, suppose no other agents change price (or wage). Then P and W are unchanged. In this situation the gross loss to firm i by not changing price tends to be small because we have:

$$\begin{aligned}\frac{dV_i}{dM}(P_i^*, P, W, M) &= \frac{\partial V_i}{\partial P_i^*}(P_i^*, P, W, M)\frac{\partial P_i^*}{\partial M} + \frac{\partial V_i}{\partial M}(P_i^*, P, W, M) \\ &= \frac{\partial V_i}{\partial M}(P_i^*, P, W, M),\end{aligned}$$

since the first term vanishes at the profit optimum $-$ the profit curve is flat at the maximizing price P_i^*. An illustration is shown in Fig. 3.1, where V_i^* denotes the maximized profit, i.e., $V_i^* = V_i(P_i^*, P, W, M)$. The result reflects a general principle, called the envelope theorem: in an interior optimum, the total derivative of a maximized function w.r.t. a parameter is equal to the partial derivative w.r.t. that parameter. Hence, the effect of a change i M on the profit is approximately the same (to a first order) whether or not the firm adjusts its price. Indeed, due to the envelope theorem, for an infinitesimal change in M, the profit of firm i is the same whether or not the firm adjusts its price optimally in response to the change in M. For *finite* changes in M this is so only approximately. That is, the gross loss by not changing price is "of second order", i.e., proportional to $(\Delta M/M)^2$, a very small number, when $|\Delta M/M|$ is small. Therefore, in view of the menu cost, say c, it may be advantageous not to change price. Indeed, the net gain ($= c -$ gross loss) by not changing price may be positive. The other firms are in a similarly situation so that no change in the general price level may be an equilibrium. But the effects on output, employment and social welfare of not changing price are "of first order", that is, proportional to $|\Delta M/M|$. This is because neither output, employment or social welfare is maximized in the initial equilibrium (under monopolistic competition).

A similar analysis applies to the wage-setting households (or crafts-unions) in the B-K model. Each worker faces a downward-sloping demand curve for her specific type of labour and each worker sets the utility maximizing wage level taking this into account

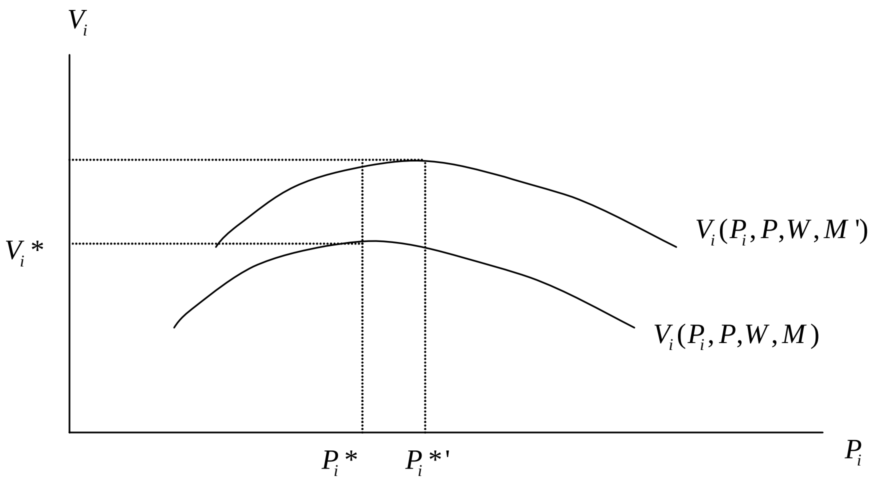

Figure 3.1:

and supplies the amount of labour demanded at that wage level. In case of a demand shock, the gross loss of not changing the wage is an increasing function of the elasticity of marginal disutility of labour. And numerical calculations for realistic parameter values tell us that a rather low elasticity of marginal disutility of labour is needed for the gross loss to be sufficiently small such that menu costs, which are inherently small, are indeed operative. But a low elasticity of marginal disutility of labour is synonymous with a high elasticity of labour supply w.r.t. the real wage. Microeconometric studies of labour supply tell us that even the compensated elasticity of labour supply w.r.t. the real wage is quite small. And if wages are adjusted, then it becomes more costly for firms not to change price. Therefore, in the Blanchard-Kiyotaki framework the menu cost theory is not really capable at providing the desired result.

It does not help to assume that workers are wage takers (perfect competition at the labour market). In that case, with low wage elasticity of labour supply, higher employment requires a considerably higher real wage. And then the gross loss to the firm by not changing price becomes non-negligible.

But there is a way out. In the Blanchard-Kiyotaki model with monopolistic competition at the labour market there is under-employment. But there is no *involuntary unemployment*. If instead we model the labour market in accordance with efficiency wage theory or collective bargaining theory, then involuntary unemployment arises. That implies that employment can easily change without much change in the wage level. That is, the elasticity of *effective* aggregate labour supply becomes much larger than that of individual labour supply as measured in the microeconometric studies of labour supply.

The conclusion is that in a model where the output market is dominated by monopolistic competition, but the labour market is governed by the principles of the efficiency wages or collective bargaining, menu costs can realistically be thought to be operative. In such a context the menu cost theory can be defended as a potent first approach to the explanation of nominal rigidities.

4. Solution to Problem 4

a) True. When $k^* < k_{GR}$, it is impossible to increase aggregate consumption in one period without decreasing aggregate consumption in another period.

b) True. In the Blanchard OLG model the consumption function for the individual (as well as at the aggregate level) is

$$c_t = (\rho + p)(a_t + h_t), \tag{4.1}$$

where ρ and p are the pure rate of time preference and the death rate, respectively (both constant), and a_t denotes financial wealth, whereas h_t is present discounted value of future labour income (the result (4.1) is due to the assumption of log utility). An increase in the rate of interest, r, makes future consumption cheaper as seen from "now". Hence there is a negative *substitution effect* on current consumption c_t. At the same time, for a given intertemporal budget (i.e., given $a_t + h_t$), an increase in r makes it possible to consume more at any time (the present discounted value of a given consumption plan becomes smaller). This is the positive *income effect* on current consumption. By (4.1) we see that these two effects exactly cancel each other. Of course, an increase in r affects h_t negatively, because h_t is discounted by r. But in our macroeconomic terminology, this is not an income infect, but a *wealth effect.* It is negative, implying that the

$$\text{total effect} = \text{substitution effect} + \text{income effect} + \text{wealth effect}$$

is negative. But still, the substitution effect and the income effect exactly offset each other (due to the log-utility function assumed in the model). (In microeconomics, the sum of our income effect and wealth effect is sometimes called the "total income effect", though in the present case they have opposite signs.)

c) Not true. The first Fischer model (that with synchronous wage setting) in the Blanchard (1990) text is an obvious counter example. In that model only unanticipated changes in money supply have real effects (and in fact these effects last only one period).

5. Appendix: Alternative derivation of (1.14)

The general requirement for fiscal sustainability is that

$$\lim_{t \to \infty} b_t < \infty. \tag{5.1}$$

Consider the differential equation (1.16) with

$$s \equiv \frac{S_t}{Y_t} = \frac{S_0 e^{gt}}{Y_0 e^{gt}} = \frac{S_0}{Y_0} \equiv s_0.$$

The solution is

$$b_t = (b_0 - b^*)e^{(r-g)t} + b^*,$$

where

$$b^* = \frac{s_0}{r-g}.$$

Since $r - g > 0$, the condition (5.1) requires $b^* \geq b_0$, i.e., $s_0 \geq (r-g)b_0$, or

$$S_0 \geq (r-g)B_0.$$

The lowest S_0 satisfying this is the $\bar{S}_0$ we are looking for. Hence,

$$\bar{S}_0 = (r-g)B_0.$$

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