Written exam for the M. Sc. in Economics Winter 2005

Advanced Macroeconomics

January 7, 2005

Four hours. No auxiliary material

To be answered in Danish or English[1]

The weighting of the problems is:

Problem 1: 40 %, Problem 2: 30 %, Problem 3: 20 %, Problem 4: 10 %.

Problem 1. We shall focus on the problems of welfare arrangements and fiscal sustainability in an "aging society". Consider a small open economy (henceforth SOE) with a government sector. For simplicity, assume:

1. Perfect financial capital mobility across borders.
2. Domestic and foreign financial claims are perfect substitutes.
3. No labour mobility across borders.
4. No uncertainty.
5. Perfect competition on all markets.

There is at the world market for financial capital a constant (real) rate of interest $r > 0$. The SOE has (adult) population equal to N and a labour force equal to L, where both N and L are constant. Due to retirement we have $L < N$. The technology of the representative firm is given by

$$Y_t = F(K_t, E_t L) \equiv E_t L f(\hat{k}_t),$$

where $\hat{k}_t \equiv K_t/(E_t L)$, F is a neoclassical production function with CRS, and Y_t and K_t are output and capital input, respectively. The whole labour force is employed. We treat time t as continuous, and the time unit is one year. The symbol E_t represents a technology factor ("E" for "efficiency of labour") growing at the constant rate $g > 0$, that is, $E_t = e^{gt}$, by choosing measurement units such that $E_0 = 1$. There are no capital adjustment costs. The rate of physical capital depreciation is $\delta \geq 0$ and is constant. Firms maximize profit.

a) Find an expression showing how the capital intensity $\hat{k}$ chosen by the firm is determined. Comment.

[1] You are also allowed to write in Swedish or Norwegian.

b) Show how the equilibrium real wage w_t is determined and that it can be written $w_t = w_0 e^{gt}$.

Let G_t denote government spending on goods and services. Suppose G_t is primarily eldercare including health services. Specifically, assume

$$G_t = \gamma(N - L)w_t, \qquad \gamma > 0,$$

where the factor of proportionality, γ, is a constant. Let O_t denote transfer payments including pensions. Assume

$$O_t = \alpha w_t(N - L), \qquad 0 < \alpha < 1,$$

where α is the "degree of compensation", a constant. Further, let $\tilde{T}_t$ denote gross tax revenue and assume

$$\tilde{T}_t = \tau(w_t L + O_t), \qquad 0 < \tau < 1,$$

where the tax rate τ is constant (capital income taxation, consumption taxes etc. are ignored). Finally, let B_t denote real public debt and assume that the budget deficit (whether positive or negative) is exclusively financed by changes in B (no money financing). Initial debt, B_0, is positive.

c) Write down an equation describing how the budget deficit and the increase per time unit in public debt are linked.

d) Determine the primary surplus, S_t, and its growth rate. How does S_t depend on L?

Assume $r > g$. Let $\bar{S}_0$ denote the minimum size of the initial primary surplus consistent with fiscal sustainability.

e) Find $\bar{S}_0$. What is the sign of $\bar{S}_0$? Comment. *Hint:* If a is a positive constant, then $\int_{t_0}^{\infty} e^{-at}dt = \frac{1}{a}$.

From now, suppose $S_0 = \bar{S}_0$.

f) Find τ. Comment.

g) Determine the path over time of the debt-income ratio $b_t \equiv B_t/Y_t$. Illustrate the time profile of b_t in a diagram. Comment. *Hint:* The differential equation $\dot{x}+ax = c$, where a and c are constants, $a \neq 0$, has the solution $x_t = (x_{t_0} - x^*)e^{-a(t-t_0)} + x^*$, where $x^* = \frac{c}{a}$.

Suppose that, in analogy with the Blanchard OLG model with age-dependent labour supply,

$$L = \frac{p}{\omega + p}N,$$

where ω is a constant "retirement rate" (prescribed by law), and p is a constant "death rate", so that $1/p$ is a rough indicator of "life expectancy", i.e., expected life time (as adult). As a crude representation of the much debated supposed increase in life expectancy of future generations, imagine that the government at time $t_0 > 0$ becomes aware that from time $t_1 = t_0 + 35$ years, life expectancy for a young person just entering the labour force will be $1/p'$ instead of $1/p$, where $p' < p$ (of course, in the real world this demographic change will not be a once for all change, but a gradual change, but for simplicity this is ignored). Population size remains equal to the constant N.

h) In a diagram draw the time profile of $\ln S_t$ as it would be in case there is no change in fiscal policy. Is the current fiscal policy sustainable? *Hint:* Consider either the present discounted value of future primary surpluses as seen from time t_1 or the time path of the debt-income ratio.

Let τ' denote the minimum size of the (constant) tax rate required for fiscal sustainability from time t_1, assuming γ and α to be unchanged for ever and no change in taxation before time t_1 (Policy I).

i) Find τ'. Determine the sign of $\tau' - \tau$. Comment.

Now assume instead that at time t_0 the government decides to incur a budget surplus (including interest payments) until time t_1 such that the debt-income ratio in the time interval (t_0, t_1) gradually falls according to

$$\dot{b}_t = -c,$$

where c is a positive constant large enough such that at time t_1 one has $b = 0$. The plan is to accomplish this not by changing τ, but by temporary and gradual adjustments of γ and/or α.

j) Find the required value of c. *Hint:* If $\dot{x} = a$, a constant, then $x_t = x_{t_0} + \int_{t_0}^{t} \dot{x}_\tau d\tau = x_{t_0} + a(t - t_0)$.

Further, the plan is, for $t \geq t_1$, to let γ and α be back at their pre t_0 level and to let τ take the minimum value, τ'', now needed to obtain fiscal sustainability from time t_1 (Policy II).

k) Find τ''. Determine the sign of $\tau'' - \tau'$. Comment.

Suppose that at time t_0 an alternative policy is proposed, namely to let τ, γ and α stay at their pre t_0 level forever and at time t_1 adjust ω such that fiscal sustainability is obtained (Policy III).

l) Find the required ω.

m) Assuming $\tau'' > \tau$, compare Policy II and Policy III w.r.t. the implied intergenerational "burden" and "benefit" distributions.

Problem 2. Consider a small open economy characterized by the features 1 - 4 in Problem 1, whereas feature 5 is replaced by

5a. Imperfect competition on output and labour markets.

Suppose the short-term behaviour of the economy can be approximately described by the following model in continuous time. Given the function $D(Y_t, R_t, \frac{eP^*}{P}, G, F)$, where $0 < D_Y < 1, D_R < 0, D_{\frac{eP^*}{P}} > 0, 0 < D_G < 1$, and $0 < D_F < 1$, the model is:

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\begin{aligned}
Y_t^d &= D(Y_t, R_t, \frac{eP^*}{P}, G, F), && (1)\\
\dot{Y}_t &= \lambda(Y_t^d - Y_t), \qquad \lambda > 0, && (2)\\
i_t &= i^*, && (3)\\
\frac{M_t}{P} &= L(Y_t, i_t), \qquad L_Y > 0,\ L_i < 0, && (4)\\
R_t &= 1/Q_t, && (5)\\
\frac{1 + E_t\dot{Q}_t}{Q_t} &= r_t, && (6)\\
r_t &\equiv i_t - E_t\pi_t. && (7)
\end{aligned}
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The endogenous variables are: $Y_t^d \equiv$ output demand, $Y_t \equiv$ output, $R_t \equiv$ long-term real interest rate, $i_t \equiv$ short-term nominal interest rate, $M_t \equiv$ money supply, $Q_t \equiv$ real price of a long-term bond (a consol), $r_t \equiv$ short-term real interest rate and $\pi_t \equiv$ (forward-looking) rate of inflation. The variables e, P^*, P, G, F and i^* are exogenous and constant; their interpretation is as follows: $e \equiv$ nominal exchange rate, $P^* \equiv$ foreign price level, $P \equiv$ domestic price level, $G \equiv$ government spending on goods and services, $F \equiv$ government budget deficit and $i^* \equiv$ foreign short-term nominal interest rate. The parameter λ is constant. The initial value, Y_0, of Y is given. The symbol E_t denotes expectation conditional on information available at time t. Expectations are rational, there is no uncertainty and no speculative bubbles.

a) Briefly interpret the model.

b) To characterize the movement over time of the economy, derive two differential equations in Y and R.

c) Construct the corresponding phase diagram and illustrate the path that the economy follows. Comment.

d) Determine the long-term real interest rate in steady state. How does Y in steady state depend on G and F, respectively?

Suppose that the economy has been in its steady state until time t_0.

e) At time t_0 an unanticipated downward shift in F occurs, but apart from this shift, everybody expects F to remain unchanged forever. Illustrate by a phase diagram and by graphical time profiles what happens to Y_t, M_t, r_t and R_t for $t \geq t_0$. Comment.

f) Assume instead that at time t_0, due to the foreseeable problems of fiscal sustainability arising in the "aging society", the government credibly announces a downward shift in F to take place at time $t_1 > t_0$. Illustrate by a phase diagram and by graphical time profiles what happens to Y_t, M_t, r_t and R_t for $t \geq t_0$. Comment. *Hint:* The answer to this question may be easier than one might immediately think.

Problem 3. Give a brief account of the main points of the menu cost theory. Doing so, relate your account to

- the concepts of "first-order effects" and "second-order effects";
- the problem that in models like the Blanchard-Kiyotaki model a low elasticity of marginal disutility of labor is needed to make menu costs operative;
- how this problem can in principle be overcome by modelling the labour market differently.

Problem 4. *Short questions*

a) Let $c_t \equiv$ consumption per member of the labour force, $k_t \equiv$ capital per member of the labour force and $k_{GR} \equiv$ the golden rule capital intensity. Consider the statement: "In the Diamond OLG model without technical progress, a technically feasible path $\{c_t, k_t\}_{t=0}^{\infty}$ such that for $t \to \infty$, $k_t \to k^* < k_{GR}$, must be dynamically efficient." True or not true? Comment.

b) "In the Blanchard OLG model, the income effect on current consumption of an increase in the rate of interest exactly offsets the substitution effect." True or not true? Comment.

c) "In any macro model with rational expectations wage setting in advance implies that not only unanticipated changes in money supply, but also anticipated changes, have real effects." True or not true? Comment.

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