Chapter 23

The open economy and alternative exchange rate regimes

In this chapter we consider simple open-economy members of the IS-LM family. Section 23.1 revisits the standard static version, the Mundell-Fleming model, in its fixed-exchange-rate as well as floating-exchange-rate adaptations. The Mundell-Fleming model is well-known from elementary macroeconomics and our presentation of the model is merely a prelude to the next sections which address dynamic extensions. In Section 23.2 we show how the *dynamic* closed-economy IS-LM model with rational expectations from the previous chapter can be easily modified to cover the case of a small open economy with fixed exchange rates. In Section 23.3 we go into detail about the more challenging topic of floating exchange rates. In particular we address the issue of exchange rate *overshooting*, first studied by Dornbusch (1976).

Both the original Mundell-Fleming model and the dynamic extensions considered here are *ad hoc* in the sense that the microeconomic setting is not articulated in any precise way. Yet the models are useful and have been influential as a means of structuring thinking about an open economy in the short run.

The models focus on short-run mechanisms in a small open economy. There is a "domestic currency" and a "foreign currency" and these currencies are traded in the foreign exchange market. In this market, nowadays, the volume of trade is gigantic.

The following assumptions are shared by the models in (most of) this chapter:

1. Free mobility across borders of financial capital (i.e., no barriers or restrictions on currency trade).

2. Domestic and foreign bonds are perfect substitutes and hence command the same expected rate of return.

3. Free mobility across borders of goods and services (i.e., no barriers or restrictions on trade in goods and services)

4. No mobility across borders of labor.

5. Domestic and foreign goods are imperfect substitutes.

6. Nominal prices are sluggish and follow an exogenous constant inflation path.

The two first assumptions together make up the case of *perfect mobility of financial capital.* As the two last assumptions indicate, we consider an economy with imperfectly competitive firms. In an asynchronous way, the firms adjust their prices when their unit costs change. The aggregate inflation rate is considered sticky.

We use the same notation as in the previous chapter, with the following clarifications and additions: Y is domestic output (GDP), P the domestic price level, P^* the foreign price level, i the domestic short-term nominal interest rate, i^* the foreign short-term nominal interest rate, and X the nominal exchange rate. Suppose UK is the "home country". Then the exchange rate X indicates the price in terms of British£ (GBP) for one US$ (USD), say. Be aware that a currency trading convention is to announce an exchange rate as, for example, "USD/GBP is X", in words: "USD through GBP is X". The intended meaning is that the exchange rate is X GBP per USD. In ordinary language (as well as in mathematics) a slash, however, means "per". Thus, writing "USD/GBP is X" ought to mean that the exchange rate is X USD per GBP, which is exactly the opposite. Whenever in this text we use a slash, it has this standard mathematical meaning. To counter any risk of confusion, when indicating an exchange rate, we therefore avoid using a slash altogether. Instead we use the unmistakable "per".

When reporting that "the exchange rate is X", our point of view is that of an *importer* in the home country. That is, if UK is the "home country", saying that "the exchange rate is X" shall mean that X GBP must be paid per $ worth of imports. And saying that "the real exchange rate is x" shall mean that x domestic goods must be paid per imported good. This convention is customary in continental Europe. Note however, that it is the opposite of the British convention which reports the home country's nominal and real "exchange rate" as $1/X$ and $1/x$, respectively.

When considering *terms of trade,* $1/x$, our point of view is that of an *exporter* in the home country. The terms of trade tell us how many foreign goods we get per exported good. In accordance with this, Table 21.1 gives a list of key open economy variables.

Table 21.1	**Open economy glossary**	
Term	*Symbol*	*Meaning*
Nominal exchange rate	X	The price of foreign currency in terms of domestic currency.
Real exchange rate	$x \equiv \frac{XP^*}{P}$	The price of foreign goods in terms of domestic goods (can be interpreted as an indicator of competitiveness).
Terms of trade	$1/x$	In this simple model terms of trade is just the inverse of the real exchange rate (generally, it refers to the price of export goods in terms of import goods).
Purchasing power parity		The nominal exchange rate which makes the cost of a basket of goods and services equal in two countries, i.e., makes $x = 1$.
Uncovered interest parity		The hypothesis that domestic and foreign bonds have the same expected rate of return, expressed in terms of the same currency.
Exports	E	
Imports	IM	
Net exports (in domestic output units)	N	$= E - xIM$.
Net foreign assets	A^f	
Net factor income from abroad (in domestic output units)	$rA^f + w^f L^f$	The present model has $L^f = 0$.
Current account surplus	CAS	$= N + rA^f + w^f L^f$. In this model $L^f = 0$.
Official reserve assets	ORA	
Private net foreign assets	A_p^f	$= A^f - ORA$.
Increase per time unit in some variable z	Δz	
Financial account surplus	FAS	$= -\Delta A_p^f - \Delta ORA = -CAS =$ current account deficit.
Net inflow of foreign exchange	$= CAS$	$= -FAS = -$ (net outflow of foreign exchange)

We simplify by talking of an exchange rate as if the "foreign country" constitutes the rest of the world and the exchange rate is thereby just a bilateral entity. A more precise treatment would center on the *effective exchange rate,* which is a trade-weighted index of the exchange rate vis-a-vis a collection of major trade partners.

23.1 The Mundell-Fleming model

Whether the Mundell-Fleming model is adapted to a fixed or floating exchange rate regime, there is a common set of elements.

23.1.1 The basic elements

Compared with the static closed-economy IS-LM model, the Mundell-Fleming model contains two new elements:

- An extra output demand component, namely a net export function $N(Y, x)$, where $x \equiv XP^*/P$ is the real exchange rate. As a higher income implies more imports, we assume that $N_Y < 0$. And as a higher real exchange rate implies better competitiveness, we assume that $N_x > 0$.[1]
- The uncovered interest parity condition (for short UIP). This says that domestic and foreign financial assets pay the same expected rate of return (measured in the same currency).

Apart from the addition of these open-economy elements, notation is as in the previous chapter. Output demand is given as

$$\begin{aligned} Y^d &= C(Y^p, r^e) + I(Y, r^e) + N(Y, x) + G + \varepsilon_D, \text{ where} \qquad (23.1) \\ 0 &< C_{Y^p} + N_Y < C_{Y^p} \leq C_{Y^p} + I_Y < 1, C_{r^e} + I_{r^e} \leq I_{r^e} < 0, N_x > 0, \end{aligned}$$

and ε_D is a demand shift parameter. This parameter could for instance reflect the level of economic activity in the world economy. Disposable income, Y^p, is

$$Y^p \equiv Y - \mathbb{T}, \tag{23.2}$$

where $\mathbb{T}$ is real net tax revenue (gross tax revenue minus transfers). We assume a quasi-linear tax revenue function

$$\mathbb{T} = \tau + T(Y), \qquad 0 \leq T' < 1, \tag{23.3}$$

[1] By assuming $N_x > 0$, it is presupposed that the Marshall-Lerner condition is satisfied, see Appendix A. Throughout the chapter we ignore that it may take one or two years for a rise in x to materialize as a rise in net exports. This is because the price of imports is immediately increased while the quantity of imports and exports only adjust with a time lag (the pattern known as the *J-curve effect*).

where τ is a constant representing the "tightness" of fiscal policy. This parameter, together with the level of public spending, G, describes fiscal policy.

Inserting (23.2) and (23.3) into (23.1), we can write aggregate demand as

$$\begin{aligned} Y^d &= D(Y, r^e, x, \tau) + G + \varepsilon_D, \quad \text{where} \qquad (23.4) \\ 0 &< D_Y = C_{Y^p}(1 - T') + I_Y + N_Y < 1, D_{r^e} = C_{r^e} + I_{r^e} < 0, \\ D_x &> 0, D_\tau = C_{Y^p} \cdot (-1) \in (-1, 0). \end{aligned}$$

The demand for money (domestic currency and checkable deposits in commercial banks) in the home country is, as in the closed economy model,

$$M^d = P \cdot (L(Y, i) + \varepsilon_L), \qquad L_Y > 0, \; L_i < 0, \qquad (23.5)$$

where i is the short-term nominal interest rate on the *domestic bond* which is denominated in the *domestic* currency. The symbol ε_L represents a shift parameter which may reflect a shock to liquidity preferences or the payment technology and thereby the money multiplier.

There is a link between r^e and i, namely $r^e = i - \pi^e$, where π^e denotes the expected value of π which is the domestic forward-looking inflation rate. Recall that with continuous interest compounding, the equation $r^e = i - \pi^e$ is an identity. In a discrete time framework the equation is a convenient approximation. Assuming clearing in the output market as well as the money market, we now have:

$$\begin{aligned} Y &= D(Y, i - \pi^e, X\frac{P^*}{P}, \tau) + G + \varepsilon_D, \qquad \text{(IS)} \\ \frac{M}{P} &= L(Y, i) + \varepsilon_L. \qquad \text{(LM)} \end{aligned}$$

In addition to the domestic short-term bond there is a short-term bond denominated in *foreign* currency, henceforth the *foreign bond.* The nominal interest rate on the foreign bond is denoted i^* and is exogenous, $i^* > 0$. The term "bonds" may be interpreted in a broad sense, including large firms' interest-bearing bank deposits.

The no-arbitrage condition between the domestic and the foreign bond is assumed given by the *uncovered interest parity* condition,

$$i = i^* + \frac{\dot{X}^e}{X}, \qquad \text{(UIP)}$$

where $\dot{X}^e$ denotes the expected increase per time unit in the exchange rate in the immediate future.[2] Imposing (UIP) amounts to assuming that arbitrage quickly

[2] As a preparation for the dynamic extensions to be considered below, we have presented the UIP condition in its continuous time version with continuous interest compounding. In discrete

brings the interest rate on the domestic bond in line with the expected rate of return on investing in the foreign bond, expressed in the domestic currency. This expected rate of return equals the foreign interest rate plus the expected rate of depreciation of the domestic currency.

By invoking the UIP condition the model assumes that asymmetric risk and liquidity aspects can be ignored in a first approximation. So domestic and foreign bonds are considered perfect substitutes. Hence only the expected rate of return matters. If we imagine that in the very short run there is, for example, a ">" in (UIP) instead of "=", then arbitrage sets in. A massive inflow of financial capital will occur (investors dispose of foreign assets and purchase domestic assets), until "=" in (UIP) is re-established. The adjustment will take the form of a lowering of i in case of a fixed exchange rate system. In case of a floating exchange rate system, the adjustment will take the form of an adjustment in X (generally both its level and subsequent rate of change). Only when (UIP) is satisfied, is the system at rest. The primary actors in the foreign exchange market are commercial banks, mutual funds, asset-management companies, insurance companies, exporting and importing corporations, and central banks.

The model assumes that (UIP) holds continuously (arbitrage in international asset markets is very fast). Thus, if for example the domestic interest rate is below the foreign interest rate, it must be that the domestic currency is expected to appreciate vis-à-vis the foreign currency, that is, $\dot{X}^e < 0$. The adjective "uncovered" refers to the fact that the return on the right-hand side of (UIP) is not guaranteed, but only an expectation. On the *covered interest parity*, see Appendix B.

The original Mundell-Fleming model is a static model describing just one short period with the price levels P and P^* set in advance. The model consists of the equations (IS), (LM), and (UIP) with the following partitioning of the variables:

- Exogenous: $P, P^*, G, \tau, \pi^e, i^*, \dot{X}^e, \varepsilon_D, \varepsilon_L$, and either X or M.
- Endogenous: Y, i, and either M or X, depending on the exchange rate regime.

We now consider the polar cases of fixed and floating exchange rates, respectively.

time with X_t denoting the exchange rate at the beginning of period t, the UIP condition reads: $1 + i_t = \frac{1}{X_t}(1 + i_t^*)X_{t+1}^e$, which can be written $1 + i_t = (1 + i_t^*)(1 + (X_{t+1}^e - X_t)/X_t) \approx 1 + i_t^* + (X_{t+1}^e - X_t)/X_t$, when i_t^* and $(X_{t+1}^e - X_t)/X_t$ are "small".

23.1.2 Fixed exchange rate

A fixed exchange rate regime amounts to a promise by the central bank to sell and buy unlimited amounts of foreign currency at an announced exchange rate. So X becomes an exogenous constant in the model.[3] The system requires that the central bank keeps foreign exchange reserves to be able to buy the domestic currency on foreign exchange markets when needed to maintain its value.

We assume that the announced exchange rate is at a sustainable level vis-a-vis the foreign currency so that credibility problems can be ignored. So we let $\dot{X}^e = 0$. Then (UIP) reduces to $i = i^*$, and output is determined by (IS), given $i = i^*$. Finally, through movements of financial capital the nominal money supply (hence also the real money supply) adjusts endogenously to the level required by (LM), given $i = i^*$ and the value of Y already determined in (IS). The system thus has a recursive structure.

There is no possibility of an independent monetary policy as long as there are no restrictions on movements of financial capital. The intuition is the following. Suppose the central bank naively attempts to stimulate output by buying domestic bonds, thereby raising the money supply. There will be an incipient fall in i. This induces portfolio holders to convert domestic currency into foreign currency to buy foreign bonds and enjoy their higher interest rate. This tends to raise X_t, however. Assuming the central bank abides by its commitment to a fixed exchange rate, the bank will have to immediately counteract this tendency to depreciation by *buying domestic assets* (domestic currency and bonds) for foreign currency in an amount sufficient to bring the domestic money supply down to its original level needed to restore both the exchange rate and the interest rate at their original values. That is, as soon as the central bank attempts expansionary monetary policy, it has to reverse it.

The model is qualitatively the same as the static IS-LM model for the closed economy with the nominal interest rate fixed by the central bank. The output and money multipliers w.r.t. government spending are, from (IS) and (LM) respectively,

$$\begin{aligned}\frac{\partial Y}{\partial G} &= \frac{1}{1-D_Y} = \frac{1}{1-C_{Y^p}(1-T')-I_Y-N_Y} > 1,\\ \frac{\partial M}{\partial G} &= PL_Y\frac{\partial Y}{\partial G} = PL_Y\frac{1}{1-C_{Y^p}(1-T')-I_Y-N_Y} > 0.\end{aligned}$$

The output multiplier can be seen as a measure of how much a unit increase in G raises aggregate demand and thereby stimulates production and income; indeed,

[3] In practice there will be a small margin of allowed fluctuation around the par value. The Danish krone (DKK) is fixed at 746.038 DKK per 100 euro +/- 2.25 percent.

$\Delta Y \approx \partial Y/\partial G \cdot \Delta G = \partial Y/\partial G$ for $\Delta G = 1$. Thereby the transaction-motivated demand for money is increased. This generates an incipient tendency for both the short-term interest rate to rise and the exchange rate to appreciate as portfolio holders worldwide buy the currency of the SOE to invest in its bonds and enjoy their high rate of return. The central bank is committed to a fixed exchange rate, however, and has to prevent the pressure for a higher interest rate and currency appreciation by *buying foreign assets* (foreign currency and bonds) for domestic currency. When the money supply has increased enough to nullify the incipient tendency for a higher domestic interest rate, the equilibrium with unchanged exchange rate is restored. The needed increase in the money supply for a unit increase in G is given by $\Delta M \approx \partial M/\partial G \cdot \Delta G = \partial M/\partial G$ for $\Delta G = 1$. One may say that it is the accommodating money supply that allows the full unfolding of the output multiplier w.r.t. government spending. Nevertheless, owing to the *import leakage* ($N_Y < 0$), both the output and money multiplier are lower than the corresponding multipliers in the closed economy where the central bank maintains the interest rate at a certain target level. The system ends up with higher Y, the same i and X, and lower net exports because of higher imports.

The output multipliers w.r.t. a demand shock, an interest rate shock, and a liquidity preference shock, respectively, are

$$\begin{aligned}\frac{\partial Y}{\partial \varepsilon_D} &= \frac{\partial Y}{\partial G} = \frac{1}{1 - C_{Y^p}(1-T') - I_Y - N_Y} > 1,\\ \frac{\partial Y}{\partial i^*} &= \frac{D_{r^e}}{1 - D_Y} = \frac{C_{r^e} + I_{r^e}}{1 - C_{Y^p}(1-T') - I_Y - N_Y} < 0,\\ \frac{\partial Y}{\partial \varepsilon_L} &= 0.\end{aligned}$$

This last result reflects that after the liquidity preference shock, the money market equilibrium is restored by full adjustment of the money supply ($dM = Pd\varepsilon_L$) at unchanged interest rate.

23.1.3 Floating exchange rate

In a floating exchange rate regime, also called a flexible exchange rate regime, the exchange rate is allowed to respond endogenously to the market forces, supply and demand, in the foreign exchange market. The model treats the money stock, M, as exogenous. More precisely, the money supply is targeted by the central bank and the model assumes this is done successfully. The exchange rate adjusts so that the available supplies of money and domestic bonds are willingly held.

In the present static portrait of the floating exchange rate regime, $\dot{X}^e$ is treated as exogenous. Following Mundell (1963), we imagine that the system has settled down in a steady state with $\dot{X}^e = 0$. The model is again recursive. First (UIP) yields $i = i^*$. Then output is determined by (LM), given $i = i^*$. And finally the required exchange rate is determined by (IS) for a given level of P^*/P. So the story behind the equilibrium described by the model is that the exchange rate has adjusted to a level such that aggregate demand and output is at the point where, given the real money supply, the transactions-motivated demand for money establishes clearing in asset markets for a nominal interest rate equal to the foreign nominal interest rate.

There is no possibility of *fiscal policy* affecting output as long as there are no restrictions on movements of financial capital. The interpretation is the following. Consider an expansive fiscal policy, $dG > 0$ or $d\tau < 0$. The incipient output stimulation increases the transaction demand for money and thereby the interest rate. The rise in the interest rate is immediately counteracted, however, by inflow of foreign exchange induced by the high interest rate. This inflow means higher demand for the domestic currency, which thereby appreciates, thus lowering competitiveness and net exports. The appreciation continues until competitiveness has decreased enough[4] to bring the interest rate back to its initial level. This state of affairs is obtained when the exchange rate has reached a level at which the fall in net exports matches the rise in G or fall in τ, thereby bringing aggregate demand and output back to their initial levels. In effect, the system ends up with unchanged output and interest rate, a lower exchange rate, X, and lower net exports.[5]

On the other hand, *monetary policy* is effective. An increase in the money supply (through an open-market operation) generates an incipient fall in the interest rate. This triggers a counteracting outflow of financial capital, whereby the domestic currency depreciates, i.e., X rises. The depreciation continues until the real exchange rate has increased enough to induce a rise in net exports and output large enough for the transaction demand for money to match the larger money supply and leave the interest rate at its original level. The system ends up with higher Y, the same i, higher X, and higher net exports.[6]

From (LM) and (IS), respectively, we find the output and exchange rate mul-

[4] Recall that, as we have defined the exchange rate, "up is down and down is up" or, perhaps with a little more transparency, "currency up is exchange rate down".

[5] This canonical result relies heavily on the idealized assumption that foreign and domestic bonds are perfect substitutes and move without restraint of any kind.

[6] The implicit assumption that a higher X does not affect the price level P is of course problematic. If intermediate goods are an important part of imports, then a higher exchange rate would imply higher unit costs of production. And since prices tend to move with costs, this would imply a higher price level. If imports consist primarily of final goods, however, it is easier to accept the logic of the model.

tipliers w.r.t. the money supply:

$$\frac{\partial Y}{\partial M} = \frac{1}{PL_Y} > 0, \tag{23.6}$$

$$\frac{\partial X}{\partial M} = \frac{1 - C_{Y^p}(1-T') - I_Y - N_Y}{D_x P^* L_Y} > 0. \tag{23.7}$$

Finally, the output multipliers w.r.t. a demand shock and a liquidity preference shock, respectively, are

$$\frac{\partial Y}{\partial \varepsilon_D} = \frac{\partial Y}{\partial G} = 0,$$

$$\frac{\partial Y}{\partial \varepsilon_L} = -\frac{1}{L_Y} < 0.$$

Like increased public spending, a positive output demand shock does not affect output. It is neutralized by appreciation of the domestic currency. A positive liquidity preference shock reduces output. The mechanism is that the increase in money demand triggers an incipient rise in the domestic interest rate. The concomitant appreciation of the currency, resulting from the induced inflow of financial capital, reduces net exports.

23.1.4 Perspectives

The message of this simple model is that for a small open economy, a fixed exchange rate regime is better at stabilizing output if money demand shocks dominate, and a floating exchange rate system is better if most shocks are output demand shocks. In any case, there is an asymmetry. Under a fixed exchange rate system, two macroeconomic short-run policy instruments are given up: exchange rate policy and monetary policy. Under a floating exchange rate system and perfect capital mobility, only one macroeconomic short-run policy instrument is given up: fiscal policy. Yet, the historical experience seems to be that an international system of floating exchange rates ends up with higher volatility in both real and nominal exchange rates than one with fixed exchange rates (Mussa, 1990; Obstfeld and Rogoff, 1996; Basu and Taylor, 1999).

The result that fiscal policy is impotent in the floating exchange rate regime does not necessarily go through in more general settings. For example, in practice, domestic and foreign financial claims may often *not* be perfect substitutes. Indeed, the UIP hypothesis tends to be empirically rejected at short forecast horizons, while it does somewhat better at horizons longer than a year (see the literature notes at the end of the chapter). And as already hinted at, treating the price level as exogenous when import prices change is not satisfactory.

Anyway, even the static Mundell-Fleming model provides a basic insight: the *impossible trinity.* A society might want a system with the following *three* characteristics:

- free mobility of financial capital (to improve resource allocation);
- independent monetary policy (to allow a stabilizing role for the central bank);
- fixed exchange rate (to avoid exchange rate volatility).

But it can have only two of them. A fixed exchange rate system is incompatible with the second characteristic. And a flexible exchange rate system contradicts the third.

In the next sections we extend the model with dynamics and rational expectations. We first consider the fixed exchange rate regime, next the flexible exchange rate regime.

23.2 Dynamics under a fixed exchange rate

We ignore the shift parameters ε_D and ε_L. On the other hand we introduce an additional asset, a long-term bond that is indexed w.r.t. domestic inflation. In the fixed exchange rate regime this extension is easy to manage. In addition we assume rational expectations. The formal structure of the model then becomes exactly the same as that of the dynamic IS-LM model for a closed economy with short- and long-term bonds, studied in the previous chapter.

With R_t denoting the real long-term interest rate at time t (defined as the internal real rate of return on an inflation indexed consol), aggregate demand is

$$\begin{aligned} Y_t^d &= C(Y_t^p, R_t) + I(Y_t, R_t) + N(Y_t, x) + G \equiv D(Y_t, R_t, x, \tau) + G, \\ &\quad \text{where} \\ 0 &< D_Y = C_{Y^p}(1 - T') + I_Y + N_Y < 1, D_R = C_R + I_R < 0, D_x > 0, \\ -1 &< D_\tau = -C_{Y^p} < 0, \end{aligned}$$

where x is the real exchange rate, XP_t^*/P_t, with X representing the given and constant nominal exchange rate and the price ratio P_t^*/P_t assumed constant. The latter assumption is equivalent to assuming the domestic inflation rate to equal the foreign inflation rate for all t. Moreover, this common inflation rate is assumed equal to a constant, π.

To highlight the dynamics between fast-moving asset markets and slower-moving goods markets, the model replaces (IS) by the error-correction specification

$$\dot{Y}_t \equiv \frac{dY_t}{dt} = \lambda(Y_t^d - Y_t) = \lambda(D(Y_t, R_t, x, \tau) + G - Y_t), \qquad Y_0 > 0 \text{ given}, \quad (23.8)$$

where $\lambda > 0$ is the constant adjustment speed. Because changing the level of production is time consuming, Y_0 is historically given.

We assume the fixed exchange rate policy is sustainable. That is, the level of X is such that no threatening cumulative current account deficits in the future are glimpsed. In view of rational expectations we then have $\dot{X}_t^e = 0$ for all $t \geq 0$. In effect the uncovered interest parity condition reduces to

$$i_t = i^*, \quad (23.9)$$

where the exogenous foreign interest rate, i^*, is for simplicity assumed constant.

The remaining elements of the model are well-known from Chapter 22:

$$\frac{M_t}{P_t} = L(Y_t, i^*), \qquad L_Y > 0, \; L_i < 0. \quad (23.10)$$

$$R_t = \frac{1}{q_t}, \quad (23.11)$$

$$\frac{1 + \dot{q}_t^e}{q_t} = r_t^e, \quad (23.12)$$

$$r_t^e \equiv i^* - \pi_t^e, \qquad \pi_t \equiv \frac{\dot{P}_t}{P_t}, \quad (23.13)$$

$$P_t = P_0 e^{\pi t}, \quad (23.14)$$

where q is the real price of the long-term bond (the consol), the superscript e denotes expected value. If a shock occurs, it fits intuition best to interpret the time derivatives in (23.8), (23.12), and (23.13) as right-hand derivatives, e.g., $\dot{Y}_t \equiv \lim_{\Delta t \to 0^+}(Y(t + \Delta t) - Y(t))/\Delta t$. The variables τ, G, i^*, x, P_0, and π are exogenous constants. The first five of these are positive, and we assume $\pi < i^*$.

As there is no uncertainty in this model (no stochastic elements), the assumption of rational expectations amounts to perfect foresight. We thus have $\dot{q}_t^e = \dot{q}_t$ and $\pi_t^e = \pi$ for all t. Therefore, the equations (23.13) and (23.9) imply $r_t^e = r_t = i^* - \pi > 0$ for all t. Combining this with (23.11) and (23.12), we end up with

$$\dot{R}_t = (R_t - i^* + \pi)R_t. \quad (23.15)$$

Assuming no speculative bubbles, the no-arbitrage condition (23.12) is equivalent to a saying that the consol has market value equal to its *fundamental value*:

$$\begin{aligned} q_t &= \int_t^{\infty} 1 \cdot e^{-\int_t^s r_\tau d\tau} ds, \qquad \text{so that} \qquad (23.16) \\ R_t &= \frac{1}{q_t} = \frac{1}{\int_t^{\infty} e^{-\int_t^s r_\tau d\tau} ds} = \int_t^{\infty} \frac{e^{-\int_t^s r_\tau d\tau}}{\int_t^{\infty} e^{-\int_t^s r_\tau d\tau} ds} r_s ds. \end{aligned}$$

In other words: the long-term rate, R_t, is an average of the (expected) future short-term rates, r_τ, with weights proportional to the discount factor $e^{-\int_t^s r_\tau d\tau}$, cf. the appendix of Chapter 22..

The evolution of the economy over time is described by the two differential equations, (23.8) and (23.15), in the endogenous variables, Y_t and R_t. Since the exchange rate X is an exogenous constant, the UIP condition is upheld by movements of financial capital providing the needed continuous adjustment of the endogenous money supply so as to satisfy $M_t = P_t L(Y_t, i^*) = P_0 e^{\pi t} L(Y_t, i^*)$, in view of (23.10) and (23.14). It is presupposed that the central bank keeps foreign exchange reserves to be able to buy the domestic currency on foreign exchange markets when needed to maintain its value.

The dynamics is essentially the same as that of a closed economy with a target short-term interest rate fixed by the central bank. In a phase diagram the $\dot{R} = 0$ locus is horizontal and coincides with the saddle path. In the absence of speculative bubbles and expected future changes in i^*, we thus get $R_t = r_t = i^* - \pi > 0$ for all $t \geq 0$. The only difference compared with the closed economy is that the short-term interest rate is not a policy variable any more, but an exogenous variable given from the world financial market.

As an example of an adjustment process, consider a fiscal tightening (increase in τ or decrease in G). This will immediately decrease output demand. Thereby output gradually falls to a new lower equilibrium level. The fall in output implies lower money demand because the amount of money-mediated transactions becomes lower. The lower money demand generates an incipient tendency for the short-term interest rate to fall and the domestic currency to depreciate. This tendency is immediately counteracted, however. To take advantage of a higher foreign interest rate, portfolio holders worldwide convert home currency into foreign currency at the given exchange rate in order to buy foreign bonds. Owing to its commitment to a fixed exchange rate, the central bank now intervenes by selling foreign currency and domestic bonds. As soon as i is restored at its original value, i^*, the downward pressure on the value of the domestic currency is nullified. By assumption, the price level stays on the time path (23.14), whereby r and R remain essentially unaffected and equal to the constant $i^* - \pi$ during the output contraction. The figures 20.12 and 20.13 of the previous chapter illustrate.

Another kind of demand shock is a shift in the exports demand due to, say, a reduced economic growth in the world economy.

23.3 Dynamics under a floating exchange rate: overshooting

The exogeneity − and in fact absence − of expected exchange rate changes in the static Mundell-Fleming model of a floating exchange rate regime is unsatisfactory. By a dynamic approach we can open up for an endogenous and time-varying $\dot{X}^e$.

The floating exchange rate regime requires one more differential equation compared to the fixed exchange rate system. To avoid the complexities of a three-dimensional dynamic system, we therefore simplify along another dimension by dropping the distinction between short-term and long-term bonds. Hence, output demand is again described as in (23.1) and depends negatively on the expected short-term real interest rate, r^e. We ignore the disturbance term ε_D.

Apart from the exchange rate now being endogenous, money supply exogenous, and long-term bonds absent, the model is similar to that of the previous section. At the same time the model is close to a famous contribution by the German-American economist Rudiger Dornbusch (1942-2002), who introduced forward-looking rational expectations into a floating exchange rate model (Dornbusch, 1976). Dornbusch thereby showed that exchange rate "overshooting" could arise. This was seen as a possible explanation of the rise in both nominal and real exchange rate volatility during the 1970s after the demise of the Bretton-Woods system. In his original article, Dornbusch wanted to focus on the dynamics between fast moving asset prices and sluggishly changing goods prices. He assumed output to be essentially unchanged in the process. In the influential Blanchard and Fischer (1989) textbook this was modified by letting output adjust gradually to spending, while goods prices were in the short run simply unaffected by demand shifts. This seems a more apt approximation, since the empirics tell us that in response to demand shifts, output moves faster than goods prices. We follow this approach and name it the *Blanchard-Fischer version* of Dornbusch's overshooting model.

23.3.1 The model

This modified Dornbusch model has three building blocks. The first building block is the output error-correction process,

$$\begin{aligned} \dot{Y}_t &= \lambda(Y_t^d - Y_t), \qquad \text{where} \qquad (23.17) \\ Y_t^d &= D(Y_t, r_t^e, x_t, \tau) + G, \end{aligned}$$

where $r_t^e \equiv i_t^e - \pi_t^e$, and $x_t \equiv X_t P_t^*/P_t$ is the real exchange rate. As in (23.4), the partial derivatives of the demand function D satisfy $0 < D_Y < 1$, $D_{r^e} < 0$, $D_x > 0$, and $-1 < D_\tau < 0$.

The second building block comes from the money market equilibrium condition, combined with a monetary policy maintaining a constant real money supply, $m > 0$. This requires that the money supply follows the path

$$M_t = mP_t = mP_0 e^{\pi t} = M_0 e^{\pi t},$$

where π is the actual inflation rate, assumed constant. The money market equilibrium condition now reduces to $m = L(Y_t, i_t)$, which defines i_t as an implicit function,

$$i_t = i(Y_t, m), \quad \text{with } \ i_Y = -L_Y/L_i > 0, \ i_m = 1/L_i < 0. \tag{23.18}$$

We assume that m is small enough so that under "normal circumstances", the interest rate, i_t, takes a value above its lower bound, nil.

The third building block is the foreign exchange market. This market is in equilibrium when the uncovered interest parity condition holds. This is the condition

$$i_t = i^* + \frac{\dot{X}_t^e}{X_t}. \tag{23.19}$$

Assuming perfect foresight, this takes the form $i_t = i^* + \dot{X}_t/X_t$, which implies

$$\dot{X}_t = (i_t - i^*)X_t. \tag{23.20}$$

Admittedly, this relationship may be questioned. As mentioned, the empirical support for the combined hypothesis of UIP and rational expectations is, at least for short forecast horizons, weak. Yet, to avoid a complicated model, we shall proceed as if (23.20) holds for all t.

In a steady state of the system, the nominal exchange rate will be a constant, X. For a steady state to be possible, we need that also the real exchange rate, XP_t^*/P_t, is constant, x. We therefore assume that the foreign inflation rate equals the domestic inflation rate, π. As an implication we have $P_t^*/P_t = P_0^*/P_0$ for all $t \geq 0$.

Because of perfect foresight, $\pi_t^e = \pi$ and $r_t^e = r_t = i_t - \pi$ for all $t \geq 0$. In view of these conditions, together with (23.18), output demand can be written

$$Y_t^d = D(Y_t, i(Y_t, m) - \pi, X_t \frac{P_0^*}{P_0}, \tau) + G. \tag{23.21}$$

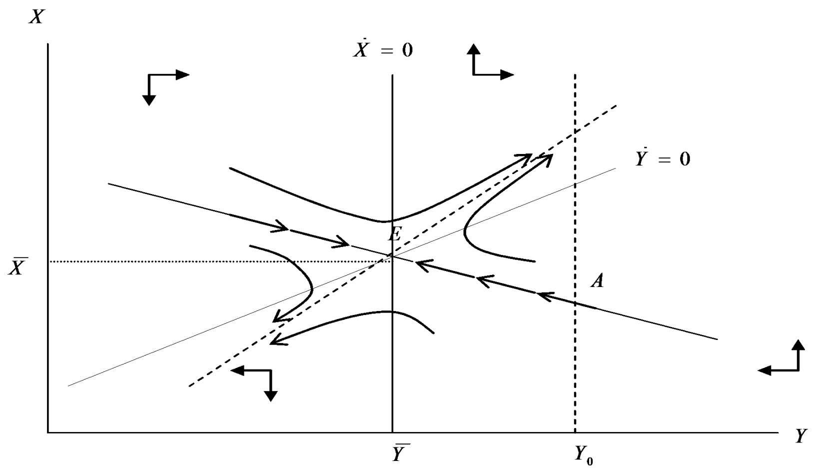

Figure 23.1: Phase diagram in case of a a floating exchange rate.

Inserting this into (23.17) and then (23.18) into (23.20), we end up with the dynamic system

$$\dot{Y}_t = \lambda \left[D(Y_t, i(Y_t, m) - \pi, X_t \frac{P_0^*}{P_0}, \tau) + G - Y_t \right], \; Y_0 > 0 \text{ given}, \quad (23.22)$$

$$\dot{X}_t = [i(Y_t, m) - i^*] X_t. \quad (23.23)$$

This system has Y_t and X_t as endogenous variables, whereas the remaining variables are exogenous and constant: $m, P_0^*/P_0, G$, τ and i^*, all positive.

The phase diagram is shown in Fig. 23.1. By (23.22), the $\dot{Y} = 0$ locus is given by the equation $Y = D(Y, i(Y, m) - \pi, XP_0^*/P_0, \tau) + G$. Taking the differential on both sides w.r.t. Y, X, and m (for later use) gives

$$\begin{aligned} dY &= (D_Y + D_{r^e} i_Y) dY + D_{r^e} i_m dm + D_x \frac{P_0^*}{P_0} dX \Rightarrow \\ (1 - D_Y - D_{r^e} i_Y) dY &= D_{r^e} i_m dm + D_x \frac{P_0^*}{P_0} dX. \quad (23.24) \end{aligned}$$

Setting $dm = 0$, we find

$$\frac{dX}{dY}|_{\dot{Y}=0} = \frac{1 - D_Y - D_{r^e} i_Y}{D_x P_0^*/P_0} > 0. \quad (23.25)$$

It follows that the $\dot{Y} = 0$ locus (the "IS curve") is upward-sloping as shown in Fig. 23.1.

Equation (23.23) implies that $\dot{X} = 0$ for $i(Y, m) = i^*$. The value of Y satisfying this equation is unique (because $i_Y \neq 0$) and is called $\bar{Y}$. That is, $\dot{X} = 0$ for

$Y = \bar{Y}$, which says that the $\dot{X} = 0$ locus (the "LM curve") is vertical. Fig. 23.1 also indicates the direction of movement in the different regions, as determined by (23.22) and (23.23). The arrows show that the steady state is a saddle point. This implies that exactly two solution paths − one from each side − converge towards E.

Since the adjustment of output takes time, Y is a predetermined variable. Thus, at time $t = 0$, the economy must be somewhere on the vertical line $Y = Y_0$. If speculative exchange rate bubbles are assumed away, the explosive or implosive paths of X in Fig. 23.1 cannot arise. Hence, we are left with the segment AE of the saddle path in the figure as the unique solution to the model for $t \geq 0$. Following this path the economy gradually approaches the steady state E. If $Y_0 > \bar{Y}$ (as in Fig. 23.1), output is decreasing and the exchange rate increasing during the adjustment process. If instead $Y_0 < \bar{Y}$, the opposite movements occur.

The steady state can be seen as a "short-run equilibrium" of the economy. Further dynamic interactions will tend to arise in the "medium run", for instance through a Phillips curve and through investment resulting in build up of fixed capital. These ramifications are ignored by the model.

How the steady state and the $\dot{X} = 0$ and $\dot{Y} = 0$ loci depend on m

In steady state we have

$$\bar{Y} = D(\bar{Y}, i^* - \pi, \bar{X}\frac{P_0^*}{P_0}, \tau) + G, \tag{23.26}$$

and

$$m = L(\bar{Y}, i^*). \tag{23.27}$$

First, (23.27) determines $\bar{Y}$ as an implicit function of m and i^* independently of (23.26). To see how $\bar{Y}$ is affected by a change in m, we take the differential on both sides of (23.27) to get $dm = L_Y d\bar{Y} + L_i di^*$. With $di^* = 0$, this gives

$$\frac{\partial \bar{Y}}{\partial m} = \frac{1}{L_Y} > 0. \tag{23.28}$$

Given $\bar{Y}$, we have $\bar{X}$ determined by (23.26). Then, to see how X is affected by a change in m, we take the differential on both sides of (23.26) to get $d\bar{Y} = D_Y d\bar{Y} + D_x P_0^*/P_0 d\bar{X}$. Combining this with (23.28), we end up with

$$\frac{\partial \bar{X}}{\partial m} = \frac{\partial \bar{X}}{\partial \bar{Y}}\frac{\partial \bar{Y}}{\partial m} = \frac{1 - D_Y}{D_x P_0^*/P_0}\frac{1}{L_Y} > 0. \tag{23.29}$$

The intuitive explanation of this sign is linked to that of (23.28). For the higher money supply to be demanded with unchanged interest rate, equilibrium in the money market requires a higher level of transactions, that is, a higher level of economic activity. In steady state this must be balanced by a sufficiently higher output demand. And since the marginal propensity to spend is less than one ($D_Y < 1$), higher net exports are needed; otherwise the rise in output demand is smaller than the rise in output. Thus, higher competitiveness and therefore depreciation of the domestic currency is required which means a higher X.

Taking into account that $dm = dM/P$, we see that the steady state multipliers of Y and X w.r.t. m are the same as the corresponding multipliers in the static model, given in (23.6) and (23.7).

It follows from (23.28) that an increase in m will shift the $\dot{X} = 0$ line to the right, cf. Fig. 23.2. Again, for a higher money supply to be matched by higher money demand at an unchanged interest rate, a higher level of economic activity is needed.

As to the effect of higher m on the $\dot{Y} = 0$ locus, consider Y as fixed at Y_0, i.e., $dY = 0$. Then (23.24) gives

$$\frac{\partial X}{\partial m}_{|\dot{Y}=0,Y=Y_0} = -\frac{D_{r^e} i_m}{D_x P_0^*/P_0} = -\frac{D_{r^e}/L_i}{D_x P_0^*/P_0} < 0.$$

Hence, an increase in m shifts the $\dot{Y} = 0$ locus downward. The intuition is that a rise in m induces a fall in the interest rate; then for output demand to remain unchanged, we need an appreciation, i.e., a fall in X.

Another way of understanding the shift of the $\dot{Y} = 0$ locus is to consider X as fixed at X_0, i.e., $dX = 0$. Then (23.24) yields

$$\frac{\partial Y}{\partial m}_{|\dot{Y}=0,X=X_0} = \frac{D_{r^e} i_m}{1 - D_Y - D_{r^e} i_Y} = \frac{D_{r^e}/L_i}{1 - D_Y + D_{r^e} L_Y/L_i} > 0. \tag{23.30}$$

Hence, we can also say that an increase in m shifts the $\dot{Y} = 0$ locus rightward, cf. Fig. 23.2. The intuition is that, given X, the fall in i induced by higher m increases output demand and therefore also the output level that matches output demand.

The conclusion is that a higher m shifts both the $\dot{X} = 0$ locus and the $\dot{Y} = 0$ locus rightward. Then it might seem ambiguous in what direction $\bar{X}$ moves. But we already know from (23.29) that $\bar{X}$ will unambiguously increase. The explanation is that, first, a higher output level is needed for money market equilibrium to obtain. Second, for the higher output level to be demanded, we need a depreciation of the domestic currency, i.e., a higher X. Fig. 23.2 illustrates.

23.3.2 Unanticipated rise in the real money supply

Now, we are ready to study the dynamic effects of an unanticipated upward shift in the real money supply, m. Suppose the economy has been in its steady

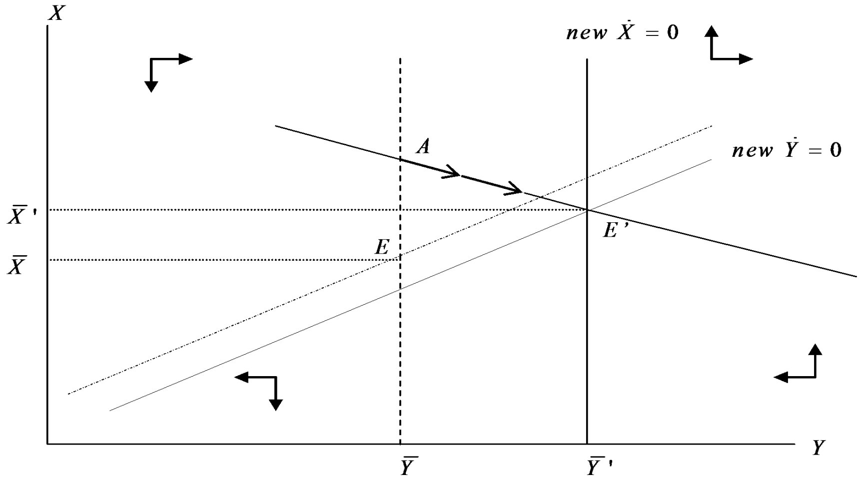

Figure 23.2: Phase portrait of an unanticipated rise in m.

state until time t_0. Then unexpectedly a discrete open-market purchase by the central bank of domestic bonds takes place. This instantly increases the monetary base which through the money multiplier leads to a larger money stock and a smaller stock of bonds held by the private sector. At the same time the nominal interest rate jumps down because output, and thereby the transactions volume, is predetermined in this "very short run" (it takes time to change output). The lower interest rate prompts arbitrage. With the aim of acquiring foreign interest-bearing assets, domestic currency will buy foreign currency until the exchange rate has jumped up to a level from which it is expected to *appreciate* at a rate such that interest parity is reestablished. Very fast, a new "very-short-run" equilibrium is formed where the given supplies of money and domestic bonds are again willingly held by the agents. The essence of the matter is that for a while we have $i_t < i_t^*$ due to the increase in the money supply. To make domestic bonds as attractive as foreign bonds, an expectation appreciation is needed. In turn this requires an *initial depreciation* in excess of the ultimate depreciation implied by the transition from the old to the new steady state.

Fig. 23.2 illustrates this exchange rate *overshooting.* We say that a variable *overshoots* if its initial reaction to a shock is greater than its longer-run response. Fig. 23.3 shows the time profiles of the exchange rate and the other key variables (D is output demand).

To really understand what is going on, let us examine the mechanics of overshooting more closely:

1. What will be the new steady state expected by the market participants? As we have just seen, when m increases, both the $\dot{X} = 0$ locus and the $\dot{Y} = 0$

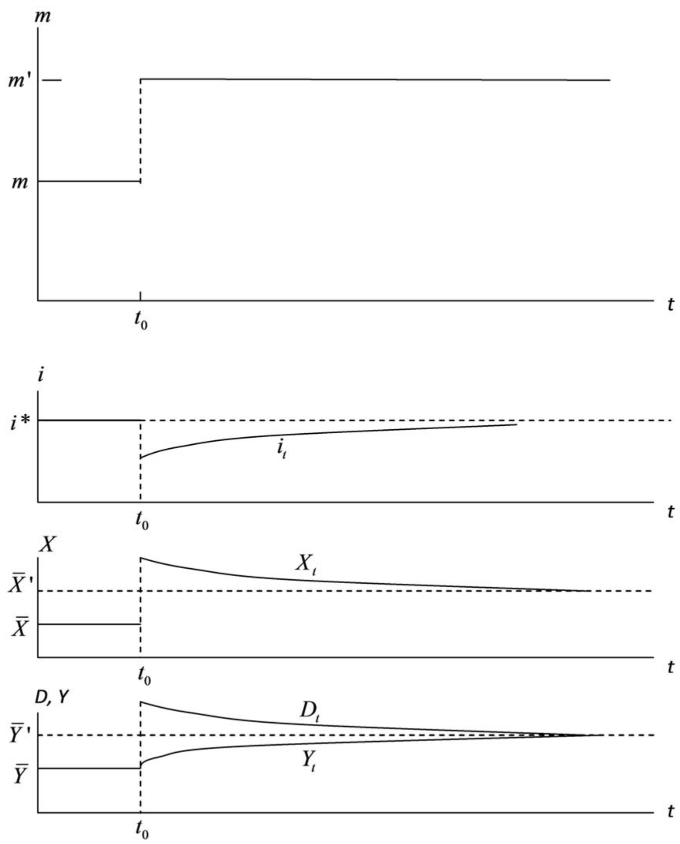

Figure 23.3: Time profiles of r, X, Y, and output demand D in response to an unanticipated rise in m (shown in the upper panel).

locus move to the right. The $\dot{X} = 0$ locus moves to the right, because the higher m tends to decrease i so that an offsetting increase in Y is needed for i to still match i^*, cf. (23.28). And the $\dot{Y} = 0$ locus moves to the right, because output demand depends negatively on the interest rate so that, through this channel, it depends positively on money supply, cf. (23.30). Nevertheless, we can be sure that the new steady-state point is associated with a *higher* exchange rate, as was explained in connection with (23.29) above.

2. Why must the initial depreciation be *larger* than that required in the new steady state? Two things are important here. First, both the outflow of financial capital, prompted by the fall in the interest rate, and the contemporaneous depreciation occur *instantly*. Imagine for a moment they occurred gradually over time. Then there would be expected depreciation of the domestic currency, implying that foreign bonds became even more attractive relative to domestic bonds. This would reinforce the outflow of financial capital and speed up the rise in X, that is, enlarge the drop in the value of the domestic currency. Thus, the financial capital outflow and the depreciation occur very fast, which mathematically corresponds to an upward *jump* in X.

The second issue is: how large will the jump be? The answer is: large enough for the concomitant *expected gradual appreciation* rate to be at the level needed to make domestic bonds not less attractive than foreign bonds *at the same time* as convergence to the new steady state is ensured. This happens where the vertical line $Y = \bar{Y}$ crosses the new saddle path, i.e., at the point A in Fig. 23.2. Ruling out bubbles, agents realize that any incipient jump to a point above or below A offers arbitrage opportunities. Exploiting these, the system is almost instantly brought back to A.

3. Why will output gradually rise and the domestic currency gradually appreciate after the initial depreciation jump? For $t > t_0$ the economy moves along the new saddle path: Y gradually responds to the high output demand generated by the low interest rate $r_t = i_t$ and the high competitiveness, X_t. As output rises, money demand increases and i_t gradually returns to "normal", see Fig. 23.3. Moreover, X_t gradually adjusts downwards and converges towards $\bar{X}'$ as the interest differential, $i_t - i^*$, declines. (Remember that the analysis presupposes that after time t_0 the market participants rightly expect no further changes in the money supply.)

The exchange rate as a forward-looking variable

It helps the interpretation of the dynamics if we recognize that the exchange rate is an asset price, hence forward-looking. Under perfect foresight the uncovered interest parity implies that the exchange rate satisfies the differential equation

(23.23), except at points of discontinuity of X_t. For convenience we repeat the differential equation here:

$$\dot{X}_t = (i_t - i^*)X_t. \tag{23.31}$$

For fixed $t > t_0$ we can write the solution of this linear differential equation as

$$X_\tau = X_t e^{\int_t^\tau (i_s - i^*)ds} \equiv X_t e^{(\bar{i}_{t,\tau} - i^*)(\tau - t)}, \qquad \text{for } \tau > t,$$

where $\bar{i}_{t,\tau}$ is the mean of the interest rates between time t and time τ, i.e., $\bar{i}_{t,\tau} \equiv \int_t^\tau i_s ds/(\tau - t)$. Being a forward-looking variable, X_t is not predetermined. It is therefore more natural to write the solution on the forward-looking form

$$X_t = X_\tau e^{-\int_t^\tau (i_s - i^*)ds} \equiv X_\tau e^{-(\bar{i}_{t,\tau} - i^*)(\tau - t)}, \qquad \text{for } \tau > t, \tag{23.32}$$

where X_τ and i_s should be interpreted as the *expected* future values as seen from time t. Thus, under the UIP hypothesis the exchange rate today equals the expected future exchange rate discounted by the mean interest differential $\bar{i}_{t,\tau} - i^*$ expected to be in force in the meantime.[7] As a consequence, new information implying anticipation of, for instance, a higher X in the future (compared with the reference path) will, for given expectations concerning the mean interest differential, show up immediately as depreciation of the domestic currency today.

From our explanation of the mechanics of overshooting, the reader might think that financial capital movements that prompt an exchange rate adjustment require a lot of exchange transactions to occur. However, what is needed for expected asset returns to be equalized is in principle just that the traders, in possession of the needed currency, in response to new information adjust their bid and ask prices to the new level at which supply and demand are equilibrated. In this way, what we see need not be much more than an international re-evaluation of domestic and foreign bonds. In highly integrated asset markets a new equilibrium may be found very fast. These circumstances notwithstanding, the volume of foreign exchange trading per day has in recent years grown to enormous magnitudes.

Returning to our specific case of a monetary expansion, in (23.32) let $\tau \to \infty$ to get

$$X_t = \lim_{\tau \to \infty} X_\tau e^{-\int_t^\tau (i_s - i^*)ds} = \bar{X}' e^{-\int_t^\infty (i_s - i^*)ds} > \bar{X}', \tag{23.33}$$

where the new steady-state value of the exchange rate after the rise in m is denoted $\bar{X}'$ as in Fig. 23.3. The inequality in (23.33) is due to $i_s < i^*$ during the adjustment process. As time proceeds, the shortfall of the domestic vis-à-vis

[7] The solution formula (23.32) presupposes absence of any jump in X between time t and time τ. Or more to the point: arbitrage prevents any such *expected* jump. Ex post, the formula (23.32) is valid only if no jumps in X actually occurred in the time interval considered.

the foreign interest rate is reduced and the exchange rate converges to its steady-state value from above. When for instance 90% of the initial distance from the steady state has been recovered, we say that the system has essentially reached its steady state. The adjustment process so far may not involve more time than a couple of years, say. Several factors that may matter for further adjustments are ignored by this model. Hence, we avoid to call $\bar{X}'$ a "long-run" value.

In (23.32) and (23.33) we consider the value of the foreign currency in terms of the domestic currency. Similar expressions of course hold for the value of the domestic currency in terms of the foreign currency. Thus, inverting (23.32) gives

$$X_t^{-1} = X_\tau^{-1} e^{-\int_t^\tau (i^* - i_s)ds} \equiv X_\tau^{-1} e^{-(i^* - \bar{i}_{t,\tau})(\tau - t)}.$$

That is, the value of the domestic currency today equals its expected future value discounted by the mean interest differential $i^* - \bar{i}_{t,\tau}$ expected to be in force in the meantime. Inverting (23.33) yields

$$X_t^{-1} = \bar{X}'^{-1} e^{-\int_t^\infty (i^* - i_s)ds} < \bar{X}'^{-1}.$$

As time proceeds, the excess of the foreign over the domestic interest rate decreases and the value of the domestic currency converges to its steady-state value from below.

23.3.3 Anticipated rise in the money supply

As an alternative scenario suppose that the economy has been in steady state until time t_0 when agents suddenly become aware that an increase in the money supply is going to take place at some future time. To be specific let us imagine that the central bank at time t_0 credibly announces that there will be discrete upward shift in M through an open-market operation at time $t_1 > t_0$, while M for a long time after t_1 will grow at the same rate, π, as before. According to the model this credible announcement immediately causes a *jump* in the exchange rate X in the same direction as the longer-run change, that is, a jump to some level like X_B in Fig. 23.4. This is due to the agents' anticipation that after time t_1, the economy will be on the new saddle path. Or, in more economic terms, the agents know that (a) from time t_1, the expansionary monetary policy will cause the interest rate to be lower than i^*, and (b) the exchange rate must therefore at time t_1 have reached a level from which it can have an expected and actual rate of appreciation such that interest parity is maintained in spite of $i < i^*$ after time t_1.

Under these circumstances, at the old exchange rate, $\bar{X}$, there would be excess supply of domestic bonds and excess demand for foreign bonds immediately after

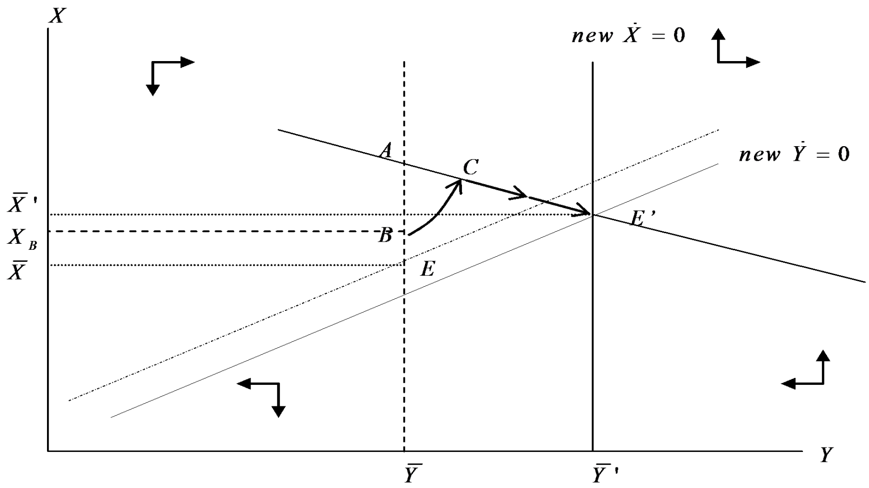

Figure 23.4: Phase portrait of an anticipated rise in m.

time t_0 and this is what instantly triggers the jump to X_B. After this initial jump the exchange rate, X, will adjust continuously. Currency is an asset, hence *anticipated* discrete jumps in the exchange rate are ruled out by arbitrage. In particular, at time t_1 there can be no jump, because no new information has arrived.

In the time interval (t_0, t_1) the movement of (Y, X) is governed by the "old" dynamics. That is, for $t_0 < t < t_1$ the economy must follow a trajectory consistent with the "old" dynamics, reflecting the operation of the no-arbitrage condition,

$$i(Y_t, m) = i^* + \frac{\dot{X}_t^e}{X_t}.$$

which rules as long as the announced policy change is not yet implemented. Under perfect foresight, the market mechanism "selects" that trajectory (BC in Fig. 23.4) along which it takes exactly $t_1 - t_0$ time units to reach the new saddle path. It is in fact this requirement that determines the *size* of the jump in X immediately after time t_0.[8]

The higher competitiveness caused by the instantaneous depreciation implies higher output demand, so that output begins a gradual upward adjustment already before monetary policy has been eased. Along with the rising Y, transaction demand for money rises gradually and so do the interest rate (recall m has not

[8]The level X_B can be shown to be unique and this is also what intuition tells us. Imagine that the jump, $X_B - \bar{X}$, was smaller than in Fig. 21.4. Then, not only would there be a longer way along the road to the new saddle path, but the system would also start from a position closer to the steady state point E, which implies an initially lower adjustment speed.

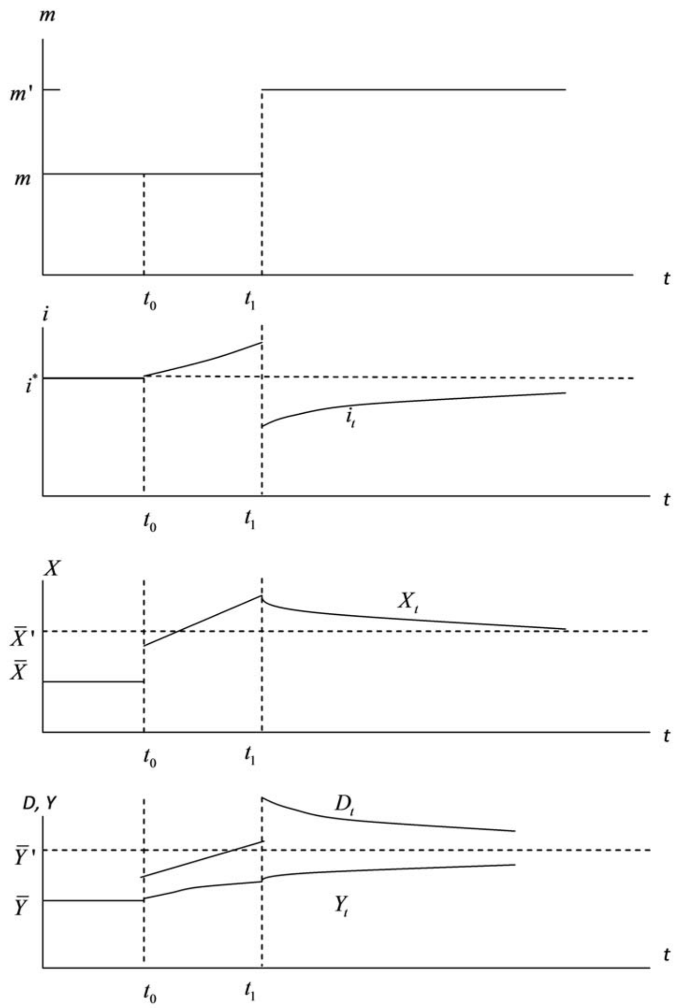

Figure 23.5: Time profiles of r, X, Y, and output demand D in response to an anticipated rise in m (shown in the upper panel).

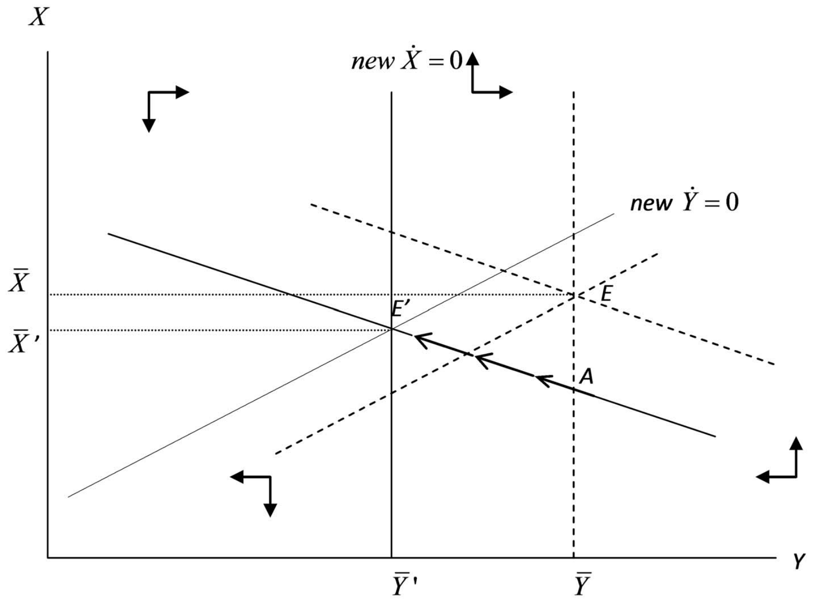

Figure 23.6: Phase portrait of an unanticipated fall in m.

changed yet) and the exchange rate. That is, in the time interval (t_0, t_1) we have both $i_t > i^*$ and $\dot{X}_t > 0$ so as to maintain interest parity. The process continues until the new monetary policy is implemented at time t_1. Exactly at this time the economy's trajectory, governed by the old dynamic regime, crosses the new saddle path, cf. the point C in Fig. 23.4. The actual rise in $m = M/P$ at time t_1 then triggers the anticipated discrete fall in the interest rate to a level $i_{t_1} < i^*$.[9] For $t > t_1$ the economy features gradual appreciation ($\dot{X} < 0$) during the adjustment along the new saddle path. Although output demand is therefore now falling, it is still high enough to pull output further up until the new steady state E' is reached.

The time profiles of Y, X, and r $(= i)$ are shown in Fig. 23.5. The tangent to the X_t curve at $t = t_0$ is horizontal. Hence, infinitely close to t_0 the size of $\dot{X}$ is vanishing. This is dictated by the "old dynamics" ruling in the time interval (t_0, t_1), which entail that the trajectory through the point B in Fig.21.4 is horizontal at B. And this is in accordance with interest parity since it takes time for Y to rise above $\bar{Y}$, hence for i to rise above i^*. Note also that if the length of the time interval (t_0, t_1) were small enough, then X might already immediately after time t_0 be above its new steady-state level, $\bar{X}'$. However, Fig. 23.4 and Fig.

[9] Since right before t_1 we have $i > i^*$, one might wonder whether the fall in the interest rate is necessarily large enough to ensure $i < i^*$ right after t_1. The fall is, indeed, large enough because the dynamics in the time interval (t_0, t_1) ensures $Y_{t_1} < \bar{Y}'$, from which follows $i(Y_{t_1}, m')$ $< i(\bar{Y}', m') = i^*$, where the inequality is due to $i_Y > 0$.

23.5 depict the opposite case, where the time interval (t_0, t_1) is somewhat larger.

23.3.4 Monetary policy tightening

Considering a *downward* shifts in the money supply path, the above processes are reversed.

Unanticipated monetary policy tightening

Suppose the system is in steady state until time t_0. Then, unexpectedly, a discrete open market sale by the central bank of domestic bonds takes place. The new steady state will have a *lower* exchange rate. Indeed, given the lower money supply and unchanged interest rate, equilibrium in the money market requires a lower level of transactions, that is, a lower level of economic activity. In steady state this must be balanced by a sufficiently lower output demand. And since the marginal propensity to spend is less than one ($D_Y < 1$), lower net exports are needed; otherwise the fall in output demand is smaller than the fall in output. Thus, lower competitiveness and therefore appreciation of the domestic currency is required which means a lower X.

In the short run, the nominal interest rate jumps up, prompting an inflow of financial capital, which in turn prompts a jump down in the exchange rate, as shown in Fig. 23.6. This appreciation must be large enough to generate the expected rate of depreciation required for domestic bonds to be no more attractive than foreign bonds in spite of $i_t > i^*$. Very fast a new "very-short-run" equilibrium is formed where the supplies of money and bonds are willingly held by the agents. The fact that there must be expected depreciation is the reason that an initial appreciation in excess of that implied by the new steady-state equilibrium is required. Again the exchange rate "overshoots", this time by taking a greater downward jump than corresponding to the new steady-state level.

For $t > t_0$ the economy moves along the new saddle path: X gradually rises and Y gradually falls in response to the low output demand generated by the high interest rate $r_t = i_t - \pi$ and the low competitiveness, X_t. In the process, money demand decreases and i_t gradually returns to "normal", see Fig. 23.7. Moreover, X_t gradually adjusts upwards and converges towards $\bar{X}'$ as the interest differential, $i_t - i^*$, gradually vanishes.

Anticipated monetary policy tightening

It may happen that the public in advance have a feeling that a monetary policy shift is on the way, due to foreseeable overheating problems, say. To be more

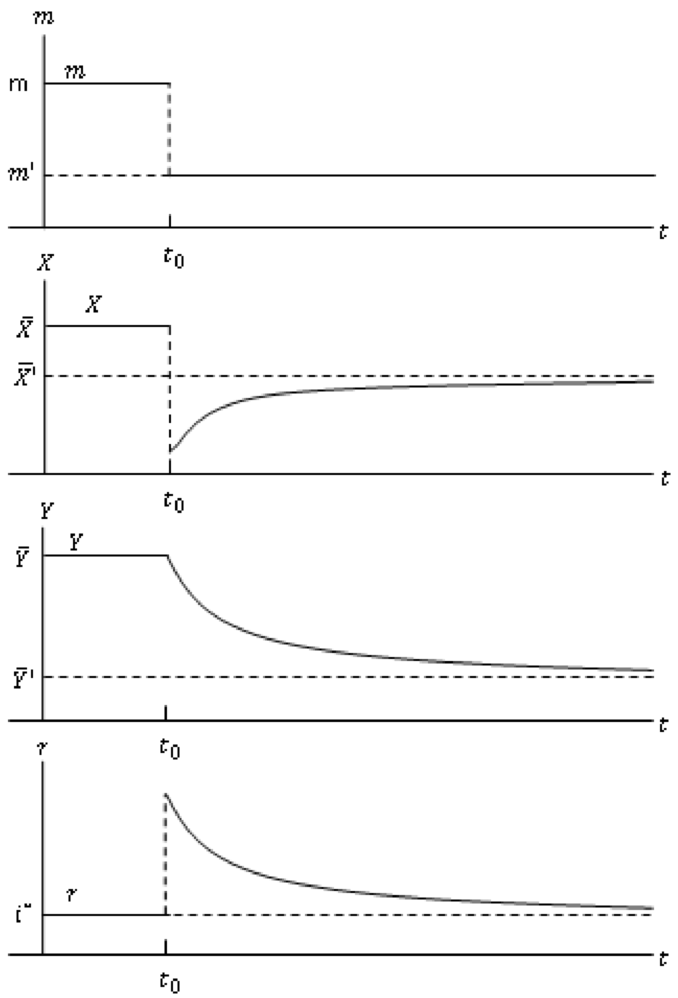

Figure 23.7: Time profiles of X, r, and Y in response to an unanticipated fall in m (shown in the upper panel).

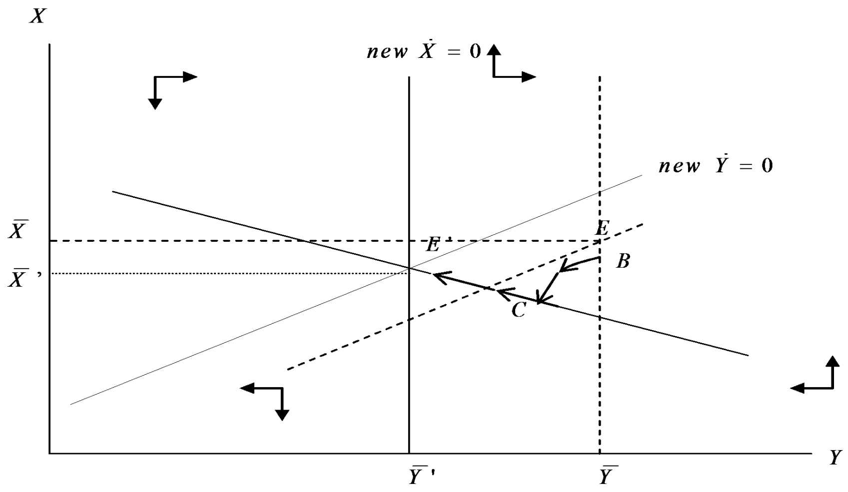

Figure 23.8: Phase portrait of an anticipated fall in m.

specific, suppose that at time t_0 a tightening of monetary policy is credibly announced by the central bank to be implemented at time $t_1 > t_0$ in the form of a reduction in real money supply to the level $m' < m$.

Fig. 23.8 illustrates what happens from time t_0. As soon as the future tight monetary policy becomes anticipated, there is an immediate effect on X in the same direction as the "longer-run" effect, i.e., X drops to some point B as in Fig. 23.8. Indeed, agents anticipate that from time t_1 the tight monetary policy will cause the interest rate to be higher than i^*, thereby engendering gradual depreciation (rise in X) along the new saddle path. Arbitrage prevents any anticipated discrete jump in the exchange rate after time t_0.

In the time interval (t_0, t_1) the economy must follow a trajectory consistent with the "old" dynamics. The market mechanism "selects" that trajectory (BC in Fig. 23.8) along which it takes exactly $t_1 - t_0$ time units to reach the new saddle path. The lower competitiveness caused by the instantaneous appreciation implies lower output demand, so that output begins a gradual downward adjustment already before monetary policy has been tightened.

The time profiles of Y, X, and $r = i - \pi$ are shown in Fig. 23.9. If the length of the time interval (t_0, t_1) is small enough, X may already immediately after time t_0 be below its new steady-state level. However, Fig. 23.8 and Fig. 23.9 depict the opposite case, where the time interval (t_0, t_1) is somewhat larger.

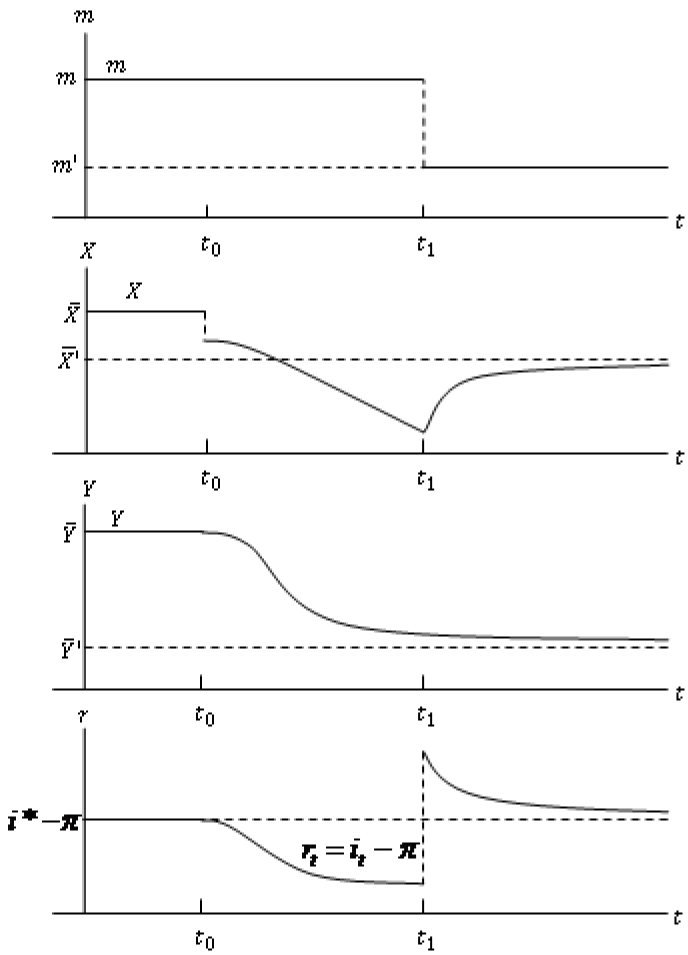

Figure 23.9: Time profiles of X, Y, and r in response to an anticipated fall in m (shown in the upper panel).

23.4 Concluding remarks

The dynamic model of a floating exchange rate regime shows that with fast moving asset markets and nominal rigidities, large volatility in exchange rates can occur. And large volatility of both nominal and real exchange rates under a floating exchange rate regime is in fact what the data show. Nevertheless, there *are* empirical problems with the model. One of them is that it *exaggerates* the exchange rate fluctuations (see Obstfeld and Rogoff, 1996, p. 621 ff.). Moreover, several empirical studies reject the UIP condition or at least they reject the combined hypothesis of UIP and rational expectations (Lewis 1995).

In the model versions considered here the price level moves along a fixed path. An extended model should incorporate that the domestic price level depends on import prices and that persistent changes in aggregate production and employment are likely to activate a Phillips curve of some sort. As they stand, the models imply that monetary shocks have *permanent* real effects, contrary to what the data in general indicates.

Incorporating a medium-run equilibrium level of the real exchange rate, anchored by some kind of expected purchasing power parity, opens up for interesting issues. With x^* denoting a medium-run equilibrium real exchange rate, $\bar{X}'$ in (23.33) would equal $x^*(P/P^*)^e$ where $(P/P^*)^e$ is the expected medium-run value of the relative price level P/P^*. Then, suppose the interest rate differential suddenly rises because the foreign country (the U.S., say) is hit by an economic recession. According to (23.33), this immediately triggers an appreciation of the home currency (China, say). Alternatively, imagine that the expected long-run value of P/P^* goes down due to a strong productivity development in the domestic economy. According to (23.33), also this triggers an appreciation of the home currency (China, say).

23.5 Literature notes

(incomplete)

The origin of the Mundell-Fleming model goes back to Robert Mundell (1963, 1964) and Marcus Fleming (1962). The over-shooting hypothesis goes back to Dornbusch (1976). What we named the *Blanchard-Fischer version* of Dornbusch's overshooting model was presented in the Blanchard and Fischer (1989) textbook. At two points our adaptation departs from Blanchard and Fischer. First, they take the exogenous inflation rate, π, to be zero. Second, they ignore the negative dependence of output demand on the interest rate. Though recognizing this dependency makes the analysis slightly more cumbersome, it is worth the trouble as it underlines the robustness of the results.

An extensive treatment of open economy macroeconomics is contained in the textbook Obstfeld, M., and K. Rogoff, 1996, *Foundations of International Macroeconomics*, The MIT Press, London. In their Chapter 8 the authors discuss the empirical difficulty that the UIP condition is rejected at short prediction horizons although it does somewhat better at horizons longer than one year. Other textbook treatments of this empirical issue include:

Feenstra and Taylor (2012).

Krugman, Obstfeld, and Melitz (2012).

Wickens, M., 2008, Macroeconomic Theory. A Dynamic General Equilibrium Approach, Princeton University Press, Oxford, Ch. 11.4.

Advanced approaches:

Isard, P., 2008, "Uncovered interest parity". In: The new Palgrave Dictionary of Economics. Second edition. Online: http://www.econ.ku.dk/English/libraries/links/

Lewis, K. K., 1995, Puzzles in international financial markets. In: Handbook of International Economics, vol. III, Elsevier, Amsterdam.

The volume of foreign exchange trading per day has in recent years increased to enormous magnitudes. This fact indicates that differences in information and expectations are prevalent. Recent contributions in macroeconomic theory and empirics are considering how to incorporate heterogeneity, imperfect knowledge, and agent's uncertainty about what is the right model of the economy. See e.g. Ellison ( ).

23.6 Appendix

A. The Marshall-Lerner condition

By assuming that net exports depends positively on the real exchange rate ($N_x > 0$), the Mundell-Fleming model presupposes that the Marshall-Lerner condition is satisfied. This is the condition that the weighted sum of the absolute elasticities of exports and imports w.r.t. the real exchange rate is large enough to offset the decrease in the terms of trade implied by a higher real exchange rate. If in the initial situation, net exports are zero, then the sum of the two absolute elasticities should be above 1. The econometric evidence is that the condition is satisfied for industrialized countries if we allow for an adjustment period of one to two years (Artus and Knight, 1984, Table 4, cf. Krugman, Obstfeld, and Melitz, 2012, p. 492). It may be argued that there should be a countervailing effect of the real exchange rate, x, in the consumption function since the purchasing power of domestic income is eroded by an increase in x. The Mundell-Fleming model assumes that the effect of this on aggregate demand is dominated by the role of

$N_x > 0$.

B. The covered interest parity

When buying foreign bonds one can avoid the uncertainty concerning the future exchange rate by entering a reverse *forward exchange* deal with someone else. Today an investor in foreign bonds thus contracts with her bank to sell in thirty days' time a certain amount of foreign currency for domestic currency at a pre-specified rate. This rate is called the thirty-day *forward exchange rate*. It is generally different from the spot exchange rate, X. But empirically the two move closely together.

The *covered interest parity* condition, CIP, is the associated no-arbitrage condition. In discrete time it reads:

$$1 + i_t = \frac{1}{X_t}(1 + i_t^*)X_{t+1}^F, \tag{CIP}$$

where X_{t+1}^F is the one-period forward exchange rate. If there is no default risk and no fear that meanwhile regulations will be imposed which restrain the movement of foreign funds, arbitrage will immediately make CIP hold. Indeed, an agent can borrow one unit of the domestic currency, buy $1/X_t$ units of the foreign currency, then for this amount buy foreign one-period bonds paying a return of $1 + i_t^*$ per bond after one period, and finally lock in the future payout in the domestic currency by selling the return forward at the rate X_{t+1}^F. As the whole undertaking can be conducted at time t, there is no risk.

Let us compare with the (UIP) in discrete time:

$$1 + i_t = \frac{1}{X_t}(1 + i_t^*)X_{t+1}^e, \tag{UIP}$$

We see that the UIP will hold if and only if $X_{t+1}^F = X_{t+1}^e$. But to the extent an asymmetric foreign exchange risk plays a role, a positive risk term should be added to one of the sides in (UIP). Anyway, while X_{t+1}^F is observable via (CIP), X_{t+1}^e is not immediately observable.

23.7 Exercises