Chapter 22

IS-LM dynamics with forward-looking expectations

A main weakness of the static IS-LM model as described in the previous chapter is the absence of dynamics and endogenous forward-looking expectations. This motivated Blanchard (1981) to develop a dynamic extension of the IS-LM model. We shall use The key elements are:

- The focus is manifestly on adjustment mechanisms in the "very short run". The model allows for a deviation of aggregate output from aggregate demand – the adjustment of output to demand takes time. In this way the model highlights the interaction between fast-moving asset markets and less-fast-moving goods markets.
- There are three financial assets, money, a short-term bond, and a long-term bond. Accordingly there is a distinction between the short-term interest rate and the long-term interest rate. Thereby changes in the term structure of interest rates, known as the yield curve, can be studied.
- Agents have forward-looking expectations. The expectations are assumed to be rational (model consistent) and thereby endogenous. Since there are no stochastic elements in the model, perfect foresight is effectively assumed.

This richer IS-LM model conveys the central message of Keynesian theory. The equilibrating role in the output market is taken by output changes generated by discrepancies between aggregate demand and production. The distinction between short- and long-term interest rates allows an account of what monetary policy can directly accomplish and what is at least more difficult to accomplish. While the central bank controls the short-term interest rate (as long as it exceeds the zero lower bound), consumption and in particular investment depend on the long-term rate. Finally, at the empirical level the incorporation of the yield curve opens up for a succinct indicator of expectations.

22.1 A dynamic IS-LM model

As in the previous chapter we consider a closed industrialized economy where manufacturing goods and services are supplied in markets with imperfect competition and prices set in advance by firms operating under conditions of abundant capacity. Time is continuous.

Let R_t denote the long-term real interest rate at time t (to be explained below). By replacing the short-term real interest rate in the aggregate demand function from the simple IS-LM model of the previous chapter by the long-term rate, we obtain a better description of aggregate demand:

$$\begin{aligned} Y_t^d &= C(Y_t - (\tau + T(Y_t)), R_t) + I(Y_t, R_t) + G \equiv D(Y_t, R_t, \tau) + G, \text{ where} \\ 0 &< D_Y < 1, D_R < 0, -1 < D_\tau = -C_{Y^p} < 0, \end{aligned}$$

Generally notation is as in the previous chapter although we shift from discrete to continuous time. We should thus interpret the flow variables as *intensities*. Disposable private income per time unit is $Y - \mathbb{T}$ where $\mathbb{T} = \tau + T(Y))$, $0 \leq T'(Y) < 1$, and τ is a constant parameter reflecting "tightness" of discretionary fiscal policy. The symbol G represents government purchases per time unit (spending on goods and services). To avoid too many balls in the air at the same time, we ignore stochastic elements both here and in the money market equation to follow.

The positive dependency of aggregate output demand on current aggregate income, Y, reflects primarily that private consumption depends positively on disposable income. That current disposable income has this role, reflects the empirically supported hypothesis that a substantial fraction of the households are credit-constrained. Perceived human wealth (the present value of the expected stream of after-tax labor income), which in standard consumer theory is a major determinant of consumption, is itself likely to depend positively on current earnings. Similarly, capital investment by demand-constrained firms will depend positively on current economic activity, Y, to the extent that this activity provides internal finance from corporate profits and signals the level of demand in the near future.

The negative dependency of aggregate demand on R reflects first and foremost that capital investment depends negatively on the expected long-term interest rate. Firm's investment in production equipment and structures is normally an endeavour with a lengthy time horizon. Similarly, the households' investment in durable consumption goods (including housing) is based on medium- or long-term considerations. A rise in R induces a negative substitution effect on current consumption and probably also, on average, a negative wealth effect. Increases in household's wealth, whether in the form of human wealth, equity shares and bonds, or housing estate, are triggered by reduction in the long-term interest rate.[1]

Because of the short-run perspective of the model, explicit reference to the available capital stock in the investment function, I, is suppressed.

The continuous-time framework is convenient because we avoid the oddity in period analysis of allowing asset markets to open only at the beginning or end of each period. The continuous-time framework is also convenient by making it easy to operate with different speeds of adjustment for different variables. Regarding the speed of adjustment to changes in demand, we shall operate with a tripartition as envisaged in Table 20.1. Output is understood to consist primarily of goods and services with elastic supply with respect to demand, in contrast to agricultural and mineral products and construction.

Table 20.1. Speed of adjustment of different variables to demand shifts

Variable	*Adjustment speed*
asset prices	high
output	medium
prices on output	low, here assumed nil

The model lets asset prices adjust immediately so that asset markets clear at any instant. The adjustment of output to demand takes time and is gradual. This is modeled as an error-correction:

$$\begin{aligned} \dot{Y}_t &\equiv \frac{dY_t}{dt} = \lambda(Y_t^d - Y_t), && (22.1) \\ &= \lambda(D(Y_t, R_t, \tau) + G - Y_t), && Y_0 > 0 \text{ given,} \end{aligned}$$

where $\lambda > 0$ is a constant adjustment speed. At any point in time the output intensity Y_t is predetermined. During the adjustment process also demand changes (since the output level and fast-moving asset prices are among the determinants of demand). The difference between demand and output is made up of changes in order books and inventories behind the scene. Indeed, the counterpart of $Y_t - Y_t^d$ in national income accounting is unintended inventory investment. In

[1] See, e.g., Case, Quigley, and Shiller (2005, 2011).

a more elaborate version of the model unintended positive or negative inventory investment should result in a feedback on subsequent demand and supply.[2]

The rest of the model consists of the following equations:

$$M_t = P_t L(Y_t, i_t), \qquad L_Y > 0,\ L_i < 0. \tag{22.2}$$

$$R_t = \frac{1}{q_t}, \tag{22.3}$$

$$r_t^e \equiv i_t - \pi_t^e, \tag{22.4}$$

$$\frac{1 + \dot{q}_t^e}{q_t} = r_t^e, \tag{22.5}$$

$$\pi_t \equiv \frac{\dot{P}_t}{P_t} = \pi. \tag{22.6}$$

Equation (22.2) is the same equilibrium condition for the money market as in the static IS-LM model. The variable, M_t, is the money stock which we may interpret either as the monetary base or money in the broader sense where private-bank-created money is included. To fix ideas, we choose the former interpretation since no private banking sector is visible in the model.

As financial markets in practice adjust very fast, the model assumes clearing in the asset markets at any instant. The real money demand function, $L(\cdot)$, depends positively on Y (viewed as a proxy for the number of transactions per time unit for which money is needed) and negatively on the short-term nominal interest rate, the opportunity cost of holding money. Like Y_t, the general price level, P_t, is treated as a *state variable*, thereby being historically determined and changing only gradually over time. For a given M_t, money market equilibrium is brought about by immediate adjustment of the short-term nominal interest rate, i_t, so that the available stock of money is willingly held.

In equation (22.3) appears the important "new" variable q_t, which is the real price of a long-term bond, here identified as an inflation-indexed *consol* (a perpetual bond) paying to the owner a stream of payments worth one unit of output per time unit in the indefinite future (no maturity date). The equation tells us that the *long-term* real interest rate at time t is the reciprocal of the real market price of a consol at time t. This is just another way of saying that the long-term rate, R_t, is defined as the internal rate of return on the consol. Indeed,

[2] In the post-war period changes in inventory stocks (inventory investment) account for less than 1% of GDP in the U.S. (Allesandria et al., 2010, Wen, 2011).

That the adjustment of output takes time and is gradual is empirically underpinned by, for instance, Sims (1998) and Estralla and Fuhrer (2002).

the internal rate of return is that number, R_t, which satisfies the equation

$$q_t = \int_t^{\infty} 1 \cdot e^{-R_t(s-t)} ds = \left[\frac{e^{-R_t(s-t)}}{-R_t} \right]_t^{\infty} = \frac{1}{R_t}. \tag{22.7}$$

Thus the long-term interest rate is that discount rate, R_t, which transforms the payment stream on the consol into a present value equal to the real market price of the consol at time t.[3] Inverting (22.7) gives (22.3). An alternative way of presenting the inflation-indexed consol is shown in Appendix A.

Equation (22.4) defines the ex ante *short-term* real interest rate as the short-term nominal interest rate minus the expected inflation rate. Next, equation (22.5) can be interpreted as a no-arbitrage condition saying that the expected real rate of return on the consol (including a possible expected capital gain or capital loss, depending on the sign of $\dot{q}_t^e$) must in equilibrium equal the expected real rate of return, r_t^e, on the short-term bond. We may think of both the short-term bond and the long-term bond as being government bonds. For given expectations ($\dot{q}_t^e$ and r_t^e), the real price of the consol instantaneously adjusts so as to make the available stock of consols willingly held. In general, in view of the higher risk associated with long-term claims, presumably a positive risk premium should be added on the right-hand side of (22.5). We shall ignore uncertainty, however, so that there is no risk premium.[4] Finally, equation (22.6) says that within the relatively short time perspective of the model, the inflation rate is constant at an exogenous level, π. The interpretation is that price changes mainly reflect changes in units costs and that these changes are relatively smooth.

We assume agents' expectations are rational (model consistent). As there is no uncertainty in the model (i.e., no stochastic elements), this assumption amounts to perfect foresight. We thus have $\dot{q}_t^e = \dot{q}_t$ and $\pi_t^e = \pi_t = \pi$. Therefore, equation (22.4) reduces to $r_t^e = i_t - \pi = r_t$ for all t. The wedge between the nominal short-term rate, i_t, which is relevant for the money market equilibrium in (22.2), and the nominal short-term rate that households and firms typically face in a credit market, is absent in the model because uncertainty and default risk are ignored.

[3] Similarly, in discrete time, with coupon payments at the end of each period, we would have

$$q_t = \sum_{s=t+1}^{\infty} \frac{1}{(1+R_t)^{s-t}} = \frac{\frac{1}{1+R_t}}{1 - \frac{1}{1+R_t}} = \frac{1}{R_t}.$$

Consols (though nominal) have been issued by UK governments occationally since 1751 and constitute only a small part of UK government debt. Their form, without a maturity date, make them convenient for dynamic analysis.

[4] If a *constant* risk premium were added, the dynamics of the model will only be slightly modified.

Whichever monetary policy regime to be considered below, the model can be reduced to two coupled first-order differential equations in Y_t and R_t. The first differential equation is (22.1) above. As to the second, note that from (22.3) we have $\dot{R}_t/R_t = -\dot{q}_t/q_t$. Substituting into (22.5), where $\dot{q}_t^e = \dot{q}_t$, and using again (22.3), gives

$$\frac{1}{q_t} + \frac{\dot{q}_t}{q_t} = R_t - \frac{\dot{R}_t}{R_t} = r_t^e = r_t = i_t - \pi, \tag{22.8}$$

in view of (22.4) with $r_t^e = r_t = i_t - \pi$. By reordering,

$$\dot{R}_t = (R_t - (i_t - \pi))R_t, \tag{22.9}$$

where the determination of i_t depends on the monetary regime.

Before considering alternative policy regimes, we shall emphasize an equation which is useful for the economic interpretation of the ensuing dynamics. Assuming absence of asset price bubbles (see below), the no-arbitrage formula (22.5) is equivalent to a statement saying that the market value of the consol equals the *fundamental value* of the consol. By fundamental value is meant the present value of the future dividends from the consol, using the (expected) future short-term interest rates as discount rates:

$$q_t = \int_t^\infty 1 \cdot e^{-\int_t^s r_\tau d\tau} ds, \qquad \text{so that} \tag{22.10}$$

$$R_t = \frac{1}{q_t} = \frac{1}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds} = \int_t^\infty w_{t,s} r_s ds,$$

$$\text{where } w_{t,s} \equiv \frac{e^{-\int_t^s r_\tau d\tau}}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds} \quad \text{and} \quad \int_t^\infty w_{t,s} ds = 1. \tag{22.11}$$

The fundamental value is the same as the solution to the differential equation for q given in (22.8), presupposing that there are no asset price bubbles (see Appendix B). The formula (22.11) follows by integration (see Appendix C). This formula shows that the long-term rate, R_t, is a weighted average of the expected future short-term rates, r_s, with weights proportional to the discount factor $e^{-\int_t^s r_\tau d\tau}$. The higher are the expected future short-term rates the lower is q_t and the higher is R_t.

If r_τ is expected to be a constant, r, then (22.10) simplifies to

$$R_t = \frac{1}{\int_t^\infty e^{-r(s-t)} ds} = \frac{1}{1/r} = r.$$

And if for example r_τ is expected to be increasing, we get

$$R_t = \frac{1}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds} > \frac{1}{1/r_t} = r_t.$$

22.2 Monetary policy regimes

We shall consider three alternative monetary policy regimes. The two first are *regime* m (money stock rule), where the real money supply is the policy tool, and *regime* i (fixed interest rate rule), where the short-term nominal interest rate is the policy tool. This second regime is by far the simplest one and is closer to what present-day monetary policy is about. Nevertheless regime m is also of interest, both because it has some historical appeal and because it yields impressive dynamics. In addition, regime m has partial affinity with what happens under a contra-cyclical interest rate rule. Our third monetary policy regime is in fact an example of such a rule, and we name it *regime* i'.

The assumption of perfect foresight means that the agents' expectations coincide with the prediction of our deterministic model. Once-for-all shocks may occur, but only so rarely that agents ignore the possibility that a new surprise may occur later. When a shock occurs, it fits intuition best to interpret the time derivative of a variable as a *right-hand* derivative, e.g., $\dot{Y}_t \equiv \lim_{\Delta t \to 0^+}(Y(t + \Delta t) - Y(t))/\Delta t$. This is also the way $\dot{q}_t$ and $\dot{P}_t$ should be interpreted if a shock at time t results in a kink on the otherwise smooth time path of q and P, respectively. In this interpretation $\dot{P}_t/P_t$ and $\dot{q}_t/q_t$ stand for *forward-looking* growth rates of the nominal price of goods and the real price of the consol, respectively.

Throughout the analysis the following variables are exogenous: the inflation rate, π, and the fiscal policy variables, τ and G. Depending on the monetary policy regime, an additional variable relating to the money market may be exogenous. The initial values, P_0 and Y_0, are historically given since in this short-run model it takes time not only for the price level but also the output level to change.

22.2.1 Policy regime m: Money stock rule

Here we assume that the central bank is capable of controlling the money stock. More specifically, the central bank finds the going inflation rate tolerable and pursues a monetary policy of maintaining the real money stock, M_t/P_t, at a constant level $m > 0$, by letting the nominal money supply follow the path:

$$M_t = P_0 e^{\pi t} m = M_0 e^{\pi t}.$$

A natural interpretation is that a part of the government budget deficit is financed by seigniorage: $\dot{M}_t/P_t = (M_t/P_t)\dot{P}_t/P_t = m\pi$.

Equation (22.2) then reads $L(Y_t, i_t) = m$. This equation defines i_t as an implicit function of Y_t and m, i.e.,

$$i_t = i(Y_t, m), \quad \text{with } i_Y = -L_Y/L_i > 0, \quad i_m = 1/L_i < 0. \tag{22.12}$$

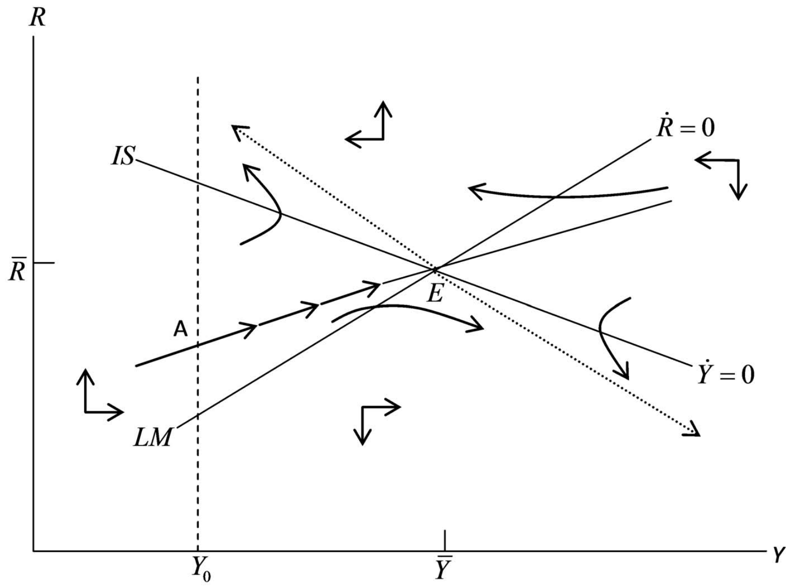

Figure 22.1: Phase diagram when m is the policy instrument.

Inserting this function into (22.9), we have

$$\dot{R}_t = [R_t - i(Y_t, m) + \pi] R_t. \tag{22.13}$$

This differential equation together with (22.1) constitutes a dynamic system in the two endogenous variables, Y_t and R_t. For convenience, we repeat (22.1) here:

$$\dot{Y}_t = \lambda(D(Y_t, R_t, \tau) + G - Y_t), \quad \lambda > 0, 0 < D_Y < 1, D_R < 0, D_\tau \in (-1, 0) \tag{22.14}$$

Phase diagram

As long as $R > 0$, (22.13) implies

$$\dot{R} \gtreqless 0 \qquad \text{for} \qquad R \gtreqless i(Y, m) - \pi, \qquad \text{respectively.} \tag{22.15}$$

We have $\frac{\partial R}{\partial Y}|_{\dot{R}=0} = i_Y = -L_Y/L_i > 0$, that is, for real money demand to equal a given real money stock, a higher volume of transactions must go hand in hand with a higher nominal short-term interest rate which in turn, for given inflation, requires a higher real interest rate. The $\dot{R} = 0$ locus is illustrated as the upward sloping curve, LM, in Fig. 22.1.

From (22.14) we have

$$\dot{Y} \gtreqless 0 \qquad \text{for} \qquad D(Y, R, \tau) + G \gtreqless Y, \qquad \text{respectively.} \tag{22.16}$$

We have $\frac{\partial R}{\partial Y}|_{\dot{Y}=0} = (1 - D_Y)/D_R < 0$, that is, higher aggregate demand in equilibrium requires a lower interest rate. The $\dot{Y} = 0$ locus is illustrated as the downward sloping curve, IS, in Fig. 22.1. In addition, the figure shows the direction of movement in the different regions, as described by (22.15) and (22.16).

The $\dot{R} = 0$ and $\dot{Y} = 0$ loci intersect at the point E with coordinates $(\bar{Y}, \bar{R})$. For this point to be a steady state obtainable by the economic system, it is required that $\bar{R} > 0$, since $R = 1/q$, and q is the real price of an inflation-indexed consol. Now, $\bar{R} = i(\bar{Y}, m) - \pi$. So, we assume that, given $\bar{Y}$ and π, m is small enough to make $i(\bar{Y}, m) - \pi > 0$, i.e., $\pi < i(\bar{Y}, m)$. If $i(\bar{Y}, m)$ is close to the lower bound, nil,[5] this requires that inflation is essentially negative, which amounts to deflation. To maintain m constant with $\pi < 0$ requires $\dot{M}_t/M_t < 0$ in this quasi- or short-run steady state. We use the the qualifier "short run" because presumably the economy will be subject to further dynamic feedbacks in the system (through a Phillips curve, changed capital stock due to investment, technological change etc.). Owing to the short time horizon, such feedbacks are ignored by the model. The short-run equilibrium may also be called a "short-run equilibrium".

In order to distinguish the short-run steady states from a "genuine" long-run steady state of an economy, we mark the steady-state values by a bar rather than an asterisk. We see that the steady state point, E, with coordinates $(\bar{Y}, \bar{R})$, is a saddle point.[6] So exactly two solution paths − one from each side − converge towards E. These two saddle paths, which together make up the stable arm, are shown in the figure (the slope of the stable arm must be positive, according to the arrows). Also the unstable arm is displayed in the figure (the negatively sloped stippled line which attracts the diverging paths).

The initial value of output, Y_0, is in this model predetermined, i.e., determined by Y's previous history; relative to the short time horizon of the model, output adjustment takes time. Hence, at time $t = 0$, the economy must be somewhere on the vertical line $Y = Y_0$. The question is then whether there can be rational asset price bubbles. An *asset price bubble,* also called a speculative bubble, is present if the market value of an asset for some notable stretch of time differs from its *fundamental value* (the present value of the expected future dividends from the asset, as defined in (22.10)). A *rational* asset price bubble is an asset price bubble that is consistent with the no-arbitrage condition (22.5) under rational expectations.

[5]The nominal interest rate can not go below 0 because agents prefer holding cash at zero interest (or slightly below to cover trivial safe-keeping costs associated with cash holding) rather than short-term bonds at negative interest.

[6]The determinant of the Jacobian matrix for the right-hand sides of the two differential equations, evaluated at the steady-state point, is $\lambda \left[\bar{R}(D_Y - 1) + \bar{R} i_Y D_R\right] < 0$. Hence, the two associated eigenvalues are of opposite sign. This is the precise general mathematical criterion for the steady state to be a saddle point.

Because consols have no terminal date and might be of a unique historical kind available in limited amount, a rational asset price bubble, driven by self-fulfilling expectations, not be ruled out within the model as it stands. In Fig. 22.1 any of the diverging paths with R ultimately falling, and therefore the asset price q ultimately rising, could in principle reflect such a bubble. A *negative* rational bubble can be ruled out, however. Essentially, this is because negative bubbles presuppose that the market price of the consol initially drops below the present value of future dividends (the right-hand side of (22.10)). But in such a situation everyone with rational expectations would want to buy the consol and enjoy the dividends. The resulting excess demand would immediately drive the asset price back to the fundamental value.

In view of its simplistic nature, the model does not provide an appropriate framework for bubble analysis. Here we will simply *assume* that the market participants never have bubbly expectations. An easy way to justify that assumption is to interpret the consols as just a convenient approximation to bonds with long but finite time to maturity (as most bonds in the real world). Now, when market participants never expect a bubbly asset price evolution, bubbles will not arise, hence the implosive paths of R in Fig. 22.1 can not materialize. The explosive paths of R in Fig. 22.1 have already been ruled out, as they would reflect negative bubbles.

We are left with the saddle path, the path AE in the figure, as the unique solution to the model. As the figure is drawn, $Y_0 < \bar{Y}$. The long-term interest rate will then be relatively low so that demand exceeds production and gradually pulls production upward. Hereby demand is stimulated, but less than one-to-one so, both because the marginal propensity to spend is less than one and because also the interest rate rises. Ultimately, say within a year, the economy settles down at the short-run steady state of the model − the short-run equilibrium.

Impulse-response dynamics

Let us consider the effects of level shifts in G and m, respectively. Suppose that the economy has been in its steady state until time t_0. In the steady state we have $r = i = \bar{R}$. Then either fiscal or monetary policy changes. The question is what the effects on r, R, and Y are. The answer depends very much on whether we consider an *unanticipated* change in the policy variable in question (G or m) at time t_0 or an *anticipated* change. As to an anticipated change, we can imagine that the government or the central bank at time t_0 credibly announces a shift to take place at time $t_1 > t_0$. From this derives the term "announcement effect", synonymous with "anticipation effect".

To prepare the ground, consider first the question: how are the IS and LM curves affected by shifts in G and m, respectively? We have, from (22.16), $\frac{\partial R}{\partial G}|_{\dot{Y}=0}$

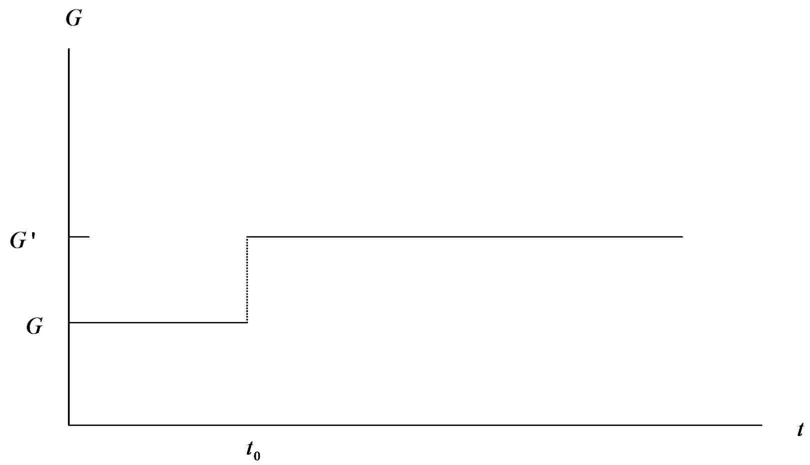

Figure 22.2: Unanticipated upward shift in G (regime m).

$= -1/D_R > 0$, that is, a shift to a higher G moves the $\dot{Y} = 0$ locus (the IS curve) upwards. But the $\dot{R} = 0$ locus is not affected by a shift in G. On the other hand, the $\dot{Y} = 0$ locus is not affected by a shift in m. But the $\dot{R} = 0$ locus (the LM curve) depends on m and moves downwards, if m is increased, since $\frac{\partial R}{\partial m}|_{\dot{R}=0} = i_m < 0$, from (22.15).

We now consider a series of policy changes, some of which are unanticipated, whereas others are anticipated.

(a) The effect of an unanticipated upward shift in G. Suppose the government is unsatisfied with the level of economic activity and at time $t_0 > 0$ decides (unexpectedly) an increase in G. And suppose people rightly expect this higher G to be maintained for a long time.

The upward shift in G is shown in Fig. 22.2.[7] When G shifts, the long-term interest rate jumps up to R_A, cf. Fig. 22.3, reflecting that the market value of the consol jumps down. The explanation is as follows. The higher G implies higher output demand, by (21.1). So an expectation of increasing Y arises (see (22.14)) and therefore also an expectation of increasing i and r, in view of (22.12). The implication is, by (22.10), a lower q_{t_0} and a higher R_{t_0}, as illustrated in Fig. 22.4. After t_0, output Y and the short-term rate r gradually increase toward their new steady state values, $\bar{Y}'$ and $\bar{r}'$, respectively, as shown by Fig. 22.4. As time proceeds and the economy gets closer to the expected high future values of r, these higher values gradually become dominating in the determination of R in

[7] Since m and τ are kept unchanged, the higher G may have to be partly debt financed and thus be associated with a higher amount of outstanding government bonds. Whether this is problematic is not our concern here.

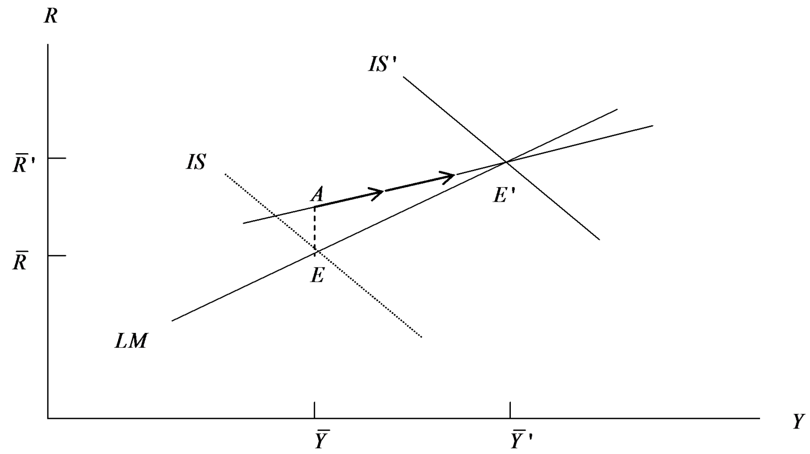

Figure 22.3: Phase portrait of an unanticipated upward shift in G.

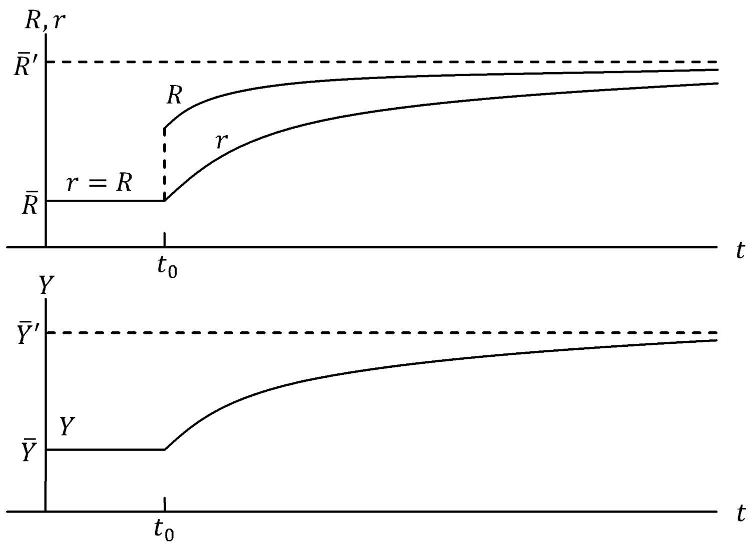

Figure 22.4: Time profiles of interest rates and output to an unanticipated shift in G (regime m).

(22.10). Hence, after t_0 also R gradually increases toward its new steady state value, the same as that for r.

By dampening output demand, the higher R implies a financial crowding-out effect on production.[8] After t_0, during the transition to the new steady state, we have $R > r$ because R "anticipates" all the future increases in r and incorporates them, cf. (22.10). Note also that (22.8) implies

$$R = r + \dot{R}/R \gtreqless r \qquad \text{for} \qquad \dot{R} \gtreqless 0, \qquad \text{respectively.}$$

For example, $\dot{R} > 0$ reflects that $\dot{q} < 0$, that is, a capital loss is expected. To compensate for this, the *level* of R (which always equals $1/q$) must be higher than r such that the no-arbitrage condition (22.5) is still satisfied.

Formulas for the steady-state effects of the change in G can be found by using the comparative statics method of Chapter 21 on the two steady-state equations

$$\bar{Y} = D(\bar{Y}, \bar{R}, \tau) + G \quad \text{and} \quad m = L(\bar{Y}, \bar{R})$$

with the two endogenous variables $\bar{Y}$ and $\bar{R}$ (Cramer's rule). Given the preparatory work already done, a more simple method is to substitute $\bar{R} = i(\bar{Y}, m) - \pi$ into the first-mentioned steady-state equation to get $\bar{Y} = D(\bar{Y}, i(\bar{Y}, m), \tau) + G$. Taking the differential on both sides gives $d\bar{Y} = D_Y d\bar{Y} + D_R i_Y d\bar{Y} + dG$, from which follows, by (22.12),

$$\frac{\partial \bar{Y}}{\partial G} = \frac{1}{1 - D_Y + D_R L_Y / L_i} > 0.$$

From $L(\bar{Y}, \bar{R}) = m$ we get $0 = L_Y d\bar{Y} + L_i d\bar{R} = L_Y(\partial \bar{Y}/\partial G)dG + L_i d\bar{R} = 0$ so that

$$\frac{\partial \bar{R}}{\partial G} = -\frac{L_Y / L_i}{1 - D_Y + D_R L_Y / L_i} > 0.$$

Since our steady-state equations corresponds exactly to the IS and LM equations for the static IS-LM model of Chapter 21, the output and interest rate multipliers w.r.t. G are the same.

As alluded to earlier, one should think about the steady state as only a quasi-steady state. That is, the role of the point $(\bar{Y}, \bar{R})$ is to act as an "attractor" in the short-run dynamics after a policy shift although the point itself would in a larger model be moving slowly due to medium-term dynamics coming from a Phillips curve and/or an increased capital stock. With appropriate parameter values in the model, its adjustment time will be "short". As a rough guess about the order

[8] The crowding out is only partial, because Y still increases.

of magnitude, eliminating 95% of the initial distance to the steady state point might take about a year, say.

Our treatment of the shift in G as permanent should not be interpreted literally. It is only meant to indicate that the fiscal stimulus is durable enough to really matter. A really permanent increase in G in this economy without economic growth might endanger fiscal sustainability, if the automatic budget reaction is not sufficient to, after a while, fully finance the increase in G.

(b) The effect of an anticipated upward shift in G. We assume that the private sector at time t_0 becomes aware that G will shift to a higher level at time t_1, cf. the upper panel of Fig. 22.4. The implied expectation that the short-term interest rate will in the future rise towards a higher level, $\bar{r}'$, immediately triggers an upward jump in the long-term rate, R. To what level? In order to find out, note that the market participants understand that from time t_1, the economy will move along the new saddle path corresponding to the new steady state, E', in Fig. 22.5. The market price, q, of the consol cannot have an *expected* discontinuity at time t_1, since such a jump would imply an infinite expected capital loss (or capital gain) per time unit immediately before $t = t_1$ by holding long-term bonds. Anticipating for example a capital loss, the market participants would want to sell long-term bonds in advance. The implied excess supply would generate an adjustment of q downwards until no longer a jump is expected to occur at time t_1. If instead a capital gain is anticipated, an excess demand would arise. This would generate in advance an upward adjustment of q, thus defeating the expected capital gain. This is the general principle that arbitrage prevents an expected jump in an asset price.

In the time interval (t_0, t_1) the dynamics are determined by the "old" phase diagram, based on the no-arbitrage condition which rules up to time t_1. In this time interval the economy must follow that path (AB in Fig. 22.5) for which, starting from a point on the vertical line $Y = \bar{Y}$, it takes precisely $t_1 - t_0$ units of time to reach the new saddle path. At time t_0, therefore, R jumps to exactly the level R_A in Fig. 22.5.[9] This upward jump has a contractionary effect on output demand. So output starts falling as shown by figures 20.5 and 20.6. This is because the potentially counteracting force, the increase in G, has not yet taken place. Not until time t_1, when G shifts to G', does output begin to rise. In the "long run" both Y, R, and r are higher than in the old steady state.

There are two interesting features. First, in regime m a credible announcement

[9] Note that R_A is unique. Indeed, imagine that the jump, $R_A - \bar{R}$, was smaller than in Fig. 22.5. Then, not only would there be a longer way along the road to the new saddle path, but the system would also start from a position closer to the "old" steady-state point, E. This implies an initially lower adjustment speed.

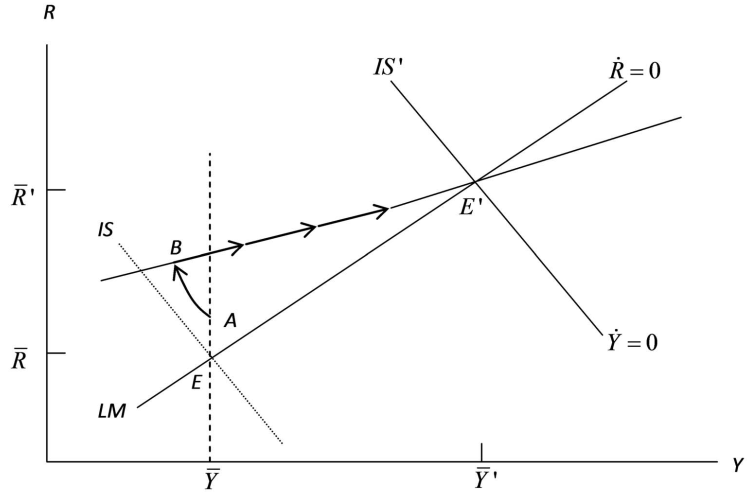

Figure 22.5: Phase portrait of an anticipated upward shift in G (regime m).

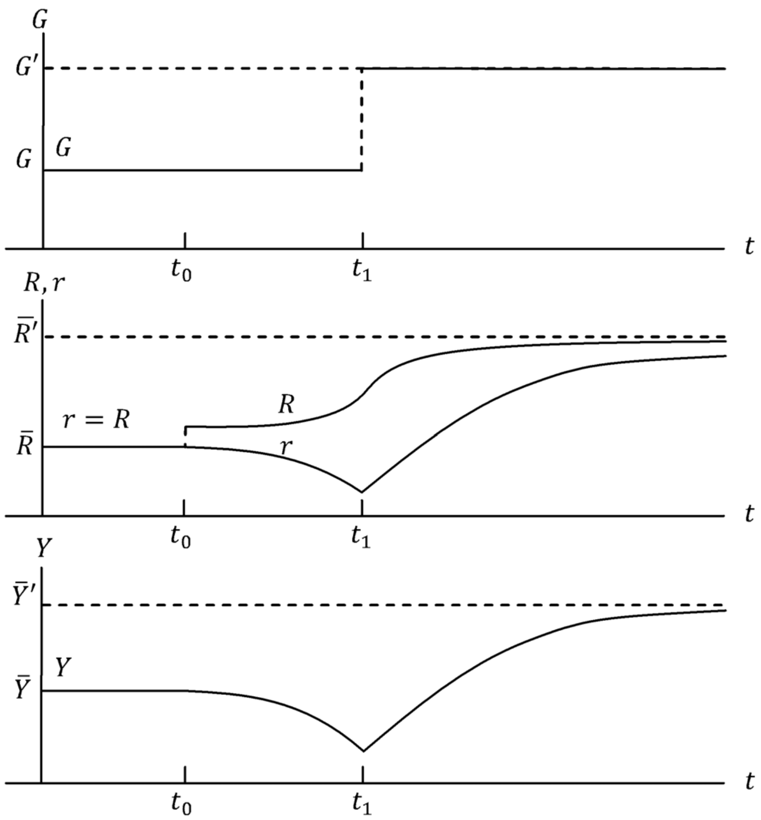

Figure 22.6: An anticipated upward shift in G and time profiles of interest rates and output (regime m).

of future expansive fiscal policy can have a temporary contractionary effect when the announcement occurs. This is due to financial crowding out. There is a way of dampening the problem, namely by letting the central bank announce prolonged open market operations to maintain i low for several years after time t_1, cf. policy regime i below. The second feature relates to the *term structure of interest rates*, also called the *yield curve.* The relationship between the internal rate of return on financial assets and their time to maturity is called the term structure of interest rates. Fig. 22.6 shows that the term structure "twists" in the time interval (t_0, t_1). The long-term rate R rises, because the time where a higher Y (and thereby a higher r) is expected to show up, is getting nearer. But at the same time the short-term rate r is falling because of the falling transaction need for money implied by the initially falling Y, triggered by the rise in the long-term interest rate.[10]

The theory of the term structure

What we have just seen is the *expectations theory of the term structure* in action. Empirically, the term structure of interest rates tends to be upward-sloping, but certainly not always and it may suddenly shift. The theory of the term structure of interest rates generally focuses on two explanatory factors. One is *uncertainty* and this factor tends to imply a positive slope because the greater uncertainty generally associated with long-term bonds generates a risk premium, known as the *term premium*, on these. The present model has nothing to say about this factor since the model ignores uncertainty.

Our model *has* something to say about the other factor, namely *expectations.* Indeed, the model quite well exemplifies what is called the *expectations theory of the term structure.* In its simplest form this theory ignores uncertainty and treats various maturities as perfect substitutes. The theory says that if the short-term interest rate is expected to rise in the future, the long-term rate today will tend to be higher than the short-term rate today. This is because, absent uncertainty, the long-term rate is a weighted average of the expected future short-term rates, as seen from (22.11). Similarly, if the short-term interest rate is expected to fall in the future, the long-term rate today will, everything else equal, tend to be lower than the short-term rate today. Thus, rather than explaining the statistical

[10] A conceivable objection to the model in this context is that it does not fully take into account that consumption and investment are likely to depend positively on expected *future* aggregate income, so that the hypothetical temporary decrease in demand and output never materializes. On the other hand, the model has in fact been seen as an explanation that president Ronald Reagan's announced tax cut in the USA 1981-83 (combined with the strict monetary policy aiming at disinflation) were followed by several years in recession. The concomitant tight monetary policy is an alternative or supplementary explanation of these events.

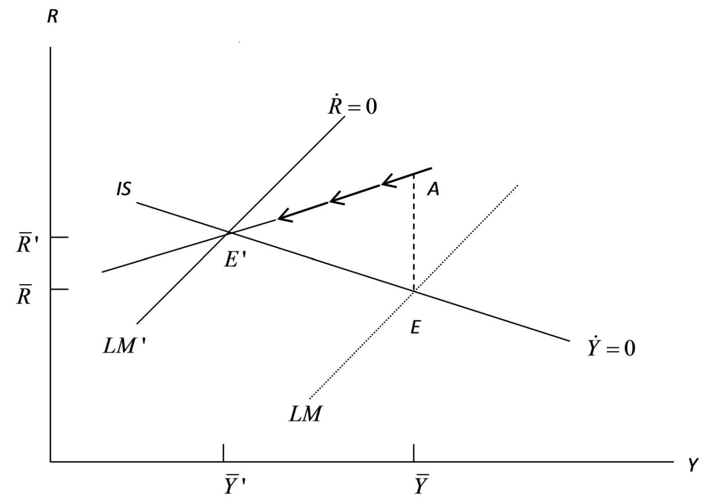

Figure 22.7: Phase portrait of an unanticipated downward shift in m (regime m).

tendency for the slope of the term structure to be positive, changes in expectations are important in explaining *changes* in the term structure. In practice, most bonds are denominated in money. Central to the theory is therefore the link between expected future inflation and the expected future short-term nominal interest rate. This aspect is not captured by the present short-run model, which ignores changes in the inflation rate.

(c) The effect of an unanticipated downward shift in m. The shift in m, brought about by sales of short-term bonds in the open market, is shown in the upper panel of Fig. 22.8. The shift triggers, at time t_0, an upward jump in the long-term rate R to the level of the new saddle path (point A in Fig. 22.7). The explanation is that the fall in money supply implies an upward jump in the short-term rate i, hence also R, at time t_0, cf. (22.10). As indicated by Fig. 22.8, the short-term rate will be expected to *remain* higher than before the decline in m. The rise in R triggers a fall in output demand and so output gradually adjusts downward as depicted in Fig. 22.8. The resulting decline in the transactions-motivated demand for money leads to the gradual fall in the short-term rate towards the new steady state level. This fall is "anticipated" by the long-term rate, which therefore, at every point in time after t_1, is lower than the short-term rate.

It is interesting that when the new policy is introduced, both R and r "overshoot" in their adjustment to the new steady-state levels. This happens, because,

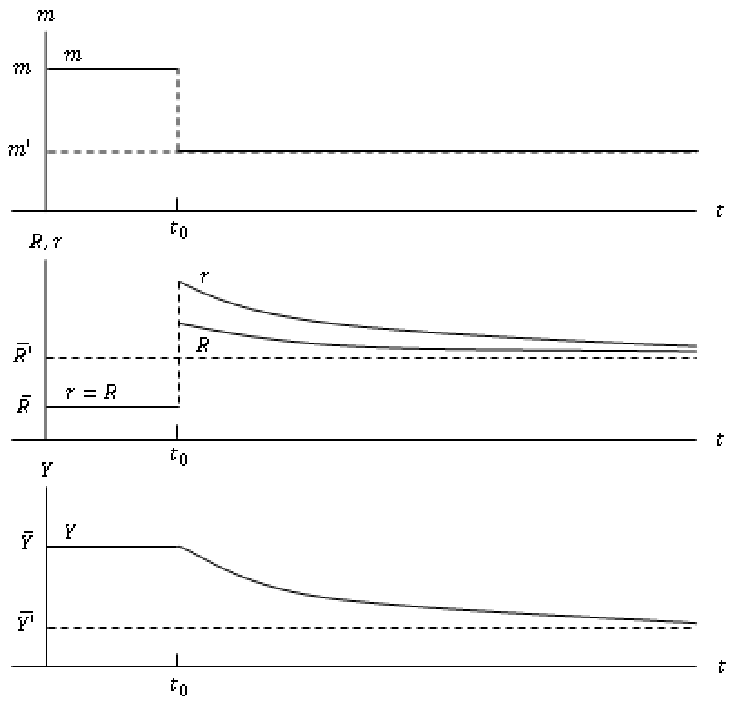

Figure 22.8: An unanticipated downward shift in m and time profiles of interest rates and output (regime m).

after t_0, both R and r have to be decreasing, parallel with the decreasing Y which implies lower money demand. Another noteworthy feature is that the yield curve is negatively sloped for some time after t_0.

Not surprisingly, there is not money neutrality. This is due, of course, to the price level being unaffected in the short run.

To find expressions for the steady-state effects of the change in m, we first take the differential on both sides of $D(\bar{Y}, i(\bar{Y}, m), \tau) + G = \bar{Y}$ to get $(1 - D_Y - D_R i_Y)d\bar{Y} = D_R i_m dm$. By (22.12), this gives

$$\frac{\partial \bar{Y}}{\partial m} = \frac{D_R/L_i}{1 - D_Y + D_R L_Y/L_i} > 0.$$

Hence, $d\bar{Y} = (\partial \bar{Y}/\partial m)dm < 0$ for $dm < 0$. From $m = L(\bar{Y}, \bar{R})$ we get $dm = L_Y d\bar{Y} + L_i d\bar{R} = L_Y(\partial \bar{Y}/\partial m)dm + L_i d\bar{R}$ so that

$$\frac{\partial \bar{R}}{\partial m} = \frac{(1 - D_Y)/L_i}{1 - D_Y + D_R L_Y/L_i} < 0.$$

Hence, $d\bar{R} = (\partial \bar{R}/\partial m)dm > 0$ for $dm < 0$. These multipliers are the same as those for the static IS-LM model of Chapter 21.

(d) The effect of an anticipated downward shift in m. The shift in m is announced at time t_0 to take place at time t_1, cf. Fig. 22.9. At the time t_0 of "announcement", R jumps to R_A and then gradually increases until time t_1. This is due to the expectation that the short-term rate will in the longer run be higher than in the old steady state. The higher R implies a lower output demand and so output gradually adjusts downward. Then also the short-term rate moves downward until time t_1. In the time interval (t_0, t_1) the dynamics are determined by the old phase diagram and the economy follows that path (AB in Fig. 22.9) for which, starting from a point on the vertical line $Y = \bar{Y}$, it takes precisely $t_1 - t_0$ units of time to reach the new saddle path. Since in the time interval (t_0, t_1), R increases, while r decreases, we again witness a "twist" in the term structure of interest rates, cf. Fig. 22.10.

Owing to the principle that arbitrage prevents an expected jump in an asset price, exactly at the time t_1 of implementation of the tight monetary policy, the economy reaches the new saddle path generated by the lower money supply (cf. the point B in Fig. 22.9). The fall in m triggers a jump upward in the short-term rate r. This is foreseen by everybody, but it implies no capital loss because the bond is short-term. Output Y continues falling towards its new low steady state level, cf. Fig. 22.10. The transactions-motivated demand for money decreases and therefore r gradually decreases towards the new steady-state level which is

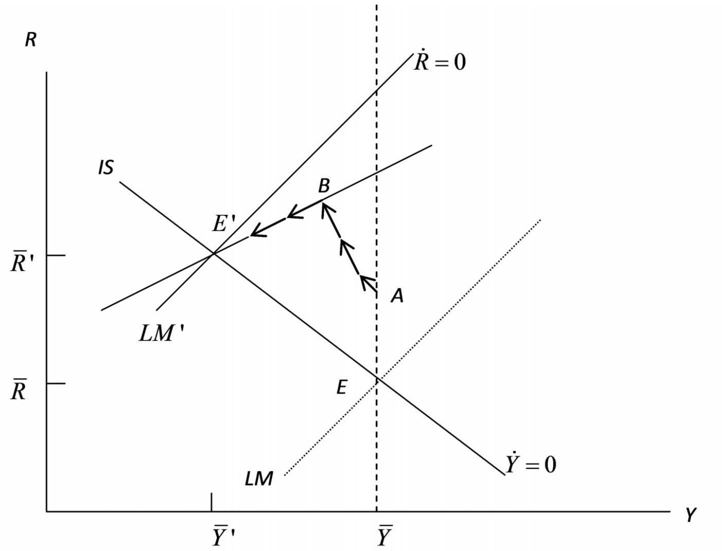

Figure 22.9: Phase portrait of an anticipated downward shift in m (regime m).

above the old because m is smaller than before. The long-term rate in advance "discounts" this gradual fall in r and is therefore, after t_1, always lower than r, implying a negatively sloped yield curve. Nevertheless, over time the long-term rate approaches the short-term rate in this model where there is no risk premium.

22.2.2 Policy regime i: Fixed short-term interest rate

Here we shall analyze a monetary policy regime where the short-term interest rate is the instrument. The model now takes i, the nominal interest rate on short-term government bonds, as an exogenous but adjustable constant. Thus, i is a policy instrument, together with the fiscal instruments, G and τ. Then the real money stock, m, has to be endogenous, which reflects that the central bank through open market operations adjusts the monetary base so that the actual short-term rate equals the one desired (and usually explicitly announced) by the central bank. Common names for this rate are the "target rate", the "policy rate", or "the official interest rate". In the real world, where there usually is a commercial banking sector, the central bank's target rate is often the so-called *interbank rate*. This is the interest rate charged on short-term (typically day-to-day) loans from one bank to another in the private banking sector, cf. Chapter 16. In the Euro area the ECB accordingly announces a certain target for the EONIA (euro overnight index average) and in the US the central bank announces a target for the Federal Funds Rate. Fig. 22.11 shows the evolution of the announced target for the Federal Funds Rate 1978-2013, stating dates of important economic and

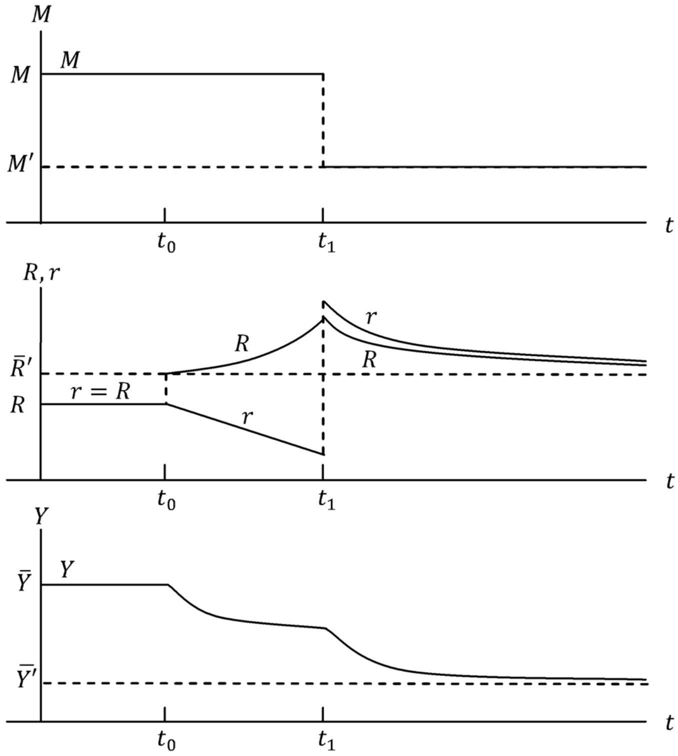

Figure 22.10: An anticipated downward shift in m and time profiles of interest rates and output (regime m).

political events over the period.

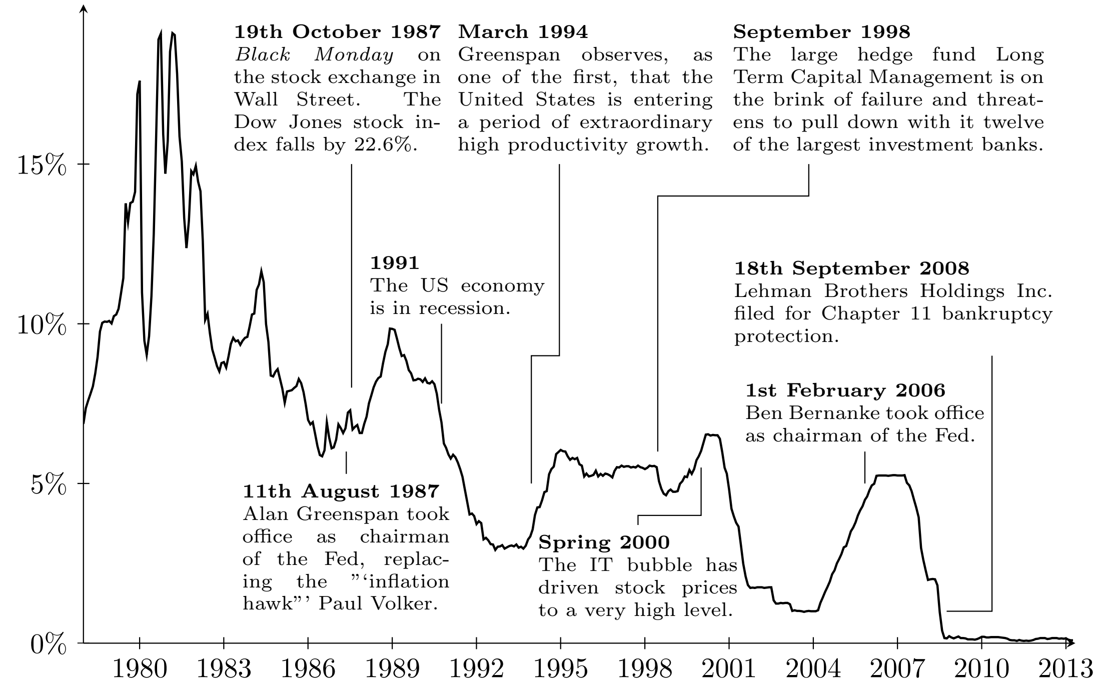

Figure 22.11: The evolution of the US federal funds rate 1978-2013. Source: Federal Reserve Bank of St. Louis.

The dynamic system

With $i > 0$ exogenous and m_t endogenous, the dynamic system consists of (22.9) and (22.1), which we repeat here for convenience:

$$\dot{R}_t = (R_t - i + \pi)R_t, \quad (22.17)$$
$$\dot{Y}_t = \lambda(D(Y_t, R_t, \tau) + G - Y_t), \quad \text{where} \quad (22.18)$$
$$0 < D_Y < 1, D_R < 0, -1 < D_\tau < 0.$$

Because Y_t does not appear in (22.17), the system (22.17) - (22.18) is simpler. The system determines the movement of R_t and Y_t. In the next step the required movement of M_t is determined by $M_t = P_t L(Y_t, i) = P_0 e^{\pi t} L(Y_t, i)$, from (22.2). In practice, an unchanged i will not be maintained forever but is likely to be adjusted according to the circumstances. Using a similar method as before we construct the phase diagram, cf. Fig. 22.12. The $\dot{R} = 0$ locus is now horizontal. The steady state is again a saddle point and is saddle-point stable. Notice, that here the saddle path *coincides* with the $\dot{R} = 0$ locus.

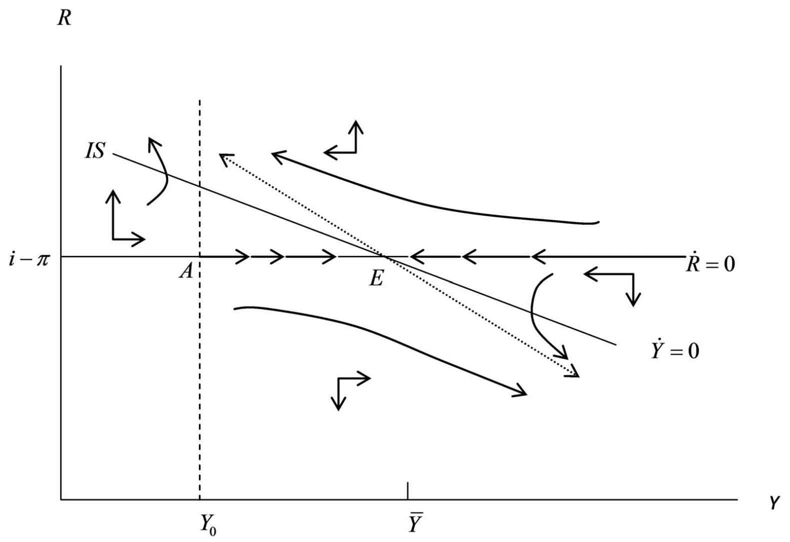

Figure 22.12: Phase diagram when i is the policy instrument.

Dynamic responses to policy changes when the short-term interest rate is the instrument

Let us again consider effects of permanent level shifts in exogenous variables, here G and i. Suppose that the economy has been in its steady state until time t_0. In the steady state we have $R = r = i - \pi$. Then either fiscal policy or monetary policy shifts. We consider the following three shifts in exogenous variables:

(a) An unanticipated decrease of G. See figures 22.13 and 22.14.

(b) An unanticipated decrease of i. See figures 22.15 and 22.16 in Appendix D.

(c) An anticipated decrease of i. See figures 22.17 and 22.18 in Appendix D.

As to the anticipated shift in i, we imagine that the central bank at time t_0 credibly announces the shift in i to take place at time $t_1 > t_0$.

The figures illustrate the responses. The diagrams should, by now, be self-explanatory. The only thing to add is that the reader is free to introduce another interpretation of, say, the exogenous variable G. For example, G could be interpreted as measuring consumers' and investors' "degree of optimism". The shift (a) could then be seen as reflecting the change in the "state of confidence" associated with the worldwide recession in 2001 or in 2008. The shift (b) could be interpreted as the immediate reaction of the Fed in the USA. As the public becomes aware of the general recessionary situation, further decreases of the federal

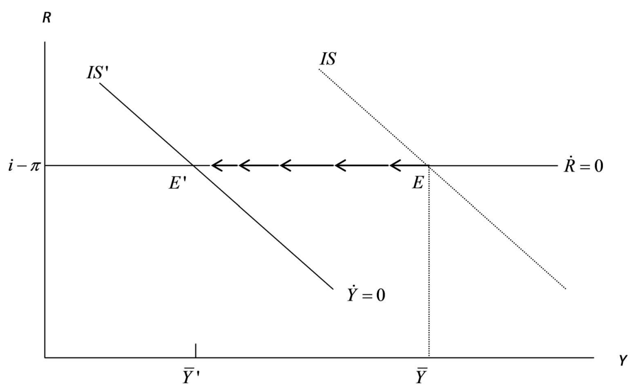

Figure 22.13: Phase portrait of an unanticipated downward shift in G (regime i).

funds rate, i, are expected and tends also to be executed. This is what point (c) is about.

22.2.3 Policy regime i': A contra-cyclical interest rate rule

Suppose the central bank conducts stabilization policy by using the interest rate rule

$$\begin{aligned} i_t &= \alpha_0 + \alpha_1(Y_t - Y^n) + \alpha_2(\pi_t - \hat{\pi}) = \alpha_0 + \alpha_1 Y_t - \alpha_1 Y^n + \alpha_2(\pi^e - \hat{\pi}) \\ &\equiv \alpha_0' + \alpha_1 Y_t, \end{aligned} \tag{22.19}$$

where α_0' is implicitly defined in (22.19), and Y^n is that level of output at which unemployment is at the NAIRU level, $\hat{\pi}$ is the desired inflation rate, and the α's are (in this model) constant policy parameters, $\alpha_1 > 0$. With a longer time horizon than in the present model, also the inflation rate would be treated as endogenous. A policy rule like (22.19) is known as a *Taylor rule.* The American economist John Taylor found the rule (with $\alpha_1 = 0.5$ and $\alpha_2 = 1.5$) to be a good description of actual U.S. monetary policy over a decade and at the same time a recommendable policy (Taylor, 1993).[11] Bernanke and Gertler (1959) present similar empirical evidence for Japan. Such a rule is called *contra-cyclical* because it dampens fluctuations in aggregate demand. This seems a better name than

[11] When inflation, π_t, and expected inflation, π_t^e, are endogenous, and one of the aims of monetary policy is to have a hold over the inflation rate, it is important to let $\alpha_2 > 1$ so that $r_t \equiv i_t - \pi_t$ goes up when π_t goes up.

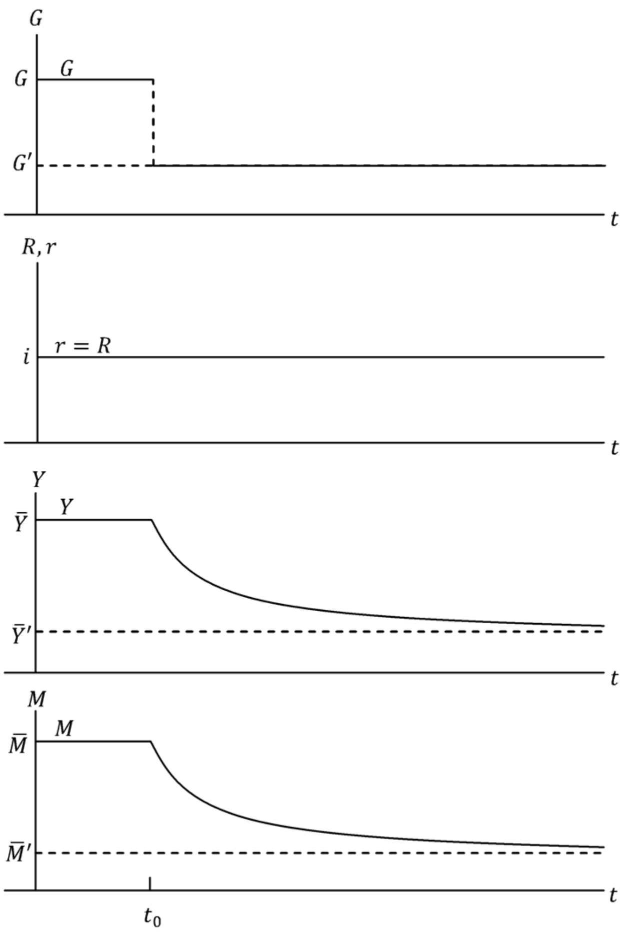

Figure 22.14: An unanticipated downward shift in G and time profiles of interest rates, output, and money supply (regime i; $\pi = 0$).

the sometimes used "counter-cyclical", which may lead to confusion because it generally refers to variables that are *negatively* correlated with aggregate output.

Given the policy rule (22.19), let us consider the dynamic system

$$\begin{aligned} \dot{Y}_t &= \lambda(D(Y_t, R_t, \tau) + G - Y_t), \qquad 0 < D_Y < 1, D_R < 0, -1 < D_\tau < 0, \\ \dot{R}_t &= [R_t - (\alpha_0' + \alpha_1 Y_t) + \pi] R_t. \end{aligned}$$

These two differential equations determine the time path of (Y_t, R_t) by a phase diagram similar to that in Fig. 22.1. Responses to unanticipated and anticipated changes in G are qualitatively the same as in regime m, where the money stock was the instrument. Qualitatively, the only difference is that the money stock is no longer an exogenous constant, but has to adjust according to

$$M_t = P_0 e^{\pi t} L(Y_t, \alpha_0' + \alpha_1 Y_t),$$

in order to let the contra-cyclical interest rule work. In Exercise 20.x the reader is asked to show that if $\alpha_1 > -L_Y/L_i$ and inflation is exogenous, this monetary policy regime is more stabilizing w.r.t. output than regime m.

22.3 Discussion

The previous chapter revisited the conventional *static* IS-LM model. Some micro-foundations for this were considered in chapters 19 and 20. In this chapter we have presented a dynamic version of the IS-LM model with endogenous forward-looking expectations. The model deals with the benchmark case of perfect foresight.

The framework captures the empirical tenet that output and employment in the short run tend to be demand-determined − with produced quantities and asset prices as the equilibrating factors, while the path of goods prices respond only little, or not at all, to changes in aggregate demand.

A limitation of simple IS-LM models, whether static or dynamic, is that they are silent about the intertemporal aspects of public and private budget constraints. In addition, aggregate behavior of the agents is postulated and not based on a weighted summation over the actions of different optimizing agent types. Yet the consumption and investment functions *can* to some extent be defended on a microeconomic basis.[12]

The forward-looking expectations in the model capture wealth effects through changes in the long-term interest rate, R. It would be an improvement if also the effect of expected future output demand on current consumption and investment were modeled. Even though we have in earlier chapters seen that Ricardian

[12] See Literature notes to Chapter 21.

equivalence is not plausible, this does not mean that expected future taxes should be ignored.

The simple process assumed for the adjustment of output to changes in demand is of course ad hoc. Nevertheless, it can be seen as a rough approximation to the theory of intended and unintended inventory investment (Wang and Wen, 2009, Wen 2011).

It is a simplification that changes in production and employment have no wage and price effects at all. At least in a medium-run perspective there should be wage and price responses within the model, i.e., some version of a Phillips curve. Then the issue arises under what conditions the dynamic interactions in the system after a disturbance tend to pull Y back to its NAIRU level or further away from it. This issue is taken up in Chapter 24 and later chapters.

The next chapter extends the present short-run framework to a small open economy.

22.4 Literature notes

Our presentation of Blanchard's dynamic IS-LM model builds on the version in Blanchard and Fischer (1989). In the original Blanchard (1981) paper, however, the key forward-looking variable is Tobin's q rather than the long-term interest rate, R. But since the (real) long-term interest rate can, in this context, be considered as inversely related to Tobin's q, there is essentially no difference. Wealth effects come true whether the source is interpreted as changes in Tobin's q or the long-term interest rate.

Treating the inflation rate as a state variable, changing only gradually, is empirically supported by, for instance, Sims (1998) and Estralla and Fuhrer (2002).

Extending the dynamic IS-LM model by some kind of a Phillips curve makes the model substantially more complicated. Blanchard (1981, last section) did in fact take a first step towards such an extension, ending up with a system of three coupled differential equations.

22.5 Appendix

A. An inflation-indexed consol

An alternative way of presenting the inflation-indexed consol is the following. The coupon per time unit at time s in the future amounts to P_s units of account, i.e., the price level at time s. This price level is related to the current price level,

P_t, via the evolution of inflation in the time interval (t, s),

$$P_s = P_t e^{\int_t^s \pi_\tau d\tau}.$$

Starting from a given nominal market value, Q_t, of the consol at time t, we thus have

$$\begin{aligned} Q_t &\equiv P_t q_t = \int_t^\infty P_s e^{-\int_t^s i_\tau d\tau} ds = P_t \int_t^\infty e^{\int_t^s \pi_\tau d\tau} e^{-\int_t^s i_\tau d\tau} ds \\ &= P_t \int_t^\infty e^{-\int_t^s (i_\tau - \pi_s) d\tau} ds = P_t \int_t^\infty e^{-\int_t^s r_\tau d\tau} ds, \end{aligned}$$

by $r_\tau \equiv i_\tau - \pi_s$. Dividing through by P_t gives (22.10).

B. Solving the no-arbitrage equation for q_t in the absence of asset price bubbles

In Section 22.1 we claimed that in the absence of asset price bubbles, the differential equation implied by the no-arbitrage equation (22.8) has the solution

$$q_t = \int_t^\infty 1 \cdot e^{-\int_t^s r_\tau d\tau} ds. \tag{22.20}$$

To prove this, we write the no-arbitrage equation on the standard form for a linear differential equation

$$\dot{q}_t - r_t q_t = -1.$$

The general solution to this is

$$q_t = q_{t_0} e^{\int_{t_0}^t r_\tau d\tau} - e^{\int_{t_0}^t r_\tau d\tau} \int_{t_0}^t e^{-\int_{t_0}^s r_\tau d\tau} ds.$$

Multiplying through by $e^{-\int_{t_0}^t r_\tau d\tau}$ gives

$$q_t e^{-\int_{t_0}^t r_\tau d\tau} = q_{t_0} - \int_{t_0}^t e^{-\int_{t_0}^s r_\tau d\tau} ds.$$

Rearranging and letting $t \to \infty$, we get

$$q_{t_0} = \int_{t_0}^\infty e^{-\int_{t_0}^s r_\tau d\tau} ds + \lim_{t\to\infty} q_t e^{-\int_{t_0}^t r_\tau d\tau}. \tag{22.21}$$

The first term on the right-hand side is the fundamental value of the consol, i.e., the present value of the future dividends on the asset. The second term on the right-hand side thus amounts to the difference between the market price, q_{t_0}, of the consol and its fundamental value. By definition, this difference represents a bubble. In the absence of bubbles, the difference is nil, and the market price, q_{t_0}, coincides with the fundamental value. So (22.20) holds (in (22.21) replace t by T and t_0 by t), as was to be shown.

C. Proof of (22.11)

CLAIM Let $q_t = \lim_{T\to\infty} \int_t^T e^{-\int_t^s r_\tau d\tau} ds < \infty$. Then:

$$\text{(i)} \quad \int_t^\infty e^{-\int_t^s r_\tau d\tau} r_s ds = 1; \qquad \text{and}$$

$$\text{(ii)} \quad \frac{1}{q_t} = \int_t^\infty w_{t,s} r_s ds, \quad \text{where } w_{t,s} \equiv \frac{e^{-\int_t^s r_\tau d\tau}}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds}.$$

Proof. The function $F(s) = e^{-\int_t^s r_\tau d\tau}$ has the derivative

$$F'(s) = -e^{-\int_t^s r_\tau d\tau} r_s.$$

Hence

$$\int_t^\infty e^{-\int_t^s r_\tau d\tau} r_s ds = -\int_t^\infty F'(s) ds = -\left. F(s)\right|_t^\infty = -\left. e^{-\int_t^s r_\tau d\tau}\right|_t^\infty = -(0-1) = 1.$$

This proves (i). We have

$$\frac{1}{q_t} = \frac{1}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds} = \frac{\int_t^\infty e^{-\int_t^s r_\tau d\tau} r_s ds}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds} = \int_t^\infty w_{t,s} r_s ds,$$

where the first equality follows from the definition of q_t, the second from (i), and the third by moving the constant $1/(\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds)$ inside the integral and then apply the definition of $w_{t,s}$. This proves (ii). □

D. More examples of dynamics in policy regime i

The figures 22.15 and 22.16 illustrate responses to an *unanticipated* lowering of the short-term interest rate, and figures 22.17 and 22.18 illustrate the responses to an *anticipated* lowering. Throughout it is assumed that $\pi = 0$.

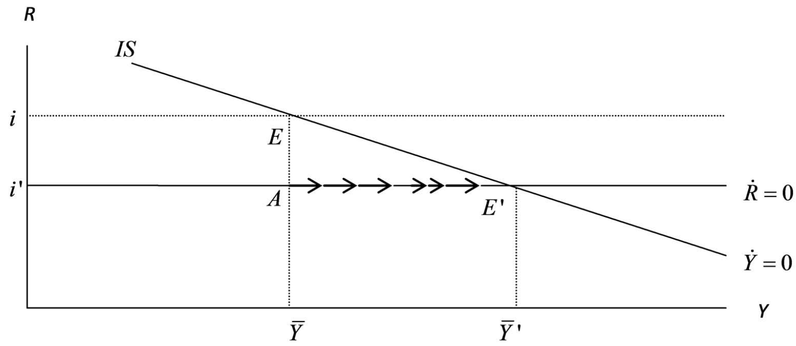

Figure 22.15: Phase portrait of an unanticipated downward shift in i (regime i, $\pi = 0$).

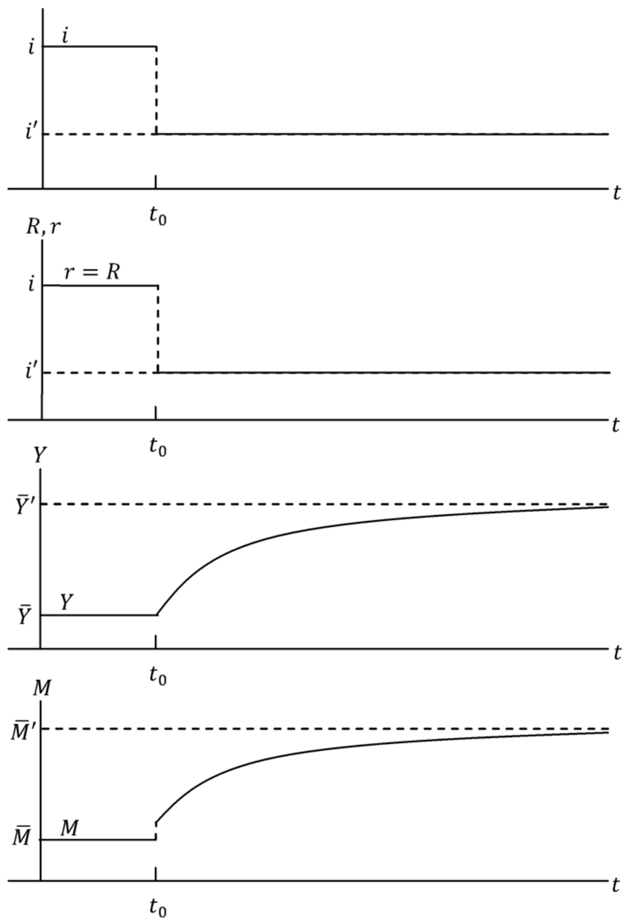

Figure 22.16: An unanticipated downward shift in i and time profiles of the long-term rate, output, and money supply (regime i, $\pi = 0$).

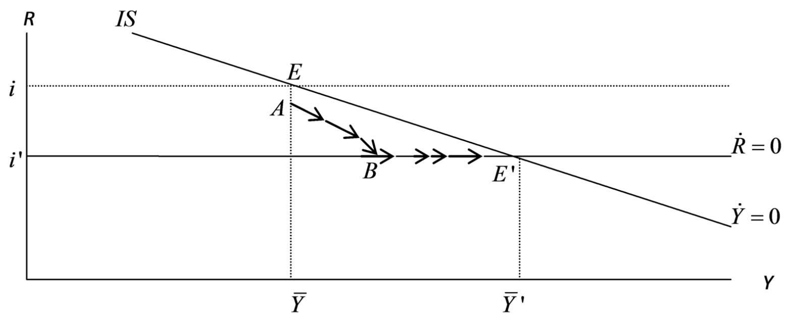

Figure 22.17: Phase portrait of an anticipated downward shift in i (regime i, $\pi = 0$).

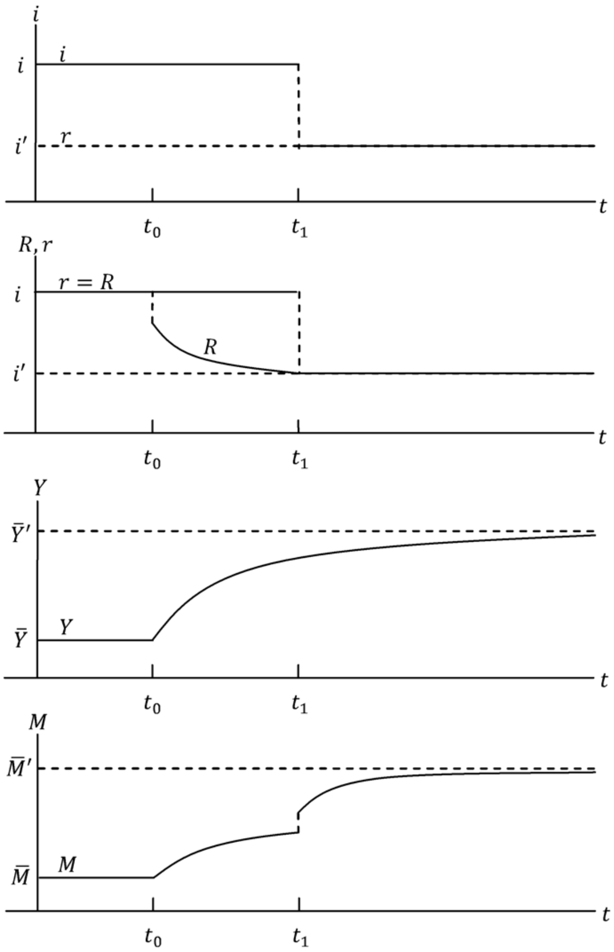

Figure 22.18: An unanticipated downward shift in i and time profiles of the long-term rate, output, and money supply (regime i, , $\pi = 0$).