Chapter 20

General equilibrium under monopolistic competition

20.1 The emergence of new-Keynesian economics

John Maynard Keynes' *General Theory of Employment, Interest and Money* (1936) came out in the midst of the Great Depression. It was an attempt to come to grips with this economic catastrophe and to find out policies for its cure and prevention in the future. On the one hand, breaking with Say's law, Keynes' book revolutionized the way economists thought about the economy as a whole. On the other hand, in many respects the analytical content of the book was incomplete.

Keynes' American followers, such as Paul Samuelson, Lawrence Klein, Franco Modigliani, Robert Solow, and James Tobin, were pragmatic and policy-oriented. Apart from incorporating a Phillips curve (linking price changes to the level of economic activity), they seemed satisfied with the basic logic of Keynes' theory. They viewed it as the relevant point of departure for the study of the short run, in particular when excess capacity and involuntary unemployment prevail (considered the normal state of affairs). The classical (pre-Keynesian) theory, relying on market clearing through flexible prices, was conceived suitable for studying not only the long run but also the short run *if and when* full employment had been achieved. This way of reconciling Keynes and the classics became known as the "neoclassical synthesis", a term coined by Paul Samuelson (1948?), or the "neoclassical-Keynesian" synthesis. We stick to the latter label, since nowadays "neoclassical" usually refers to supply-determined models with optimizing agents and flexible prices. It seems nowadays generally agreed that the "synthesis" was in fact no genuine synthesis at all, but rather a loose connection between different macroeconomic frameworks.

The *monetarists*, lead by Milton Friedman, attacked the policy activism of Keynesianism on the grounds of time lags in implementation, uncertainty about the relevant intervention, or mere government incompetence. The monetarists shared the notion that nominal rigidities are of importance for short run mechanisms, although in their view the "short run" was shorter than believed by the Keynesians. Of lasting influence were Friedman's "permanent income hypothesis" (Friedman 1957) and even more his emphatic claim that while there is usually a short-run trade-off between inflation and unemployment, there is no long-run trade-off.[1] The endogeneity of inflation expectations − in the long run it is "impossible to fool rational people" − was seen as implying this.

The *new-classical counter-revolution*, started by Robert Lucas, Thomas Sargent, and Neil Wallace in the early 1970s and later joined by Robert Barro and Edward Prescott, rejected Keynesian thinking altogether and started afresh. Or rather, they revived the classical or Walrasian line of thinking, emphasizing the equilibrating role of flexible prices under perfect competition not only as long-run theory, but also as short-run theory. Lucas' epoch-making contribution was the systematic incorporation of "rational expectations" into macroeconomics under conditions of uncertainty. When combined with the hypothesis of market clearing by price adjustment, this gave rise to the "policy-ineffectiveness proposition" claiming that systematic monetary policy designed to stabilize the economy is doomed to failure. Regarding the explanation of business cycle fluctuations, there were two different strands in this new classical approach. Lucas' *monetary misperception theory* (Lucas 1972 and 1975) emphasized shocks to the money supply as the primary driving force. In contrast, the *real business cycle theory* of Kydland and Prescott (1982) and Prescott (1986) views economic fluctuations as primarily caused by shocks to real factors, "productivity shocks". Yet, the two strands, which we consider in more detail in later chapters, were developed within the same type of stochastic modeling approach with a Walrasian foundation.

Partly in response to the challenges from this *new classical macroeconomics*, partly independently, other economists in the 1970s and the 1980s took a different line of attack. Their general perception was that the Keynesian approach, when extended by an expectations-augmented Phillips curve, performed well empirically. Money neutrality was generally seen as an acceptable approximation to long-run issues. But regarding short-run issues, refinements of the Keynesian theory along several dimensions were in need. At the same time such refinements could make use of new tools from microeconomic general equilibrium theory and the rational expectations methodology. We are here talking about a quite heterogeneous group of economists who are called *new Keynesians.* Their endeavour

[1] Friedman (1968). Almost simultaneously the same point was made by Edmund Phelps (1967, 1968).

became known as the *new Keynesian reconstruction* effort.

A first wave of contributions to this reconstruction was presented in the previous chapter, namely what is known as "macroeconomics with quantity rationing", where the focus is on the interaction between non-clearing markets when wages and prices are fixed. Little was said, however, about *by whom* and *how* wages and prices were set and why they were sticky. This is where the second wave of contributions took off by integrating agents with market power and sources of wage and price stickiness (e.g., Rotemberg, 1982, Akerlof and Yellen, 1985a, Mankiw, 1985, Blanchard, 1986, Blanchard and Kiyotaki, 1987).[2]

The previous chapter gave an introduction to some of the ideas involved. In continuation of this, the present chapter gives a systematic account of a general equilibrium model with monopolistic competition and nominal rigidities, where also a detailed picture of the household sector with endogenous labor supply is included, the Blanchard-Kiyotaki model (1987). Although the model is essentially static, it has served as an important building block in dynamic new-Keynesian models.

A reader who does not want to get too involved in analytical techniques, known as *CES gymnastics*, may in a first reading of this chapter glance over the rather long Section 20.2. This will allow concentration on the subsequent sections which deal with different types of general equilibrium, the role of menu costs, the interplay of nominal price rigidities and relative price rigidities, and macroeconomic implications.

20.2 The Blanchard-Kiyotaki model of monopolistic competition

Before going to the specifics of the Blanchard-Kiyotaki model (henceforth B-K model), let us recall what is meant by a *monopolistic competition* market structure:

1. There is a given "large" number, m, of firms and equally many (horizontally) differentiated products.

2. Each firm supplies its own differentiated product on which it has a monopoly and which is an imperfect substitute for the other products.

3. A price change by one firm has only a negligible effect on the demand faced by any other firm .

[2] A series of key articles from this period is collected in the two-volume edition Mankiw and Romer (1991).

Firms are "small" so that each good constitutes only a "small" fraction of the sales in the overall market system. Each firm faces a perceived downward-sloping demand curve and chooses a price which maximizes the firm's expected profit, thus implying a mark-up on marginal costs. There is no perceivable reaction from the firm's (imperfect) competitors. So the monopolistic competition setup abstracts from strategic interaction between firms and is in that respect different from oligopoly.

Sometimes a fourth property is included in the definition of monopolistic competition, namely that each firm makes zero profit. The interpretation is that there is a large set of as yet unexploited *possible* differentiated goods, and that there is free entry and exit. But in the present model entry and exit are considered to be costly and time consuming, and so the number of active firms is given in the short run.

The monopolistic competition framework is applied not only to firms, but also to households' labor supply. Each household is considered to be a wage setting supplier of its own specific type of labor, which is an imperfect substitute to other households' types of labor. Thus, both workers and firms have market power and face downward-sloping demand curves on the basis of which they make their pricing decisions. It may help the intuition to think of the households as organized in many small craft unions rather than as individual workers. In any case, in equilibrium each labor supplier sells a bit of her (his) labor to many firms.[3]

20.2.1 Overview of agents' decision problems

There are m firms, $i = 1, ..., m$, and m goods, one for each firm. The goods are imperfect substitutes (think of different kinds or brands of cars, beers, toothpaste etc.). Further, there are n households (or craft unions), $j = 1, ..., n$, each supplying a distinctive labor variety over which it has monopoly. These n types of labor are imperfect substitutes as inputs in the firms' production.

Fiat money is the only financial asset and is the numeraire. The model is essentially static in so far as only one period is considered. There is no private banking sector. There are "many" firms and "many" households (m and n are large).

The two basic decision problems

The decision problem of firm i is to choose a vector $(P_i, Y_i, (N_{ij})_{j=1}^n)$, where P_i is price, Y_i is output and N_{ij} is labor input of type j, $j = 1, 2, ..., n$, so as to

[3] Thus, monopsony is absent.

maximize

$$V_i = P_i Y_i - \sum_{j=1}^{n} W_j N_{ij} \qquad \text{s.t.} \tag{20.1}$$

$$Y_i = Y_i^d(P_i, ...), \tag{20.2}$$

$$Y_i = \left(\sum_{j=1}^{n} N_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{1}{\alpha}}, \tag{20.3}$$

where $Y_i^d(P_i, ...)$ is the demand function faced by the firm and the right-hand side of (20.3) is the production function.[4] The parameters describing the production function satisfy the inequalities $\sigma > 1, \alpha \geq 1$. The parameter σ is the constant elasticity of substitution between the different types of labor input, and α is the degree of *decreasing* returns to labor. When $\alpha > 1$, there are decreasing returns and when $\alpha = 1$, constant returns. Other inputs than labor are not considered.

The decision problem of household j is to choose a vector $((C_{ij})_{i=1}^m, M_j', W_j, N_j)$, where C_{ij} is consumption of good i, $i = 1, 2, ..., m$, M_j' is money holding at the end of the period, W_j is wage rate and N_j is labor supply, so as to maximize

$$U_j = \left[m^{\frac{1}{1-\theta}} \left(\sum_{i=1}^{m} C_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^{\gamma} (\frac{M_j'}{P})^{1-\gamma} - N_j^{\beta} \quad \text{s.t.} \tag{20.4}$$

$$N_j = N_j^d(W_j, ...) \tag{20.5}$$

$$\sum_{i=1}^{m} P_i C_{ij} + M_j' = M_j + W_j N_j + \sum_{i=1}^{m} V_{ij} \equiv I_j, \tag{20.6}$$

where $N_j^d(W_j, ...)$ is the labor demand function faced by household j, (20.6) is the budget constraint with I_j denoting total wealth of the household, consisting of the initial endowment of money, M_j, labor income, $W_j N_j$, and profits, V_{ij}, from firm i, $i = 1, 2, ..., m$.[5] As fiat money is the only non-human asset available (we imagine produced goods can not be stored), holding money at the end of the period is the only way to transfer purchasing power to the future. What should matter is then the *expected* real value, M_j'/P^e, of money transferred to the next period. Implicit in the formula (20.4) is thus the assumption that the expected general price level next period, P^e, equals the current price level, P.[6] Note that

[4] After having solved at least part of the *households*' decision problem below we shall be able to specify the demand function $Y_i^d(P_i, ...)$.

[5] After having solved at least part of the *firms*' decision problem below, we shall be able to specify the labor demand function $N_j^d(W_j, ...)$ in (20.5).

[6] In fact, Blanchard and Kiyotaki are not explicit about why money is demanded. Money

the consumption-and-money-holding term in the utility function is essentially of the same form as that in the previous chapter (take a logarithmic transformation and define $(1-\gamma)/\gamma \equiv \beta$).

The parameters describing the preferences satisfy the following inequalities: $0 < \gamma < 1, \theta > 1, \beta \geq 1$. The parameter θ is the constant elasticity of substitution between the different consumption goods, the parameter γ indicates the relative weight of consumption vis-a-vis money holding in the utility function. The coefficient $m^{1/(1-\theta)}$ in (20.4) just reflects a convenient normalization. The parameter β is 1+ elasticity of marginal disutility of work. When $\beta > 1$, there is increasing marginal disutility of work.

The symbol P denotes the "ideal" consumer price index corresponding to household j's preferences. Since the relevant sub-utility function, involving the m consumption goods and money, is homogeneous of degree 1, such an index exists. The index will be some function $\varphi(P_1, ..., P_m)$ of the prices of the consumption goods. This function (to be determined below) is closely related to a certain Lagrange multiplier and will depend on the parameters in the utility function. For now it suffices to note that P will be a kind of average of the actual prices − the "general price level".

20.2.2 The resulting behavior

Household j

It is convenient to define a consumption utility index C_j by

$$C_j \equiv m \left(m^{-1} \sum_{i=1}^{m} C_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} = m^{\frac{1}{1-\theta}} \left(\sum_{i=1}^{m} C_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}. \tag{20.7}$$

Such an index is called a CES index (CES stands for Constant Elasticity of Substitution). The index is normalized such that if the consumption basket contains equally much of each good, i.e., $C_{ij} = \bar{C}_j$, $i = 1, 2, ..., m$, then consumption utility is $C_j = m\bar{C}_j$.

A handy way of solving household j's decision problem is to divide the solution procedure into three steps. In the first step the choice between consumption expenditure and carrying money over to the next period is made. In the second step it is decided how to divide the consumption budget between the different consumption goods. And in the third step a decision on the supply of labor and the wage rate to claim.

holding just appears as an argument in the utility function. To fix ideas, we have chosen one possible interpretation. An alternative interpretation would be that money holding yields liquidity services.

As a preparation for step 1, let B_j be the consumption budget of household j, i.e.,

$$B_j \equiv \sum_{i=1}^{m} P_i C_{ij}. \tag{20.8}$$

Then, by definition of an ideal consumer price index P, cf. Box 20.1, in the optimal plan we must have

$$PC_j = B_j. \tag{20.9}$$

Box 20.1. An ideal price index $\varphi(P_1, ..., P_m)$
Let the function φ and the budget B_j^0 be given. Let the price vector $(P_1^0, \ldots, P_m^0)$ be such that $P^0 \equiv \varphi(P_1^0, \ldots, P_m^0) = 1$ and $(C_{1j}^0, \ldots, C_{mj}^0)$ is the forthcoming demand vector given the budget B_j^0. Then $P^0 C_j^0 = 1 \cdot C_j^0 = \sum_{i=1}^{m} P_i^0 C_{ij}^0 = B_j^0$. Then, imagine that some of the prices change and the new price vector is $(P_1, ..., P_m)$. By definition, an ideal price index is the minimum factor by which the original budget, B_j^0, must be multiplied if the consumer is to be fully compensated for the price change, i.e., the ideal price index equals the compensating budget multiplier. Hence, if the new value of the ideal price index is $P = \varphi(P_1, ..., P_m)$, then a new budget equal to $B_j = PB_j^0 = PC_j^0$ leaves the consumer as happy as before.

The CES utility function (20.7) is homogeneous of degree 1 in $C_{1j}, \ldots, C_{mj}$, reflecting that preferences are homothetic. That is, given the price vector $(P_1, \ldots, P_m)$, the corresponding demand vector $(C_{1j}, \ldots, C_{mj})$ is proportional to the consumption budget B_j. It is this property that will allow the construction of a meaningful price index, $P = \varphi(P_1, ..., P_m)$, indicating the minimum expense per unit of consumption utility, given the prices $P_1, ..., P_m$.[7] If there were ">" in (20.9), then the consumer has got higher utility than she can afford within the budget B_j, which is impossible; and if there were "<" in (20.9), then the consumer could increase utility within the given budget B_j. (END OF BOX)

Step 1(j) : Choosing between B_j and M_j' *(consumption versus holding money).* Consider the sub-utility function $\tilde{U}_j \equiv C_j^{\gamma}(M_j'/P)^{1-\gamma} = (B_j/P)^{\gamma}(M_j'/P)^{1-\gamma} = B_j^{\gamma} M_j'^{1-\gamma}/P$ and the problem: given the wealth I_j, choose B_j to maximize $\tilde{U}_j$ s.t. $B_j + M_j' = I_j$. After inserting the constraint and taking logs (assuming an interior solution), we solve the equivalent problem:

$$\max_{B_j} \widetilde{\tilde{U}_j} = \gamma \ln B_j + (1-\gamma)\ln(I_j - B_j) - \ln P.$$

[7] Because of the homogeneity of degree 1 of the CES utility function, it can be seen as an indicator of utility as well as quantity.

FOC:

$$\frac{d\widetilde{\tilde{U}_j}}{dB_j} = \gamma\frac{1}{B_j} + (1-\gamma)\frac{-1}{I_j - B_j} = 0,$$

which implies

$$B_j = PC_j = \gamma I_j$$

and, from $B_j + M_j' = I_j$,

$$M_j' = (1-\gamma)I_j.$$

It follows that the indirect utility function for consumption and money holding can be written

$$\Lambda_j = B_j^{\gamma} M_j'^{1-\gamma}/P = (\gamma I_j)^{\gamma}\left[(1-\gamma)I_j\right]^{1-\gamma}/P \equiv \mu I_j/P, \tag{20.10}$$

where μ denotes the constant marginal utility of wealth, $\mu \equiv \gamma^{\gamma}(1-\gamma)^{1-\gamma}$).

Step 2(j) : Choosing C_{ij}, $i = 1, 2, ..., m$ (the consumption bundle), given the consumption budget.
Given B_j,

$$\begin{aligned}\max_{C_{1j},\ldots,C_{mj}} C_j &= m^{\frac{1}{1-\theta}}\left(\sum_{i=1}^{m} C_{ij}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \qquad \text{s.t.}\\ \sum_{i=1}^{m} P_i C_{ij} &= B_j.\end{aligned}$$

For solving this problem, we shall apply the Lagrange method because it delivers a Lagrange multiplier which has a useful economic interpretation. We introduce the Lagrangian

$$\mathcal{L} = C_j - \lambda(\sum_{i=1}^{m} P_i C_{ij} - B_j),$$

where λ is the Lagrange multiplier.

FOCs: $\partial\mathcal{L}/\partial C_{ij} = 0$, $i = 1, 2, ..., m$, i.e.,

$$\frac{\partial C_j}{\partial C_{ij}} = \lambda P_i, \qquad i = 1, 2, ..., m. \tag{*}$$

From the definition of C_j we have

$$\begin{aligned}
\frac{\partial C_j}{\partial C_{ij}} &= m^{\frac{1}{1-\theta}} \frac{\theta}{\theta-1} \left(\sum_{i'=1}^{m} C_{i'j}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}-1} \frac{\theta-1}{\theta} C_{ij}^{\frac{\theta-1}{\theta}-1} \\
&= m^{\frac{1}{1-\theta}} \left(\sum_{i'=1}^{m} C_{i'j}^{\frac{\theta-1}{\theta}} \right)^{\frac{1}{\theta-1}} C_{ij}^{\frac{-1}{\theta}} = m^{\frac{1}{1-\theta}} \left[\left(\sum_{i'=1}^{m} C_{i'j}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^{\frac{1}{\theta}} C_{ij}^{\frac{-1}{\theta}} \\
&= m^{\frac{1}{1-\theta}} \left[\frac{C_j}{m^{\frac{1}{1-\theta}}} \right]^{\frac{1}{\theta}} C_{ij}^{\frac{-1}{\theta}} \qquad \text{(from (20.7))} \\
&= \lambda P_i \qquad \text{(from (*))}
\end{aligned}$$

so that from

$$m^{\frac{\theta}{1-\theta}} \frac{C_j}{m^{\frac{1}{1-\theta}}} C_{ij}^{-1} = m^{-1} \frac{C_j}{C_{ij}}$$

follows

$$m^{-1} \frac{C_j}{C_{ij}} = (\lambda P_i)^{\theta},$$

or

$$C_{ij} = (\lambda P_i)^{-\theta} \frac{C_j}{m}, \qquad i = 1, 2, ..., m. \tag{20.11}$$

Since we consider maximization and the Lagrange function $\mathcal{L}$ is concave, the first-order conditions (20.11) are both necessary and sufficient conditions for an interior optimum.

We see from (20.11) that

$$\frac{C_{ij}}{C_{hj}} = \left(\frac{P_i}{P_h} \right)^{-\theta} = \left(\frac{P_h}{P_i} \right)^{\theta}.$$

That is, θ is the elasticity of substitution between goods i and h.

Interestingly, the Lagrange multiplier λ is closely related to the consumer price index P. Indeed:

Claim 1. $\lambda = 1/P$.

Proof. Multiply by C_{ij} in (*) to get

$$\begin{aligned}
\frac{\partial C_j}{\partial C_{ij}} C_{ij} &= \lambda P_i C_{ij} \qquad \Rightarrow \\
\sum_{i=1}^{m} \frac{\partial C_j}{\partial C_{ij}} C_{ij} &= \lambda \sum_{i=1}^{m} P_i C_{ij} \qquad \Rightarrow \\
C_j &= \lambda \sum_{i=1}^{m} P_i C_{ij} \qquad \text{(from Euler's theorem on homogenous functions)} \\
&= \lambda B_j \qquad \text{(from (20.8))} \\
&= \lambda P C_j \qquad \text{(from (20.9))}.
\end{aligned}$$

Hence, since $C_j > 0, \lambda = 1/P$. □

Note that in view of Claim 1, we can write (20.11) as

$$C_{ij} = \left(\frac{P_i}{P}\right)^{-\theta} \frac{C_j}{m}, \qquad i = 1, 2, ..., m. \tag{20.12}$$

In this expression the factor C_j represents total real spending on consumption by household j and thus the factor C_j/m represents average real spending per consumption good. This will equal the actual demand for each good if the m prices are the same. But if for example $P_i < P$, we get $C_{ij} > C_j/m$ so that consumption of good i exceeds average consumption per good.

But how is the price index, P, determined?

Claim 2.

$$P = \left(\frac{1}{m} \sum_{i=1}^{m} P_i^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{20.13}$$

Proof. From (20.9), (20.8) and (20.12),

$$\begin{aligned}
P C_j &= B_j = \sum_{i=1}^{m} P_i C_{ij} = \sum_{i=1}^{m} P_i \left(\frac{P_i}{P}\right)^{-\theta} \frac{C_j}{m} \qquad \Rightarrow \\
P &= \left(\frac{1}{P}\right)^{-\theta} \sum_{i=1}^{m} P_i^{1-\theta} \frac{1}{m} \qquad \Rightarrow \\
P^{1-\theta} &= \frac{1}{m} \sum_{i=1}^{m} P_i^{1-\theta},
\end{aligned}$$

from which follows (20.13). □

Note that the price index in (20.13) is a kind average of the m prices in the sense that it is homogeneous of degree 1 and has the property that if $P_i = \bar{P}$ for all i, then $P = \bar{P}$.

Making use of the indirect utility function Λ_j in (20.10), we are now ready to set foot on the third step, the decision on the wage rate and the supply of labor given the labor demand function $N_j^d(W_j, ...)$. However, this decision problem is not well-defined until we have specified the labor demand function. This requires that we first turn to the firms' behavior.

Solving the problem of firm i

It is convenient to define an "effective labor input" index L_i symmetrically to the consumption utility index C_j above:

$$L_i \equiv n\left(n^{-1}\sum_{j=1}^{n} N_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = n^{\frac{1}{1-\sigma}}\left(\sum_{j=1}^{n} N_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}. \tag{20.14}$$

If $N_{ij} = \bar{N}_i$ for all j, then the definition implies $L_i = n\bar{N}_i$. The production function (20.3) can now be written

$$Y_i = (n^{\frac{1}{\sigma-1}} L_i)^{\frac{1}{\alpha}}. \tag{20.15}$$

A convenient way of solving firm i's decision problem, see (20.1) - (20.3), is to divide the solution procedure into three steps that are in principle symmetric with the three steps for the household. In the first step we find the required effective labor input, given the desired level of output. In the second step it is decided how many units of the different types of labor to use in order to make up the desired effective labor input. And in the third step the price and output supply are decided.

Step 1(i) : Finding the effective labor input, L_i, required to obtain a given output level, Y_i.
Given Y_i, find the required effective labor input L_i. From (20.15) we get the solution

$$L_i = n^{\frac{1}{1-\sigma}} Y_i^{\alpha}. \tag{**}$$

Step 2(i) : Choosing N_{ij}, $j = 1, 2, ..., n$ (the labor type mix), given the desired effective labor input $\bar{L}_i$.

Given a desired effective labor input $L_i = \bar{L}_i$, firm i solves the problem:

$$\min_{(N_{ij})_{j=1}^n} \sum_{j=1}^n W_j N_{ij} \qquad \text{s.t.}$$

$$n^{\frac{1}{1-\sigma}} \left(\sum_{j=1}^n N_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} = \bar{L}_i.$$

Again, for solving such a problem, the Lagrange method is convenient because it delivers a Lagrange multiplier which has a useful economic interpretation. Therefore, we introduce the Lagrangian

$$\mathcal{L} = \sum_{j=1}^n W_j N_{ij} - \eta \left[n^{\frac{1}{1-\sigma}} \left(\sum_{j=1}^n N_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \bar{L}_i \right],$$

where η is the Lagrange multiplier. FOCs: $\partial \mathcal{L}/\partial N_{ij} = 0$, $j = 1, 2, ..., n$, i.e.,

$$W_j = \eta \frac{\partial L_i}{\partial N_{ij}}, \qquad j = 1, 2, ..., n. \tag{***}$$

From the definition of L_i we have

$$\begin{aligned}
\frac{\partial L_i}{\partial N_{ij}} &= n^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma - 1} \left(\sum_{j'=1}^n N_{ij'}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}-1} \frac{\sigma - 1}{\sigma} N_{ij}^{\frac{\sigma-1}{\sigma}-1} \\
&= n^{\frac{1}{1-\sigma}} \left(\sum_{j'=1}^n N_{ij'}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} N_{ij}^{\frac{-1}{\sigma}} = n^{\frac{1}{1-\sigma}} \left[\left(\sum_{j'=1}^n N_{ij'}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{1}{\sigma}} N_{ij}^{\frac{-1}{\sigma}} \\
&= n^{\frac{1}{1-\sigma}} \left[\frac{L_i}{n^{\frac{1}{1-\sigma}}} \right]^{\frac{1}{\sigma}} N_{ij}^{\frac{-1}{\sigma}} \qquad \text{(from (20.14))} \\
&= \frac{W_j}{\eta}, \qquad \text{(from (***))}
\end{aligned}$$

so that from

$$n^{\frac{\sigma}{1-\sigma}} \frac{L_i}{n^{\frac{1}{1-\sigma}}} N_{ij}^{-1} = n^{-1} \frac{L_i}{N_{ij}}$$

follows

$$n^{-1} \frac{L_i}{N_{ij}} = (\frac{W_j}{\eta})^{\sigma},$$

or

$$N_{ij} = (\frac{W_j}{\eta})^{-\sigma} \frac{L_i}{n}, \qquad j = 1, 2, ..., n. \tag{20.16}$$

Since we consider a minimization problem and the Lagrange function $\mathcal{L}$ is convex, the first-order conditions (20.16) are both necessary and sufficient conditions for an interior optimum.

We see from (20.16) that

$$\frac{N_{ij}}{N_{ik}} = \left(\frac{W_j}{W_k}\right)^{-\sigma} = \left(\frac{W_k}{W_j}\right)^{\sigma}.$$

That is, σ is the elasticity of substitution between labor types j and k.

Let W denote the "ideal" wage level index, i.e., the minimum cost per unit of effective labor. Then, in the optimal plan,

$$WL_i = \sum_{j=1}^{n} W_j N_{ij}.$$

By the same method of proof as with Claim 1 and 2, it is easy to show that

$$\eta = W = \left(\frac{1}{n}\sum_{j=1}^{n} W_j^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{20.17}$$

Now, (20.16) can be written

$$\begin{aligned} N_{ij} &= \left(\frac{W_j}{W}\right)^{-\sigma} \frac{L_i}{n} \qquad (20.18) \\ &= \left(\frac{W_j}{W}\right)^{-\sigma} n^{\frac{\sigma}{1-\sigma}} Y_i^{\alpha}, \qquad j = 1, 2, ..., n. \end{aligned}$$

by (**). In the expression (20.18) the factor L_i represents total effective employment in firm i and thus L_i/n is average employment per labor type. This will be the actual employment of each labor type if they demand the same wage. But if for instance $W_j < W$, we get $N_{ij} > L_i/n$. That is, employment in firm i of labor type j will exceed average employment per labor type.

Note that the ideal wage level index and the labor demand function are symmetric to the consumer price index and the consumption demand functions, respectively, found above.[8]

[8] In step 2 for the household, we maximized utility for a given budget, while in step 2 for the firm, we minimized costs for a given output level. Therefore, in the first case we got $\lambda = 1/P$, while in the second case we got $\eta = W$. In fact, also in the household's problem one could formulate step 2 as a minimization problem, namely that of minimizing consumption expenditure, $\sum_{i=1}^{m} P_i C_{ij}$, for a given utility level C_j. The corresponding Lagrange multiplier, say λ', would satisfy $\lambda' = P$.

We are now ready to look at firm i's third step:

Step 3(i) : Setting the price P_i and the supply of output Y_i.
The problem is:

$$\begin{aligned}
\max_{P_i,Y_i} V_i &= P_iY_i - WL_i \qquad \text{s.t.} \\
Y_i &= Y_i^d = \sum_{j=1}^{n} C_{ij} = \left(\frac{P_i}{P}\right)^{-\theta} \frac{\gamma\sum_{j=1}^{n} I_j}{mP}, \\
L_i &= n^{\frac{1}{1-\sigma}} Y_i^{\alpha}, \qquad (**)
\end{aligned}$$

that is, maximize profits subject to the demand function (from (20.12) and step 1(j)) and the inverse of the production function. For this decision problem it is important that m is "large" so that the effect of P_i on the "average" price level P is negligible.

Before solving the problem it is convenient to introduce the dependence of demand on national income. We define (as usual for a closed economy) national income, Y, as aggregate value added in real terms, i.e.,

$$\begin{aligned}
Y &\equiv \frac{\sum_{i=1}^{m} P_iY_i}{P} = \frac{\sum_{i=1}^{m} P_i(\sum_{j=1}^{n} C_{ij})}{P} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} P_iC_{ij}}{P} \\
&= \frac{\sum_{j=1}^{n}\sum_{i=1}^{m} P_iC_{ij}}{P} = \frac{\sum_{j=1}^{n} B_j}{P} = \frac{\gamma\sum_{j=1}^{n} I_j}{P}, \qquad (20.19)
\end{aligned}$$

where the last equality comes from the solution for B_j in step 1(j). By setting $Y_i = \sum_{J=1}^{n} C_{ij}$, $i = 1,\ldots,m$, we have here assumed equilibrium in the goods markets. In view of (20.19), the demand function faced by firm i can be written

$$Y_i = Y_i^d = \left(\frac{P_i}{P}\right)^{-\theta} \frac{Y}{m}. \qquad (20.20)$$

We are now ready to solve the supply and price setting problem of firm i. Let $P_i(Y_i)$ denote the maximum price at which output Y_i can be sold; this $P_i(Y_i)$ is given as the inverse of the demand function (20.20). Now, after inserting this and (**) into the objective function of firm i, the problem is:

$$\max_{Y_i} V_i = P_i(Y_i)Y_i - Wn^{\frac{1}{1-\sigma}}Y_i^{\alpha} = TR - TC, \qquad (20.21)$$

where TR is total revenue and TC is total cost.

FOC:

$$\begin{aligned}
\frac{dV_i}{dY_i} &= P_i + Y_i\frac{dP_i}{dY_i} - Wn^{\frac{1}{1-\sigma}}\alpha Y_i^{\alpha-1} = MR - MC = 0. \qquad \text{Now,} \\
MR &= P_i\Big(1+\frac{Y_i\frac{dP_i}{dY_i}}{P_i}\Big) = P_i\Big(1+\frac{1}{\frac{P_i}{Y_i}\frac{dY_i}{dP_i}}\Big) = P_i\Big(1-\frac{1}{\theta}\Big) \qquad \text{(by (A5))} \\
&= MC \Rightarrow \\
P_i &= \frac{\theta}{\theta-1}MC. \qquad (20.22)
\end{aligned}$$

This is the standard pricing principle from monopoly theory: the profit maximizing price is a mark-up on marginal cost. The mark-up is higher, the lower is the substitutability between the consumption goods as measured by θ. We have assumed $\theta > 1$ because otherwise no equilibrium with price setters can exist.

When $\alpha > 1$, there are decreasing returns to labor, hence, MC is itself endogenous:

$$\begin{aligned}
MC &= Wn^{\frac{1}{1-\sigma}}\alpha Y_i^{\alpha-1} \\
&= Wn^{\frac{1}{1-\sigma}}\alpha\left[\left(\frac{P_i}{P}\right)^{-\theta}\frac{Y}{m}\right]^{\alpha-1} \qquad \text{(from (20.20)).}
\end{aligned}$$

Inserting into (20.22), dividing through by P, and solving for P_i/P gives

$$\frac{P_i}{P} = \left[\frac{\theta}{\theta-1}\frac{W}{P}n^{\frac{1}{1-\sigma}}\alpha\left(\frac{Y}{m}\right)^{\alpha-1}\right]^{\frac{1}{1+\theta(\alpha-1)}}, \qquad i = 1, ..., m. \qquad (20.23)$$

This is the *price rule* for firm i. It reflects that a higher W implies a higher MC, which implies a higher optimal price, given the constant mark-up, $\theta/(\theta-1)$, cf. Fig. 20.1. Similarly, an increase in the general price level P improves the competitive position of firm i. This invites an increase in P_i but not all the way up if $\alpha > 1$. That is, P_i/P falls, because part of the improved competitive position is taken out as higher supply of output.[9] Finally, an increase in national income Y implies, ceteris paribus, an outward shift in the demand curve faced by the firm; satisfying the higher demand implies higher MC (when $\alpha > 1$), hence, higher P_i, given the constant mark-up.

Hereby, we have finished the solution of firm i's problem. We are now ready to solve the third step in household j's problem.

[9] To see this notice that, from (20.23), $P_i = P\left[\frac{\theta}{\theta-1}\frac{W}{P}n^{\frac{1}{1-\sigma}}\alpha\left(\frac{Y}{m}\right)^{\alpha-1}\right]^{\frac{1}{1+\theta(\alpha-1)}}$, so that the exponent on P on the right hand side is $1-\frac{1}{1+\theta(\alpha-1)} \in (0,1)$, when $\alpha > 1$.

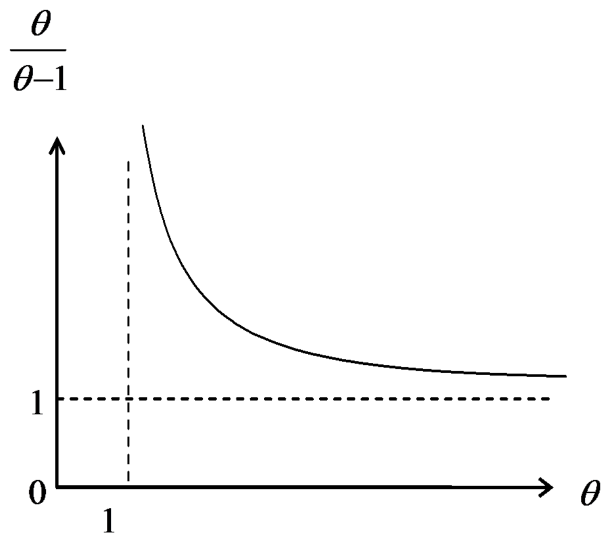

Figure 20.1: The relationship between elasticity of substitution and firms' markup.

Back to the household decision on labor supply and wage claim

The problem for household j ($j = 1,\dots,n$) is:

$$\begin{aligned}
\max_{W_j,N_j} U_j &= \mu\frac{I_j}{P} - N_j^{\beta} \qquad \text{s.t.} \\
N_j &= N_j^d = \sum_{i=1}^{m} N_{ij} = \left(\frac{W_j}{W}\right)^{-\sigma}\frac{\sum_{i=1}^{m} L_i}{n} = \left(\frac{W_j}{W}\right)^{-\sigma}\frac{N}{n}, \qquad (20.24) \\
I_j &= M_j + W_jN_j + \sum_{i=1}^{m} V_{ij}, \qquad (****)
\end{aligned}$$

where μ is the marginal utility of wealth, cf. (20.10). At the right-hand side of the constraint (****) only the term W_jN_j is endogenous. The constraint (20.24) comes from (20.18) and our definition of *aggregate labor demand*, N :

$$\begin{aligned}
N &\equiv \frac{\sum_{j=1}^{n} W_jN_j^d}{W} = \frac{\sum_{j=1}^{n} W_j(\sum_{i=1}^{m} N_{ij})}{W} = \frac{\sum_{j=1}^{n}\sum_{i=1}^{m} W_jN_{ij}}{W} \\
&= \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} W_jN_{ij}}{W} = \frac{\sum_{i=1}^{m} WL_i}{W} = \sum_{i=1}^{m} L_i.
\end{aligned}$$

By setting $N_j^d = \sum_{i=1}^{m} N_{ij}$, $j = 1,\dots,n$, we have here assumed equilibrium in the labor markets. The second last equality holds, in view of the definition of L_i and W, when firms optimize.

For this decision problem it is important that n is "large" so that the effect of W_j on the "average" wage rate W is negligible.

Step 3(j) : Setting W_j and N_j *(the wage claim and the supply of labor).*
To solve the problem above, let $W_j(N_j)$ denote the maximum wage rate at which the labor supply N_j can be sold; this $W_j(N_j)$ is given as the inverse of the demand constraint (20.24). Now, after inserting this and the constraint (****) into the objective function of household j, the problem is:

$$\max_{N_j} U_j = \mu \frac{W_j(N_j)N_j + \text{constant}}{P} - N_j^{\beta} = TR - TC,$$

where TR is total revenue of labor and TC is total cost, both in utility terms. We may call TC *total disutility of labor.*

FOC:

$$\frac{dU_j}{dN_j} = \frac{\mu}{P}(W_j + N_j \frac{dW_j}{dN_j}) - \beta N_j^{\beta-1} = MR - MC = MR - MDL = 0,$$

where MDL denotes *marginal disutility of labor.* We have

$$\begin{aligned} MR &= \frac{\mu}{P} W_j (1 + \frac{N_j \frac{dW_j}{dN_j}}{W_j}) = \frac{\mu}{P} W_j (1 + \frac{1}{\frac{W_j}{N_j}\frac{dN_j}{dW_j}}) = \frac{\mu}{P} W_j (1 - \frac{1}{\sigma}) \qquad \text{(by (8'))} \\ &= MDL \Rightarrow \\ W_j &= \frac{\sigma}{\sigma - 1}\frac{P}{\mu} MDL. \qquad (20.25) \end{aligned}$$

This is the standard wage setting principle for a monopolist supplier of labor: the utility maximizing wage rate is a mark-up, $\sigma/(\sigma - 1)$, on marginal disutility of labor. The markup is higher, the lower is the substitutability between the different types of labor as measured by σ. We have assumed $\sigma > 1$ because otherwise equilibrium with wage setters cannot exist.

When $\beta > 1$, MDL itself is endogenously increasing in labor supply:

$$MDL = \beta N_j^{\beta-1} = \beta \left[\left(\frac{W_j}{W} \right)^{-\sigma} \frac{N}{n} \right]^{\beta - 1} \qquad \text{(by (20.24))}.$$

Inserting into (W_j), dividing through by W, and solving for W_j/W gives

$$\frac{W_j}{W} = \left[\frac{\sigma}{\sigma - 1} \frac{P}{W} \frac{\beta}{\mu} \left(\frac{N}{n} \right)^{\beta - 1} \right]^{\frac{1}{1+\sigma(\beta-1)}}, \qquad j = 1, ..., n. \qquad (20.26)$$

This is the *wage rule* for labor of type j. It reflects that a higher P makes consumption more expensive, which implies substitution towards more leisure, i.e., less labor supply, so that a higher wage rate W_j can be claimed. Similarly, an increase in the general wage level W improves the competitive position of household or craft union j. This invites an increase in W_j but not all the way up if $\beta > 1$. That is, W_j/W falls, because part of the improved competitive position is taken out as higher supply of labor.[10] Finally, an increase in aggregate demand for labor, N, implies, everything else equal, an outward shift in the demand curve faced by the household; to satisfy the higher demand, a higher MDL must be accepted (when $\beta > 1$) and to compensate for this, given the constant mark-up, a higher W_j is demanded.

Hereby, we have finished the analysis of decision making.

20.3 General equilibrium

We first consider the case where there are no forces that induce nominal wage and price stickiness.

20.3.1 The case with flexible wages and prices

Suppose that no wage and price adjustment costs are present. In this case, named the "flexible-price case", in spite of monopolistic competition, *money is neutral.* But in contrast to perfect competition, monopolistic competition leads to a Pareto-inferior general equilibrium with *underutilization of resources.* This is a simple consequence of the supply behavior of isolated optimizing agents with market power.

A general equilibrium with flexible prices and wages is a price-wage vector $(P_1, ..., P_m,\ W_1, ..., W_n)$ and a quantity vector $(Y_1, ..., Y_m,\ N_1, ..., N_n)$ such that (a) the price and wage rules are satisfied and (b) demand equals supply on all markets. We shall call such an equilibrium a *flex price-flex wage equilibrium.* As we shall see, in a flex price-flex wage equilibrium money is neutral.

Aggregate demand for money can be written

$$M' \equiv \sum_{j=1}^{n} M'_j = (1-\gamma)\sum_{j=1}^{n} I_j = \frac{1-\gamma}{\gamma}PY, \tag{20.27}$$

from step $1(j)$ and equilibrium in the goods markets, (20.19). When we also take equilibrium in the labor markets together with the budget constraints (****)

[10]To see this notice that, from (20.26), $W_j = W\left[\frac{\sigma}{\sigma-1}\frac{P}{W}\frac{\beta}{\mu}\left(\frac{N}{n}\right)^{\beta-1}\right]^{\frac{1}{1+\sigma(\beta-1)}}$, so that the exponent on W on the right hand side is $1-\frac{1}{1+\sigma(\beta-1)} \in (0,1)$, when $\beta > 1$.

into account, we find that M' must equal the aggregate supply of money, which is $\sum_{j=1}^{n} M_j$, that is,

$$M' = \sum_{j=1}^{n} M_j \equiv M. \tag{20.28}$$

This reflects Walras' law: equilibrium in the goods and labor markets, together with the budget constraints, imply equilibrium in the last market, the "money market". We use citation marks because in this model money is the only asset and so the corresponding "market" has very limited similarity with the money market in a setup where money is traded for other other financial assets.

Substituting (20.28) into (20.27), we see that

$$P = \frac{\gamma}{1-\gamma}\frac{M}{Y}. \tag{20.29}$$

This tells us that in general equilibrium the variables P and Y are linked in a very simple way. It remains to find the solution for P and Y.

In the absence of price adjustment costs, prices and wages are flexible and follow the price rule and wage rule, respectively. Since all firms have the same price rule, (20.23), they all set the same price, say $\bar{P}$. Then $P_i = \bar{P}$ for all i. Hence, also the price index P equals $\bar{P}$, in view of (20.13). Using $P_i/P = 1$ for all i in (20.23), we can solve for P/W to get

$$\frac{P}{W} = \frac{\theta}{\theta - 1} n^{\frac{1}{1-\sigma}} \alpha m^{1-\alpha} Y^{\alpha-1}. \tag{20.30}$$

This is the *aggregate price rule,* which gives the price-wage ratio consistent with firms' pricing and production decision. By inverting and taking logs in (20.30), we get the downward-sloping solid line with slope $1 - a^{-1}$ in Fig. 20.2. A higher level of aggregate output is associated with higher $MC = W/(\partial Y/\partial N)$ because of a lower marginal product of labor. Given the mark-up, this leads to a higher price-wage ratio and a lower W/P. In the limiting case of $\alpha = 1$, the aggregate price rule is represented by a horizontal line.

The analogue aggregate wage rule can also be formulated as a relation between W/P and aggregate income Y, since there is a link between aggregate labor demand and aggregate income (output). Indeed, we have

$$\begin{aligned} N &= \sum_{i=1}^{m} L_i = n^{\frac{1}{1-\sigma}} \sum_{i=1}^{m} Y_i^{\alpha} \qquad \text{(by (**))} \\ &= n^{\frac{1}{1-\sigma}} \sum_{i=1}^{m} \left[\left(\frac{P_i}{P} \right)^{-\theta} \frac{Y}{m} \right]^{\alpha} \qquad \text{(by (20.20))} \\ &= n^{\frac{1}{1-\sigma}} m \left(\frac{Y}{m} \right)^{\alpha} \quad \text{(in view of symmetry between the firms, } P_i = P, \ \forall i) \\ &= n^{\frac{1}{1-\sigma}} m^{1-\alpha} Y^{\alpha}. \end{aligned} \tag{20.31}$$

Inserting into (20.26) gives

$$\frac{W_j}{W} = \left[\frac{\sigma}{\sigma - 1} \frac{P}{W} \frac{\beta}{\mu} n^{\frac{\sigma}{1-\sigma}(\beta-1)} m^{(1-\alpha)(\beta-1)} Y^{\alpha(\beta-1)} \right]^{\frac{1}{1+\sigma(\beta-1)}}, \tag{20.32}$$

for $j = 1, ..., n$. Now, since all households have the same wage rule (11), they all set the same wage, say $\bar{W}$. Then $W_j = \bar{W}$ for all j. Hence, also the "average" wage, W, is equal to $\bar{W}$, in view of (20.17). Using $W_j/W = 1$ for all j in (20.32) we can solve for W/P to get

$$\frac{W}{P} = \frac{\sigma}{\sigma - 1} \frac{\beta}{\mu} n^{\frac{\sigma}{1-\sigma}(\beta-1)} m^{(1-\alpha)(\beta-1)} Y^{\alpha(\beta-1)}. \tag{20.33}$$

This is the *aggregate wage rule*, which gives the real wage consistent with households' wage setting and labor supply decision. By taking logs in (20.33) we get the upward-sloping solid line with slope $(\beta - 1)/a$ in Fig. 20.2. A higher level of aggregate output requires higher employment and therefore higher marginal disutility of labor and, given the mark-up, this leads to a higher real wage. In the limiting case of $\beta = 1$ (i.e. perfectly elastic labor supply), the aggregate wage rule is represented by a horizontal line.

Inverting (20.30) and using (20.33) gives the equilibrium level of aggregate output,

$$Y = \left[\frac{\theta - 1}{\theta} \frac{\sigma - 1}{\sigma} (K_p K_w)^{-1} \right]^{\frac{1}{\alpha\beta - 1}} \qquad \text{for} \quad a > 1 \text{ or } \beta > 1, \tag{20.34}$$

where $K_p \equiv n^{\frac{1}{1-\sigma}} \alpha m^{1-\alpha}$ and $K_w \equiv \frac{\beta}{\mu} n^{\frac{\sigma}{1-\sigma}(\beta-1)} m^{(1-\alpha)(\beta-1)}$. Inserting this into (20.33) and (20.29), respectively, gives the equilibrium real wage

$$\frac{W}{P} = \left(\frac{\sigma}{\sigma - 1}\right)^{\frac{\alpha-1}{\alpha\beta-1}} K_w^{\frac{\alpha-1}{\alpha\beta-1}} \left(\frac{\theta - 1}{\theta} K_p{}^{-1}\right)^{\frac{\alpha(\beta-1)}{\alpha\beta-1}},$$

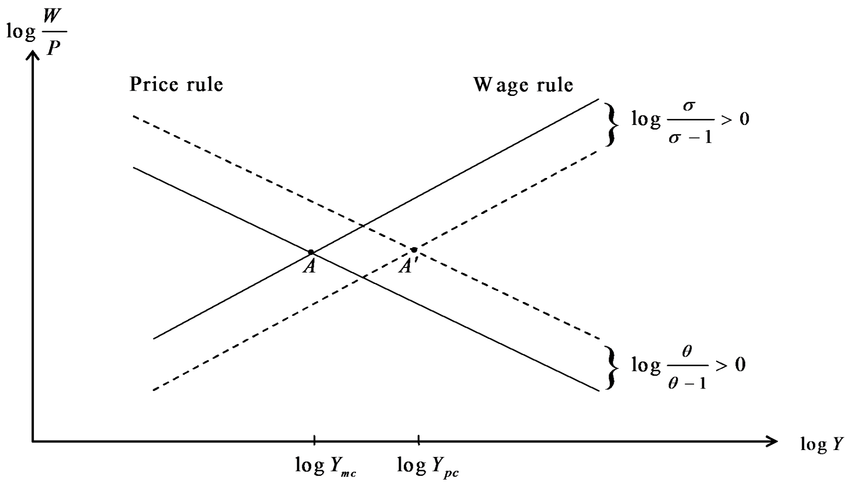

Figure 20.2: Monopolistic competition equilibrium and perfect competition equilibrium compared (the case $\alpha > 1$, $\beta > 1$).

and the equilibrium price level

$$P = \frac{\gamma}{1-\gamma} M \left[\frac{\theta - 1}{\theta} \frac{\sigma - 1}{\sigma} (K_p K_w)^{-1} \right]^{\frac{-1}{\alpha\beta - 1}}.$$

A unique equilibrium exists if either $\beta > 1$ or $\alpha > 1$ ($a < 1$). In case $\beta = a = 1$, generally no equilibrium exists because firms' markup claims are incompatible with households' wage claims.[11] Unless otherwise indicated, we will from now assume $\alpha \geq 1$, but $\beta > 1$, so that there is a unique flex price-flex wage equilibrium.

Results

We see that in the absence of price adjustment costs the model has the classical features:

- The real variables (output and the real wage) are determined by technology and preferences independently of the stock of money.
- The price level is proportional to the stock of money.

In brief: in the flex price-flex wage equilibrium *money is neutral.* Yet, we see from Fig. 20.2, comparing with the corresponding equilibrium under perfect

[11] In the knife edge case $(\theta - 1)\theta^{-1} n^{-1/(1-\sigma)} = \sigma\mu^{-1}/(\sigma - 1)$ there exist infinitely many equilibria. Indeed, for $W/P = \sigma\mu^{-1}/(\sigma - 1)$ the $(Y, W/P)$ will be an equilibrium for any $Y > 0$.

competition, that monopolistic competition leads to *underutilization of resources*. The effect of market power is to move the economy from point A′ in Fig. 20.2 to point A. The reason is that market power gives an incentive to withhold supply. The underutilization of resources shows up as *underemployment* (labor is the only input). In any case, the degree of underutilization can be large. Indeed, under perfect competition in goods and labor markets firms and households are price takers. By eliminating the markup factors $\theta/(\theta-1)$ and $\sigma/(\sigma-1)$ from (20.34) we find the corresponding aggregate output level to be

$$Y_{pc} = (K_p K_w)^{-1/(\alpha\beta-1)}.$$

Hence the distortion, measured in forgone output, is

$$\frac{Y}{Y_{pc}} = \left(\frac{\theta}{\theta-1}\frac{\sigma}{\sigma-1}\right)^{-1/(\alpha\beta-1)} < 1.$$

We see that the Y/Y_{pc} ratio is an decreasing function of the market power of firms, as measured by the markup $\theta/(\theta-1)$, as well as of workers, as measured by the markup $\sigma/(\sigma-1)$. The higher these markups are, the higher the degree of underemployment. But whereas the real wage is lowered by increased market power of firms, it is raised by increased market power of workers. Therefore, going from perfect competition to monopolistic competition in all markets has an ambiguous effect on the equilibrium real wage.

Sometimes a simplified version of the B-K model is used, where monopolistic competition rules only in the goods markets, whereas there is perfect competition in the labor market and usually only one type of labor. Then the stippled wage-rule curve in Fig. 20.2 becomes a labor supply and as long as it is not vertical (inelastic labor supply), the underutilization-of-resources conclusion again comes true. This is because firms will still set prices with a mark-up. But without the additional monopolistic behavior on labor markets, the degree of underutilization will be less.[12]

The underutilization of resources can also be illustrated as in Fig. 20.3, which depicts equilibrium from the perspective of product line i. For fixed M and $P = P_{mc}$ ("mc" for monopolistic competition), the demand curve faced by firm i is shown as the downward-sloping solid curve $D(P_i/P, M/P_{mc})$ to which corresponds the marginal revenue curve, MR. For fixed P and W, the marginal costs faced by firm i is shown as the upward-sloping marginal cost curve, MC (note that both MR and MC are measured in real terms, i.e., relative to the general price level

[12] An analogue conclusion would appear in a model with only monopolistic competition in the labor markets. In any case, when it comes to the incorporation of menu costs, a model with monopolistic competition in both goods and labor markets works best (see below).

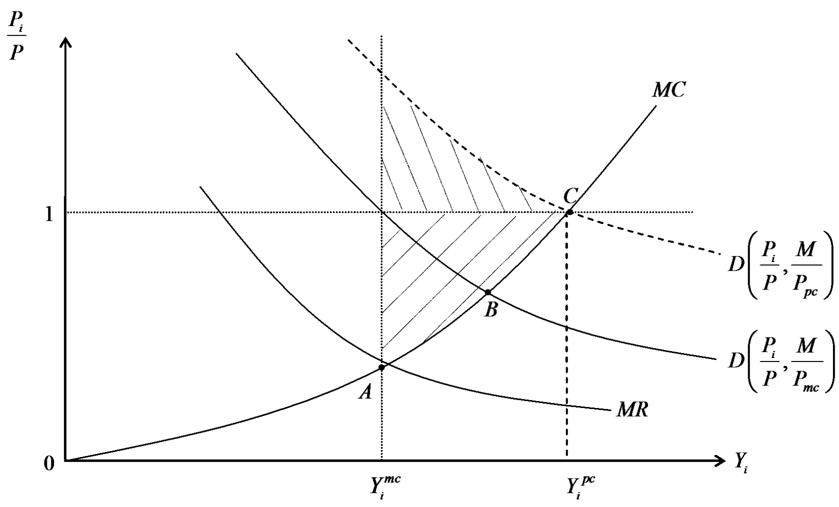

Figure 20.3: Equilibrium from the perspective of product line i.

P). All prices and wages are set optimally in accordance with the rules derived above. Hence, $MR = MC$ and $P_i/P = 1$. Under perfect competition, however, firms produce up to the point C, where marginal costs in real terms equal 1. Then Y will be higher. As indicated by (20.29), this results in a lower general price level P_{pc} ("*pc*" for perfect competition), hence a greater real value of the money stock. To this corresponds the dashed demand curve in Fig. 20.3. To the additional producer and consumer surplus displayed as the hatched areas in Fig. 20.3 corresponds the additional welfare, going from monopolistic competition, Y_i^{mc}, to perfect competition, Y_i^{pc}.[13]

The Pareto-inferior underemployment that arise under monopolistic competition is an example of *coordination failure.* Any agent does the best she can, given what the others do, but the outcome is socially inefficient. A coordinated action could improve the outcome for everybody like in the prisoners' dilemma (see Cooper 1999, Benassy 2002).

20.3.2 The case with sticky wages and prices

The neutrality of money in the above analysis derives from assuming the price and wage setters face neither pecuniary nor non-pecuniary costs when they change prices and wages, respectively. Following Blanchard and Kiyotaki, we now assume the presence of such adjustment costs in the form of *menu costs* as described in

[13] In this model the marginal utility of wealth is constant, as noted in connection with equation (20.10). Hence, the sum of the producer surplus and consumer surplus for the representative producer is indeed an appropriate measure for welfare.

Chapter 19. Owing to the *envelope theorem*, even *small* menu costs may under certain conditions be enough to prevent firms from changing their price − and craft unions from changing their wage claim − in response to a demand shock brought about by an income transfer financed by money issue. Then the price and wage rules are suspended. As long as marginal cost is below the price and marginal disutility of labor (measured in equivalent money units) is below the wage, output and employment respond to changes in aggregate nominal demand, while prices and wages are kept unchanged.

We assume that before the demand shock prices and wages are set at their profit and utility maximizing levels, respectively. Initially, the price and wage rules described above are thus satisfied. Hence, (20.29) still holds but it is natural to write it on the form

$$Y = \frac{\gamma}{1-\gamma}\frac{M}{P}, \tag{20.35}$$

as long as prices remain fixed at their pre-determined level whereby also the general price level P remains fixed. Given this price level, aggregate output is proportional to aggregate nominal demand, which is proportional to the money stock.

Let us call this kind of equilibrium a *fix price-fix wage equilibrium*. It has many features in common with "old-Keynesian" models. The demand for labor by the firms depends not only on the "price signals", but also on *quantity signals*, namely the level of demand faced by the single firm which in turn depends on the aggregate quantity signal Y. In the next instance, there is a feedback from laborers, whose consumption demand depends *not* so much on how much labor they would *prefer* to sell at the going wage rate, but on how much they are *able* to sell. Summing over all firms and households we see that actual aggregate demand is determined not by fully adjusted equilibrium prices and wages, but by the given prices and wages and by quantity signals from the market. The quantity signals come from the constraints on how much the different agents can sell or buy at the going prices and wages. This provides a foundation for the Keynesian concept of *effective demand,* presented in the previous chapter.

Menu costs in action

To clarify the key role of menu costs in this story, we consider a change in M in the form of a lump-sum "helicopter drop" (income transfer financed by money issue) at the beginning of the considered period. Although the agents are active in several periods, we focus on one period and assume the decision making is myopic.

The profit function of firm i is, from (20.20), (20.21), and (20.29),

$$\begin{aligned} V_i &= P_i \left(\frac{P_i}{P}\right)^{-\theta} \frac{\gamma}{1-\gamma} \frac{M}{mP} - W n^{1/(1-\sigma)} \left(\frac{P_i}{P}\right)^{-\theta\alpha} \left(\frac{\gamma}{1-\gamma} \frac{M}{mP}\right)^{\alpha} \\ &\equiv V(P_i, P, W, M), \end{aligned}$$

where we remember that P_i = the output price of firm i, P = the general price level, W = the general wage level and M = the money stock. Facing a downward sloping demand curve, firm i chooses P_i so as to maximize profit. Suppose that initially, $P_i = P_i^*$, where P_i^* is the price that maximizes V_i for given given P, W, and M.[14] Thus, maximum profit is

$$V(P_i^*, P, W, M) \equiv V_i^*,$$

Let the money stock shift to the new level $M' > M$. *Suppose no other agents respond to this by changing price (or wage).* Then P and W are unchanged. In this situation the opportunity cost to firm i of not changing price tends to be small. Indeed, considering the marginal effect on V of the rise in M when not changing price, we have:

$$\begin{aligned} \frac{dV}{dM}(P_i^*, P, W, M) &= \frac{\partial V}{\partial P_i}(P_i^*, P, W, M)\frac{\partial P_i}{\partial M} + \frac{\partial V}{\partial M}(P_i^*, P, W, M) \qquad (20.36) \\ &= 0 + \frac{\partial V}{\partial M}(P_i^*, P, W, M). \end{aligned}$$

The first term on the right-hand side of (20.36) vanishes at the profit optimum because $\frac{\partial V}{\partial P_i}(P_i^*, P, W, M) = 0$, i.e., the profit curve is flat at the maximizing price P_i^*. An illustration in a similar setup is shown in Fig. 19.4 of the previous chapter where the menu cost theory is dealt with in more detail. The result reflects the *envelope theorem*: in an interior optimum, the total derivative of the maximized function w.r.t. a parameter is equal to the partial derivative w.r.t. that parameter; the relevant parameter here is the aggregate money stock, M. Hence, the effect of a change in M on the profit is approximately the same (to a first order) whether or not the firm adjusts its price. Therefore, in view of the menu cost, say c, it may be advantageous not to change price. Indeed, the net gain ($= c -$ opportunity cost) by not changing price may be positive. Each individual firm is in the same situation as long as the other firms have not changed price. The outcome that no firm changes its price is thus an equilibrium. Since there is no change in the general price level in this equilibrium, a higher output level results.

[14] P_i is thus the solution obtained by substituting (20.35) into (20.23).

These considerations presuppose that the *households* do not increase their wage demands in response to the increased demand for labor. But in principle this raises no new problem because the wage-setting households (or crafts-unions) also face menu costs.[15] As we have seen, each worker faces a downward-sloping demand curve for her specific type of labor and each worker sets the utility maximizing wage taking the demand curve into account and supplies then the amount of labor demanded at that wage level. If there are menu costs associated with changing the wage claim and they are not too small, an increase in demand need not have any effect on the wage claims. Again this follows from the envelope theorem. The utility curve in a (W_j, U_j) diagram is flat at the utility maximizing wage W_j^*. Thus, all in all, no agent in the economy may want to change price or wage, given that none of the agents change price and wage. And instead, output, employment and social welfare respond.

When menu costs are operative in both output and labor markets, output and employment adjust to demand, while prices and wages are unchanged. The demand functions for goods and labor, and the relation between aggregate demand and real money balances, derived in Section 20.2, were derived without use of the now suspended price and wage rules. Hence these functions and relations still hold.

Consider again the "money market" where aggregate demand is

$$M' \equiv \sum_{j=1}^{n} M_j' = (1-\gamma)\sum_{j=1}^{n} I_j = \frac{1-\gamma}{\gamma}PY,$$

from (20.27). Aggregate supply of money is the available money stock $M = \sum_{j=1}^{n} M_j$. In the flex price-flex wage equilibrium the general price level, P, satisfied $P = \{\gamma/[(1-\gamma)Y]\}\, M$ and changed in proportion to the change in M.

But in the fix price-fix wage equilibrium, since P is pre-determined, we write our solution

$$Y = \frac{\gamma}{1-\gamma}\frac{M'}{P}. \tag{20.37}$$

As long as menu costs are operative, aggregate output is thus proportional to aggregate nominal demand, which is proportional to the money stock.

The point of the menu-cost theory is that even *small* menu costs can be enough to prevent firms from changing their price in response to a change in demand; and this can have sizeable effects on aggregate output, employment, and social welfare. Indeed, under monopolistic competition and endogenous labor supply neither output, employment or social welfare are maximized in an initial equilibrium. Therefore the envelope theorem does not apply to these variables.

[15] The motivation for the proviso "in principle" will become clear below.

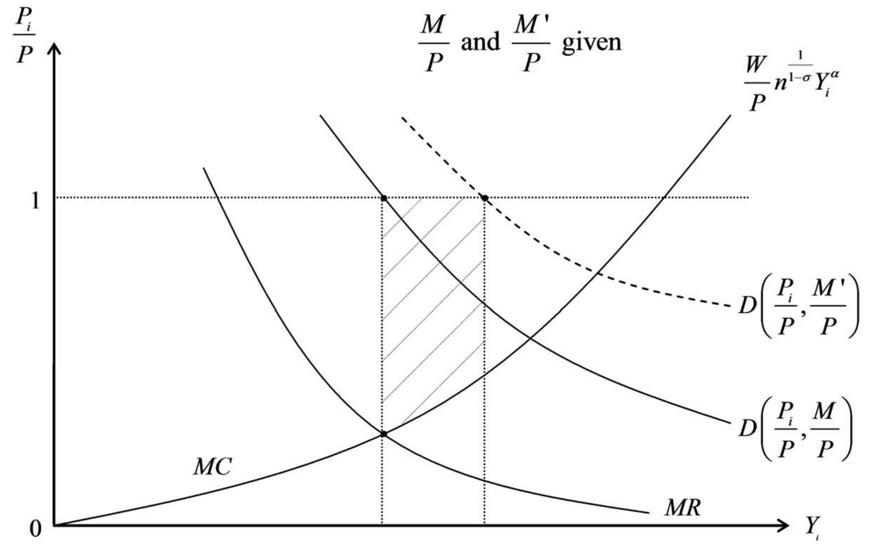

Figure 20.4: The effect of a shift to $M' > M$ when menu costs are operative (the case $\alpha > 1$).

The point is illustrated by Fig. 20.4. For fixed M and P, the demand curve faced by firm i is shown as the solid downward-sloping curve $D(P_i/P, M/P)$ to which corresponds the marginal revenue curve, MR. For fixed P and W, the marginal costs faced by the firm are shown as the marginal cost curve, MC (note that both MR and MC are measured in real terms, i.e., relative to the general price level P). Suppose that initially all prices are set optimally in accordance with the rule derived above. Hence, $MR = MC$ and $P_i/P = 1$ initially.

Consider a "moderate" shift in the money stock from M to $M' > M$. If there were no menu costs, prices would increase and leave the real money stock and output unchanged. But when menu costs exist, it is possible that neither prices nor wages change. Then, the larger nominal money stock translates into a larger *real money stock* and the demand curve is shifted to the right. As long as we still have $MC < P_i/P = 1$, the firm willingly produces and sells the extra output corresponding to the higher demand. The extra profit obtained this way is marked by the hatched area in Fig. 20.4. Firms in the other production lines are in the same situation and willingly increase output. As a result, aggregate output increases according to (20.37) which, via (20.31), translates into higher employment. The final outcome is a new general equilibrium with unchanged wages and prices and higher production, consumption and higher welfare.

The effects on aggregate output, employment, and social welfare of not changing price in response to the rise in nominal aggregate demand can thus be substantial. As was shown in a similar setting in Section 19.3, these quantity effects are of "first order", namely proportional to $|\Delta M/M|$.

Suppose instead that the increase in the money stock is not "moderate" but "large". Consider the case where $\alpha > 1$, so that the MC curve is upward-sloping. Then the new demand curve may cross the MC curve at an output level above the perfect-competition level Y^c where $MC = 1$. If so, the *rule of the minimum*, cf. Section 19.3, hinders that output level to be realized. If the menu costs remains operative, we get $Y_i = Y^c$ because producing more than Y^c would result in lower profit.

In the real world, nominal aggregate demand (here proportional to the money stock) fluctuates up and down around some expected level. Sometimes the welfare effects of menu costs will be positive, sometimes negative. Hence, *on average* the welfare effects tend to cancel out to a first order. This does not affect the basic point of the menu cost theory, however, which is that changes in aggregate nominal demand can have first-order real effects (in the same direction) because the opportunity cost by not changing price is only of second order.

Labor supply and labor markets revisited

It is noteworthy that the menu-cost theory does not go through in this straightforward way if monopolistic competition rules only in the goods markets, whereas the labor market is competitive. With upward-sloping labor supply curves ($\beta - 1 > 0$) an increase in production (and therefore employment) presupposes an increase in the wage level, W. Then the opportunity cost for the firms of not changing price when demand increases becomes higher and the "critical" size of the menu cost, c, therefore smaller. Only if the elasticity of marginal disutility of labor, $\beta - 1$, is zero so that labor supply is perfectly elastic, will menu costs come through to the same extent as in the general model above with monopolistic competition both on goods and labor markets.

There is a further problem, however. Even though the *qualitative* logic of the general B-K model with menu costs may be sound enough, there is a not negligible *quantitative* problem. This quantitative problem was in fact emphasized by Blanchard and Kiyotaki (1987) themselves. The problem is that microeconometric studies of labor supply find that even the compensated elasticity of labor supply w.r.t. the real wage is quite small.

To see the implication, note that in the model each worker faces a downward-sloping demand curve for her specific type of labor. The opportunity cost of not changing the claimed wage is an increasing function of the elasticity of marginal disutility of labor. Numerical calculations for realistic parameter values tell us, however, that a rather low elasticity of marginal disutility of labor is needed for the opportunity cost of not changing the wage claim to be small enough so as to not exceed the menu cost of changing the wage claim. As underlined time and again this menu cost itself is inherently small.

Now, a low elasticity of marginal disutility of labor is synonymous with a high elasticity of "operative" labor supply w.r.t. the real wage. Indeed, we found the real wage claim of labor type j to be

$$\frac{W_j}{P} = \frac{\sigma}{\sigma - 1}\frac{1}{\mu}MDL = \frac{\sigma}{\sigma - 1}\frac{1}{\mu}\beta N_j^{\beta-1},$$

where $\beta - 1$ indicates the elasticity of the marginal disutility of labor, hence also of the wage claim, w.r.t. the operative labor supply, N_j (recall that $\sigma > 1$ and $\beta \geq 1$). Reordering the equation gives

$$N_j = \left(\frac{\mu(\sigma - 1)}{\beta\sigma}\right)^{1/(\beta-1)} \left(\frac{W_j}{P}\right)^{1/(\beta-1)}.$$

This can be interpreted as saying that the elasticity of labor supply w.r.t. the real wage is $1/(\beta - 1)$. Requiring a low $\beta - 1$ is thus equivalent to requiring a high elasticity of labor supply w.r.t. the real wage. But the microeconometric evidence tells us that in reality this elasticity is *small* (for a temporary real wage increase it is typically estimated to be between 0.1 and 0.4, and for a permanent wage increase possibly nil or even negative, cf. Chapter 5).

Higher demand for labor can thus easily lead to upward wage adjustment. Then it becomes more costly for firms not to change price. The conclusion is that in the B-K framework the menu cost theory is not really capable of providing the desired result.

As alluded to above, it clearly does not help to assume that workers are wage takers (perfect competition in the labor market). In that case, with low wage elasticity of labor supply, it takes a considerably higher real wage to allow higher employment. And then the opportunity cost to the firm by not changing price again becomes significant.

There is a way out, however. In the B-K framework with monopolistic competition there is under-employment in the labor market. But there is no involuntary unemployment. Recall the definition of *involuntary unemployment* as being present when there are people around without a job although they are as qualified as those employed and are ready and willing to take a job at the going wage or even a lower wage. If instead we model the labor markets in accordance with efficiency wage theory, social norms and fairness theory, insider-outsider theory or collective bargaining theory, then wages tend to be above the individual reservation wage. Thus involuntary unemployment arises. As an implication, employment can easily change without much change in the wage level. That is, our fix price-fix wage equilibrium becomes a *Keynesian equilibrium* where aggregate *employment* is very elastic with respect to demand at unchanged wage level in the short run.

We infer that in a model where the output market is dominated by monopolistic competition, but the labor market is governed by efficiency wages, social norms, insider-outsider or collective bargaining principles, menu costs can realistically be thought to sustain nominal rigidities.[16] The point is that for nominal price rigidities to come to the fore, "support" from *real wage insensitivity* is needed. We say that real wage insensitivity is present when a large shift in aggregate employment can occur without much change in the real wage.

A warning about terminology Ball and Romer (1990) introduced the term "real rigidity" for a situation where a relative price, for example the real wage, is not very sensitive to a change in the corresponding quantity, say employment. This term may lead to misunderstanding, however, and therefore we prefer the term "real price rigidity", or even better "real price insensitivity", for the phenomenon in question. There is a risk that "real rigidity" is interpreted to mean inflexibility of real quantities like output or employment. But it is exactly when employment is very flexible (highly elastic employment level) that "real rigidity" in the Ball and Romer meaning is present, namely when it takes very little, if any, rise in the real wage to permit a large expansion of employment. This ambiguity is avoided when we replace "real ..." by "real price ..." and in the present case "real wage ...".

A further terminological problem is, perhaps, that also the term "rigidity" can be misunderstood, being associated with the presence of particular barriers that *hinder* changes which would otherwise take place. The essential point of the theory is merely that some relative prices are *insensitive* to quantity variations. Whether this is because there is no incentive to change a relative price or there is such an incentive, but barriers block the corresponding action, is immaterial. Using the term "real price insensitivity", we avoid such misunderstanding.

20.4 Spillover complementarity and multiple equilibria

The B-K model exhibits spillover complementarity. Generally, *spillover complementarity* is said to be present if many agents' action (in the same direction) has a positive feedback on the individual agent's action.[17] To illustrate the idea,

[16] This was a main point in Yellen (1984) and Akerlof and Yellen (1985).

[17] See Cooper and John (1988). For evidence, see Cooper and Haltiwanger (1996). If the feedback is negative, there is *spillover substitutability.*

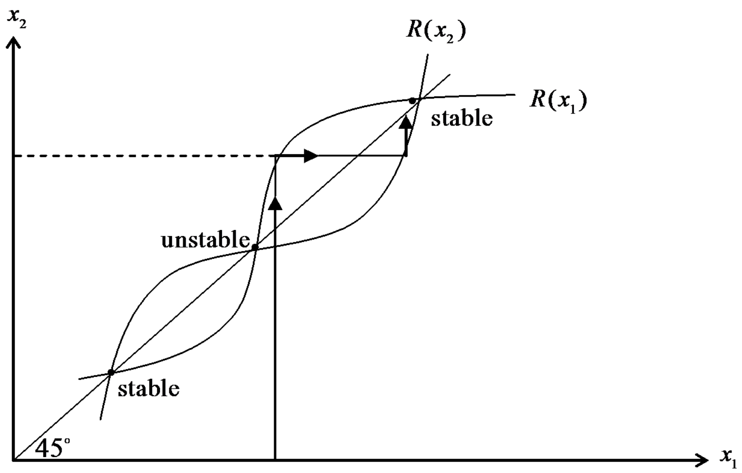

Figure 20.5: Complementarity resulting in two stable equilibria.

suppose for simplicity there are only two agents and that they are symmetric. In 20.5 agent 2's action, x_2, is given as a function, $R(x_1)$, of the action, x_1, of agent 1 and vice versa. Then *multiple equilibria* may arise. In Fig. 20.5 there are three equilibria, two of which are stable, the third being unstable as indicated by the arrows.

To discuss the role of spillover complementarity and multiple equilibria in the B-K model, we will consider the convenient special case where the marginal disutility of work is constant ($\beta = 1$) and there are no menu costs in wage setting. The real wage is thus constant and we can concentrate on price setters. Let π denote the proportion of firms that increase their price in response to an increase in M. The opportunity cost, the "loss" L_i, for firm i of not adjusting its price can be shown (see Appendix) to be an increasing function of π, $L_i = L(\pi)$, $L' > 0$, $c = L(\tilde{\pi})$. Then, as illustrated in Fig. 20.6, for values of the menu cost, c, in a certain range there are three equilibria, $E_0, \tilde{E}$, and E_1. If $\pi < \tilde{\pi}$, firm i will not change price. The situation is the same for all the other firms, so that there can be no equilibrium with $0 < \pi < \tilde{\pi}$. To put it differently, the expectation that no one will change price is self-fulfilling − the equilibrium E_0 results. Similarly, If $\pi > \tilde{\pi}$, firm i *will* change price. And so will all the other firms, from which follows that there can be no equilibrium with $\tilde{\pi} < \pi < 1$. The expectation that all will change price is self-fulfilling, so that the equilibrium E_1 results. An asymmetric equilibrium at $\tilde{E}$ could be realized by a very improbable coincidence, but it is fragile (a small deviation leads to further divergence). Interestingly, the two stable equilibria can be Pareto-ordered. Everybody are better off at the high activity equilibrium E_0 than at the low activity equilibrium E_1. This is a additional example of *coordination failure*. The first example appeared already in

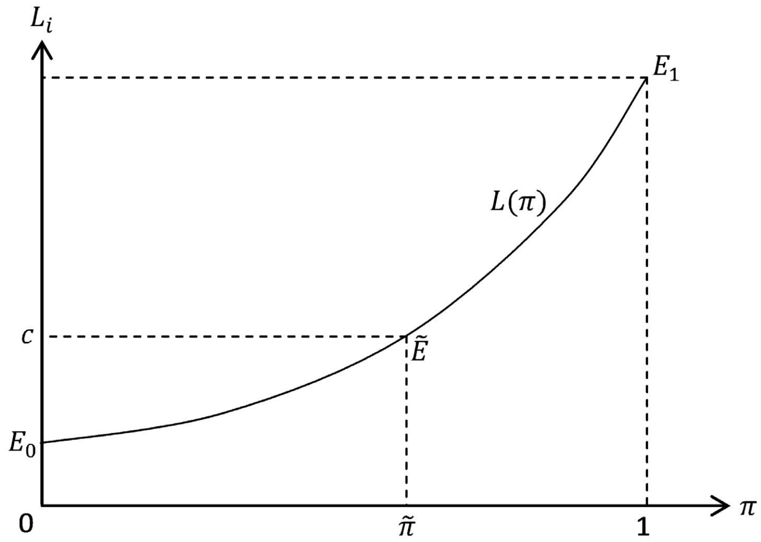

Figure 20.6: Multiple equilibria in price adjustments ($\Delta M/M$ given).

the flex-price case, namely the general underemployment referred to at the end of Section 20.3.1.

20.5 Concluding remarks

(incomplete)

We have presented the Blanchard-Kiyotaki model which has become one of the cornerstones of new-Keynesian thinking. Both labor and goods markets are monopolistically competitive. In combination with presence of menu costs this provides a solid basis for the Keynesian distinction between *effective demand,* in the sense of "active" demand, and "notional" (or "classical" or "Walrasian") demand. In contrast to the IS-LM model of the next chapter, the Blanchard-Kiyotaki model pays attention to the supply side no less than the demand side.

We have concentrated on the case of rising marginal cost, the theoretically challenging case. We first considered the subcase where there are no forces that induce nominal wage and price stickiness. In this case, named the "flexible-price case", in spite of monopolistic competition, *money is neutral.* But in contrast to perfect competition, monopolistic competition leads to existence of a Pareto-inferior general equilibrium with *underutilization of resources.* This is a simple consequence of the supply behavior of isolated optimizing agents with market power.

Next, we addition price adjustment costs in the form of menu costs. These costs may lead price setters to abstain from adjusting their price when demand changes. As a result, *money is not neutral*. Even small adjustment costs can have large real consequences at the aggregate level.

Limitations of the B-K model:

Owing to the way the labor markets are modelled by Blanchard and Kiyotaki, their model lacks the real wage insensitivity needed to make weighty nominal price rigidity be realistic (the labor supply elasticity problem).

Another limitation, inherited from the simpler framework of the previous chapter, is that the menu costs are not framed in an intertemporal perspective where present values of costs matter rather than instantaneous or "static" costs.

The Blanchard-Kiyotaki model has served as one of the building blocks for what is known as new-Keynesian economics. In combination with elements from the IS-LM model (see next chapter), the Blanchard-Kiyotaki framework has in different ways been made dynamic and brought to data. One variety of these extensions is known as the new-Keynesian DSGE model, where DSGE stands for Dynamic Stochastic General Equilibrium. We return to that model in Part VII of this book.

20.6 Literature notes

(incomplete)

An early contribution in the field is Benassy (1978) where non-Walrasian fix-price allocations are generated as imperfect-competition equilibria with price-setters.

Zhelobodko et al., 2012) study free-entry monopolistic equilibrium beyond the constant elasticity of substitution assumption of Dixit and Stiglitz (1977) and Blanchard and Kiyotaki (1987).

20.7 Appendix

Not yet available.

20.8 Exercises