Chapter 13

General equilibrium analysis of public and foreign debt

This chapter reviews long-run dynamics of public and foreign debt in the light of the continuous time OLG model of the previous chapter. Section 13.1 reconsiders the *Ricardian equivalence* issue. In Section 13.2 we extend the enquiry to a *general equilibrium analysis of budget deficits and debt dynamics* in a closed economy. Section 13.3 addresses general equilibrium aspects of public and foreign debt in a small open economy. Issues of *twin deficits* and the current account of a growing economy are considered. In Section 13.4 the assumption of lump-sum taxes is replaced by income taxation in order to examine the relationship between *debt and distortionary taxation.* The theme of *optimal debt* is addressed in Section 13.5, and the concluding Section 13.6 addresses the *time-inconsistency problem* faced by economic policy when outcomes depend on private sector expectations.

13.1 Reconsidering the issue of Ricardian equivalence

Recall that Ricardian equivalence is the claim that, given the (expected) future path of government spending, it does not matter for aggregate private consumption and saving whether the government finances its current spending by lump-sum taxes or borrowing. Whether this claim is an acceptable approximation or not is still a subject of debate among macroeconomists.

As we know from earlier chapters, the representative agent approach and the life-cycle-OLG approach lead to opposite conclusions regarding the issue. In models with a representative household with infinite horizon (the Barro and Ramsey dynasty models) a change in the timing of lump-sum taxes does not change the present value of the infinite stream of taxes imposed on the individual dynasty by a government satisfying its intertemporal budget constraint. A cut in current taxes is offset by the expected higher future taxes. Private saving goes up just as much as current taxes are reduced. This is exactly what is needed for paying the higher taxes in the future and maintain the preferred time path of consumption. Current consumption is thus not affected. And aggregate saving in society as a whole stays the same (the higher government dissaving being matched by higher private saving).

It is otherwise in the life-cycle-OLG models (without an operative Barro-style bequest motive). For instance the Diamond OLG model with a public sector reveals how taxes levied at different times are levied on different sets of agents. In the future some of the currently alive will be gone and there will be newcomers to bear part of the higher future tax burden. A current tax cut thus makes current tax payers feel wealthier and this leads to an increase in their current consumption. So current private consumption in the economy ends up higher. The present generations consequently benefit and future generations bear the cost in the form of smaller national wealth than otherwise.

Because of the more refined notion of time in the Blanchard OLG model from Chapter 12 and its capability of treating wealth effects more aptly, let us see what this model precisely says about the issue. A simple book-keeping exercise will show that the size of the public debt *does* matter. By affecting private wealth, it affects private consumption.

To keep things simple, we ignore retirement ($\lambda = 0$). To avoid notational confusion of the birth rate with the debt-income ratio, the former will in this chapter be denoted β while we still denote the latter by b. As in the previous chapters, B_t will denote net government debt, G_t government spending on goods and services, and T_t net tax revenue, $\tilde{T}_t - X_t$, where $\tilde{T}_t$ is gross tax revenue while X_t is transfers, all in real terms. We assume that the interest rate is in the long run higher than the output growth rate. Hence, to remain solvent the government has to satisfy its intertemporal budget constraint. Ignoring seigniorage and presupposing the government does not plan to procure more tax revenue than needed to satisfy its intertemporal budget constraint, as seen from time 0 (interpreted as "now"), we have the condition

$$\int_0^\infty T_t e^{-\int_0^t r_s ds} dt = \int_0^\infty G_t e^{-\int_0^t r_s ds} dt + B_0, \tag{GIBC}$$

where the expected future time paths of G_t and r_t are considered given and B_0 is historically given. In brief, (GIBC) says that the present value of future net tax revenues must equal the sum of the present value of future spending on goods and services and the current level of debt. A temporary cut in taxes in an early time interval after time 0 must be offset in a later time interval by a rise in taxes of the same present value.

Given aggregate private financial wealth, A_0, and aggregate human wealth, H_0, aggregate private consumption is

$$C_0 = (\rho + m)(A_0 + H_0). \tag{13.1}$$

Because of the logarithmic specification of instantaneous utility, the propensity to consume out of wealth is a constant equal to the sum of the pure rate of time preference, ρ, and the mortality rate, m. Human wealth is the present value of expected future net-of-tax labor earnings of those currently alive:

$$H_0 = N_0 \int_0^\infty (w_t - \tau_t) e^{-\int_0^t (r_s + m) ds} dt. \tag{13.2}$$

Here, τ_t is the per capita lump-sum net taxation at time t, i.e., $\tau_t \equiv T_t/N_t \equiv (\tilde{T}_t - X_t)/N_t$, where N_t is the size of the population (here equal to the labor force, which in turn equals employment). The discount rate is the sum of the risk-free interest rate, r_t, and the actuarial compensation which is identical to the mortality rate, m.

To fix ideas, consider a closed economy. In view of the presence of government debt, aggregate private financial wealth in the closed economy is $A_0 = K_0 + B_0$, where K_0 is aggregate (private) physical capital and B_0 is assumed positive. Thus, (13.1) can be written

$$C_0 = (\rho + m)(K_0 + B_0 + H_0), \tag{13.3}$$

where ρ is the pure rate of time preference and m is the mortality rate. We ask whether B_0 is net wealth , for a given K_0, the sum $B_0 + H_0$ depends on the size of B_0, given the expected future path of G_t in (GIBC). We will see that the answer is yes. This is because, contrary to the Ricardian equivalence hypothesis, a higher B_0 is *not* offset by an equally reduced H_0 brought about by the higher future lump-sum taxes. Such a fully offsetting reduction of H_0 will not occur. Therefore C_0 *is* increased. Aggregate consumption depends positively on B_0.

The argument is the following. Rewrite (13.2) as

$$\begin{aligned}
H_0 &= N_0 \int_0^\infty \frac{w_t N_t - T_t}{N_t} e^{-\int_0^t (r_s + m) ds} dt \qquad \text{(from } \tau_t = T_t/N_t\text{)} \\
&= \int_0^\infty (w_t N_t - T_t) e^{-nt} e^{-\int_0^t (r_s + m) ds} dt \qquad \text{(since } N_0 = N_t e^{-nt}\text{)} \\
&= \int_0^\infty (w_t N_t - T_t) e^{-\int_0^t (r_s + n + m) ds} dt = \int_0^\infty (w_t N_t - T_t) e^{-\int_0^t (r_s + \beta) ds} dt,
\end{aligned}$$

using that the population growth rate, n, equals $\beta - m$. Therefore,

$$\begin{aligned} H_0 + B_0 &= \int_0^\infty (w_t N_t - T_t) e^{-\int_0^t (r_s+\beta)ds} dt + B_0 = \int_0^\infty (w_t N_t - G_t) e^{-\int_0^t (r_s+\beta)ds} dt \\ &\quad - \int_0^\infty (T_t - G_t) e^{-\int_0^t (r_s+\beta)ds} dt + B_0. \end{aligned} \tag{13.4}$$

Note that the first integral on the right-hand side of (13.4) is given (independent of a changed time profile of τ_t).

Reordering (GIBC), we have

$$B_0 = \int_0^\infty (T_t - G_t) e^{-\int_0^t r_s ds} dt. \tag{13.5}$$

Hence, the last line of (13.4) can be written

$$\begin{aligned} & - \int_0^\infty (T_t - G_t) e^{-\int_0^t (r_s+\beta)ds} dt + \int_0^\infty (T_t - G_t) e^{-\int_0^t r_s ds} dt \\ = & \int_0^\infty \left((T_t - G_t) e^{-\int_0^t r_s ds} - (T_t - G_t) e^{-\int_0^t r_s ds} e^{-\int_0^t \beta ds} \right) dt \\ = & \int_0^\infty (T_t - G_t) e^{-\int_0^t r_s ds} \left(1 - e^{-\int_0^t \beta ds} \right) dt. \end{aligned} \tag{13.6}$$

From (13.6) then follows

$$H_0 + B_0 = \int_0^\infty (w_t N_t - G_t) e^{-\int_0^t (r_s+\beta)ds} dt + \int_0^\infty (T_t - G_t) e^{-\int_0^t r_s ds} \left(1 - e^{-\int_0^t \beta ds} \right) dt. \tag{13.7}$$

There are two cases regarding the birth rate β to consider: $\beta = 0$ versus $\beta > 0$. The first case turns the Blanchard model into a representative agent model, since the second term on the right-hand side of (13.7) vanishes. Then the remaining term indicates that $H_0 + B_0$ is independent of the time profile of taxes. Only the given time path of G_t matters. A higher B_0 does not affect the $w_t N_t - G_t$ flow, and so the *sum* $H_0 + B_0$ is unaffected. The only effect of a higher B_0 is thus to make H_0 equally much lower so as to leave $H_0 + B_0$ unchanged. The case $\beta = 0$ thus implies Ricardian equivalence.

When $\beta > 0$ (positive birth rate), however, the second term on the right-hand side of (13.7) becomes decisive. In view of (13.5), the primary surplus, $T_t - G_t$, has to be positive for a substantial time interval, when $B_0 > 0$, the more so the larger is B_0. Consequently, the right-hand side of (13.7) is larger the larger is B_0. We conclude:

$$\begin{cases} H_0 + B_0 \text{ is independent of } B_0, \text{ if } \beta = 0, \text{ while} \\ H_0 + B_0 \text{ depends positively on } B_0, \text{ if } \beta > 0. \end{cases} \tag{13.8}$$

The intuition is that when the birth rate is positive, the tax burden in the future falls partly on new generations. Larger holdings of government bonds thus make the current generations feel wealthier in spite of future taxes being raised.

EXAMPLE Let $B_0 > 0$. Suppose T_0 is proportional to G_0 for all $t \geq 0$ with the factor of proportionality $1+\xi$. Then, inserting $T_0 = (1+\xi)G_0$ into (13.7) gives

$$H_0 + B_0 = \int_0^\infty (w_t N_t - G_t) e^{-\int_0^t (r_s + \beta) ds} dt + \xi \int_0^\infty G_t e^{-\int_0^t r_s ds} \left(1 - e^{-\int_0^t \beta ds}\right) dt,$$

which for $\beta > 0$ is an increasing function of ξ. In turn, ξ is an increasing function of B_0 because inserting $T_0 = (1+\xi)G_0$ into (13.5) and solving for ξ gives $\xi = B_0 / \int_0^\infty G_t e^{-\int_0^t r_s ds} dt > 0$. So, for $\beta > 0$, $H_0 + B_0$ depends positively on B_0. □

The result may be seen in the light of the different discount rates involved. The discount rate relevant for the government when discounting future tax receipts and future spending is just the market interest rate, r. But the discount rate relevant for the households currently alive is $r + \beta$. This is because the present generations are, over time, a decreasing fraction of the tax payers, the rate of decrease being larger the larger is the birth rate. In the Barro and Ramsey models the "birth rate" is effectively zero in the sense that no *new* tax payers are born. When the bequest motive (in Barro's form) is operative, those alive today will take the tax burden of their descendents fully into account.

This takes us to the distinction between *new individuals* and *new decision makers*, a distinction related to the fundamental difference between representative agent models and overlapping generations models.

It is neither finite lives nor population growth

It is sometimes claimed that finite lives or the presence of population growth are basic theoretical reasons for the absence of Ricardian equivalence. This is a misunderstanding, however. The distinguishing feature is whether new decision makers continue to enter the economy or not.

To sort this out, let $\bar{\beta}$ be a constant birth rate of *decision makers.* That is, if the population of decision makers is of size N, then $N\bar{\beta}$ is the inflow of new decision makers per time unit.[1] Given the assumption of a perfect credit market, we claim:

$$\text{there is Ricardian equivalence if and only if } \bar{\beta} = 0. \tag{13.9}$$

Indeed, with (13.8) in mind, when $\bar{\beta} = 0$, future taxes have to be paid by those current tax payers who are still alive in the future. In the absence of credit

[1] In view of the law of large numbers, we do not distinguish between expected and actual inflow.

market imperfections the current tax payers will thus respond to deficit finance (deferment of taxation) by increasing current saving out of the currently higher after-tax income. This increase in saving matches the expected extra taxes in the future. So current private consumption is unaffected by the deficit finance.

If $\bar{\beta} > 0$, however, deficit finance means shifting part of the tax burden from current tax payers to new tax payers in the future whom current tax payers do not care about. Even though representative agent models like the Ramsey and Barro models may include population growth in a demographic sense, they have a *fixed* number of dynastic families (decision makers) and whether the *size* of these dynastic families rises (population growth) or not is of no consequence for the question of Ricardian equivalence.

Another implication of (13.9) is that it is not the *finite lifetime* that is decisive for absence of Ricardian equivalence in OLG models. Indeed, even if we imagine the agents in a Blanchard-style model have a zero death rate, there will still be a *positive* birth rate. New decision makers continue to enter the economy through time. When deficit finance occurs, part of the tax burden is shifted to these newcomers.

To be specific, let $\bar{m}$ be a constant and age-independent death rate of existing decision makers. Then $\bar{n} \equiv \bar{\beta} - \bar{m}$ is the growth rate of the number of decision makers. With β, m, and n denoting the birth rate, death rate, and population growth rate, respectively, in the usual *demographic* sense, we have in Blanchard's model $\bar{\beta} = \beta$, $\bar{m} = m$, and $\bar{n} = n$. In the Ramsey model, however, $\bar{\beta} = \bar{m} = \bar{n} = 0 \leq n = \beta - m$. With this interpretation, both the Blanchard and the Ramsey model fit into (13.9). In the Blanchard model every new generation consists of new decision makers, i.e., $\bar{\beta} = \beta > 0$. In that setting, whether or not the population grows, the generations now alive know that the higher taxes in the future implied by deficit finance today will in part fall on the new generations. We therefore have $n \geq 0$, $\bar{\beta} = \bar{n} + \bar{m} \geq \bar{m} > 0$, and in accordance with (13.9) there is not Ricardian equivalence. In the Ramsey model where, in principle, the new generations are not new decision makers since their utility were already taken care of through bequests by their forerunners, there is Ricardian equivalence. This is in accordance with (13.9), since $\bar{\beta} = 0$, whereas $n \geq 0$.

The assumption in the Blanchard model that $\bar{m}$ $(= m)$ is independent of age might be more acceptable if we interpret $\bar{m}$ not as a biological mortality rate but as a *dynasty mortality rate*.[2] Thinking in terms of dynasties allows for *some* intergenerational links through bequests. In this interpretation $\bar{m}$ is the approximate probability that the family dynasty "ends" within the next time interval of unit length (either because members of the family die without children or because the preferences of the current members of the family no longer incorporate a be-

[2] This interpretation was suggested already by Blanchard (1985, p. 225).

quest motive). Then, $\bar{m} = 0$ corresponds to the extreme Barro case where such an event never occurs, i.e., that all existing families are infinitely-lived through intergenerational bequests. Even in this limiting case we can interpret statement (13.9) as telling that if new families still enter the economy ($\bar{\beta} > 0$), then Ricardian equivalence does not hold. How could *new* families enter the economy? One could imagine that immigrants are completely cut off from their relatives in their home country or that a parent only loves the first-born. In that case children who are not first-born, do not, effectively, belong to any preexisting dynasty, but may be linked forward to a chain of their own descendants (or perhaps only their first-born descendants). So *in spite of the infinite horizon of every family alive*, there are *newcomers*; hence, *Ricardian equivalence does not hold.*

Statement (13.9) also implies that if $\bar{\beta} = 0$, then $\bar{m} > 0$ does *not* destroy Ricardian equivalence. It is the difference between the public sector's future tax base (including the resources of individuals yet to be born) and the future tax base emanating from the individuals that are alive today that in the above analysis accounts for non-neutrality of variations over time in the pattern of lump-sum taxation. This reasoning also reminds us that it is immaterial for the validity of (13.9) whether there is productivity growth in the economy or not.

Additional sources of Ricardian non-equivalence

While the above demographic argument against Ricardian equivalence seems logically convincing, it is another question how large *quantitative* deviations from Ricardian equivalence it can deliver. Taking into account the sizeable life expectancy of the average citizen, Poterba and Summers (1987) point out that demography alone delivers only modest deviations if the issue is timing of taxes over the business cycle. Additional sources of deviation that have been put forward in the literature include:

1. *Short-sightedness.* There is evidence that households on average are not as forward-looking as required by the Ricardian equivalence hypothesis. Behavioral economists and experimental economics question that people conform to the assumption of full intertemporal rationality. People seem to have strong "present bias" (Laibson, 1997). With a limited planning horizon (up to five years, say) the effective discount rate becomes high and thereby capable of generating substantial deviation from Ricardian equivalence.

2. *Failure to leave bequests.* Though the bequest motive is certainly of empirical relevance, it is operative for only a minority of the population[3] (primarily

[3] Wolf (2002).

the wealthy families) and it need not have the altruistic form hypothesized by Barro, cf. Chapter 7.

3. *Imperfections in credit markets.* In practice there are imperfections in the credit markets. Many people can not borrow against expected future earnings. When you are credit rationed, you effectively face an interest rate higher than that faced by the government. Then, even if these people expect higher taxes in the future, the present value of the additional taxes is for these people less than the current reduction of taxes. Incurring a debt-financed tax cut the government helps credit-constrained people to tilt their intertemporal consumption by doing what these people would like to do but cannot, namely borrow - and in fact usually the government can do so at a comparatively low interest rate.

4. *Most taxes are not lump sum.* This should *not* be seen as an argument against the possible *theoretical* validity of the Ricardian equivalence hypothesis. Indeed, what the hypothesis claims is that there are no allocational effects of changes in the timing of *lump-sum* taxes. Nevertheless, widening the discussion to distortionary taxes is of course relevant. Towards the end of Chapter 6 we briefly considered both income taxes and consumption taxes.

5. *The Keynesian view.* The Keynesian point is that deviations from Ricardian equivalence tend to be amplified in situations with unemployment and slack aggregate demand. The reason is that otherwise un-utilized resources may be activated by a budget deficit resulting from a tax cut. By stimulating aggregate consumption in the "first round", a temporary tax cut stimulates aggregate demand and thereby production. The higher level of production amounts to higher income and thereby a further rise in consumption in the "second round" - and so on in the Keynesian multiplier process. In a recession also investment may be stimulated in the process due to increased sales. All in all a positive demand spiral arises: $T \downarrow \Rightarrow C \uparrow \Rightarrow Y \uparrow \Rightarrow I \uparrow \Rightarrow Y \uparrow \Rightarrow C \uparrow$ etc.[4]

To sum up, there are good reasons to believe that Ricardian equivalence fails. Of course, this could in some sense be said about nearly all theoretical abstrac-

[4]According to Keynesian theory, a similar multiplier process takes off as a result of a deficit-financed increase in government *spending* on goods and services: $G \uparrow \Rightarrow Y \uparrow \Rightarrow I \uparrow \Rightarrow Y \uparrow \Rightarrow I \uparrow \Rightarrow Y \uparrow$. Here, however, more than just a change in the timing of taxes is involved, namely a change in government spending on goods and services. So, we are outside the domain of the Ricardian Equivalence controversy in the narrow sence. The broader issue of the size of the government spending multiplier in alternative situations is treated later in this book.

tions. But the prevalent view among macroeconomists is that Ricardian equivalence systematically fails in *one* direction: it over-estimates the offsetting reaction of private saving in response to budget deficits. Moreover, relaxing the restrictive assumptions on which the Ricardian equivalence hypothesis rests, tends to *strengthen* the deviation from Ricardian equivalence implied by the simple demographic argument from OLG models.[5]

13.2 Dynamic general equilibrium effects of lasting budget deficits

The above discussion of effects of public debt is *partial equilibrium* analysis. We treated K, r, and w as unaffected by the changes in government debt. But when aggregate saving changes in a closed economy, so does K and generally also r and w. This should be taken into account. We therefore turn to a quantitative assessment of the full dynamic effects of public debt. This requires *general equilibrium* analysis.

We will apply the Blanchard OLG model from Chapter 12. To simplify, we ignore technological progress, population growth, and retirement all together. Therefore $g = n = \lambda = 0$, so that birth rate = mortality rate = m, and employment = population = N (a constant) for all t. Let public spending on goods and services be a constant $\bar{G} > 0$, assumed not to affect marginal utility of private consumption. Suppose all this spending is (and has always been) public *consumption*. There is thus no public capital. Let taxes and transfers be *lump sum* so that we need keep track only of the *net* tax revenue, T, and the consumption-saving trade-off is not affected by taxes.

We consider a closed economy described by

$$\dot{K}_t = F(K_t, N) - \delta K_t - C_t - \bar{G}, \qquad K_0 > 0, \text{ given,} \tag{13.10}$$

$$\dot{C}_t = (F_K(K_t, N) - \delta - \rho)C_t - m(\rho + m)(K_t + B_t), \tag{13.11}$$

$$\dot{B}_t = [F_K(K_t, N) - \delta] B_t + \bar{G} - T_t, \qquad B_0 > 0, \text{ given,} \tag{13.12}$$

where we have used the equilibrium relation $r_t = F_K(K_t, N) - \delta$. Here (13.10) is essentially just accounting for a closed economy; (13.11) describes changes in aggregate consumption, taking into account the generation replacement effect; and (13.12) describes how budget deficits give rise to increases in government debt. All government debt is assumed to be short-term and of the same form as a variable-rate loan in a bank. Hence, at any point in time B_t is historically determined and independent of the current and expected future interest rates.

[5] Some empirical evidence was briefly discussed in chapters 6 and 7.

As we shall see, the long-run interest rate will exceed the long-run output growth rate (which is nil). We know from Chapter 6 that in this case, to remain solvent, the government must satisfy its No-Ponzi-Game condition which, as seen from time zero, is

$$\lim_{t\to\infty} B_t e^{-\int_0^t [F_K(K_s,N)-\delta]ds} \leq 0. \tag{13.13}$$

This says that the debt is not in the long run allowed to grow at a rate as high as the long run interest rate. So, a permanent debt-rollover is ruled out.

In addition we assume that households satisfy their transversality conditions. Thereby the aggregate consumption function will be

$$C_t = (\rho + m)(K_t + B_t + H_t), \tag{13.14}$$

with

$$H_t = N \int_t^\infty (w_s - \tau_s) e^{-\int_t^s (r_z+m)dz} ds, \tag{13.15}$$

as in Section 13.1. These formulas will be useful when it comes to interpretation of the dynamics in the economy. For ease of exposition, we let the aggregate production function satisfy the Inada conditions $\lim_{K\to 0} F_K(K,N) = \infty$ and $\lim_{K\to\infty} F_K(K,N) = 0$. We assume $\delta > 0$ and $\rho \geq 0$.

So far the model is incomplete in the sense that there is nothing to pin down the time profile of T_t, except that ultimately the stream of taxes should conform to (13.13). Let us first consider a permanently balanced government budget.

Dynamics under a balanced budget

Suppose that from time 0 the government budget is balanced. Therefore, $\dot{B}_t = 0$ and $B_t = B_0$ for all $t \geq 0$. So (13.12) is reduced to

$$T_t = (F_K(K_t, N) - \delta)B_0 + \bar{G}, \tag{13.16}$$

giving the tax revenue required for the budget to be balanced, when the debt is B_0. This time path of T_t is determined *after* we have determined the time path of K_t and C_t through the two-dimensional system

$$\dot{K}_t = F(K_t, N) - \delta K_t - C_t - \bar{G}, \qquad K_0 > 0, \text{ given}, \tag{13.17}$$

$$\dot{C}_t = [F_K(K_t, N) - \delta - \rho]C_t - m(\rho + m)(K_t + B_0). \tag{13.18}$$

This system is independent of T_t. The implied dynamics can usefully be analyzed by a phase diagram.

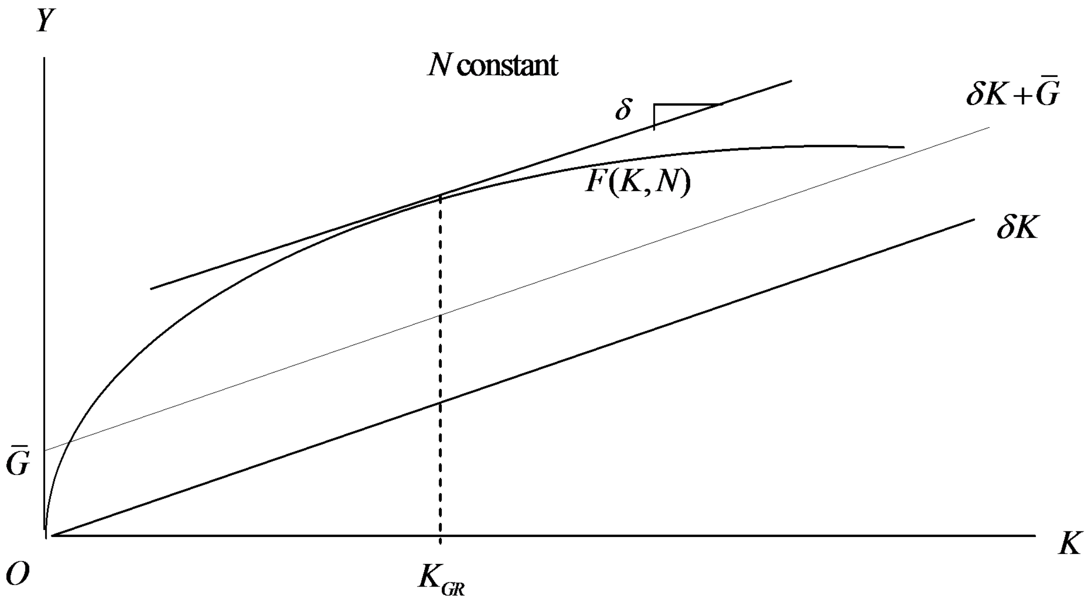

Figure 13.1: Building blocks for a phase diagram.

Phase diagram Equation (13.17) shows that

$$\dot{K} = 0 \quad \text{for} \quad C = F(K, N) - \delta K - \bar{G}. \tag{13.19}$$

The right-hand side of (13.19) is the vertical distance between the $Y = F(K, N)$ curve and the $Y = \delta K + \bar{G}$ line in Fig. 13.1. On the basis of this we can construct the $\dot{K} = 0$ locus in Fig. 13.2. We have indicated two benchmark values of K in the figure, namely the golden rule value K_{GR} and the value $\bar{K}$. These values are defined by

$$F_K(K_{GR}, N) - \delta = 0, \qquad \text{and} \qquad F_K\left(\bar{K}, N\right) - \delta = \rho,$$

respectively.[6] We have $\bar{K} \leq K_{GR}$, since $\rho \geq 0$ and $F_{KK} < 0$.

From equation (13.18) follows that

$$\dot{C} = 0 \quad \text{for} \quad C = \frac{m(\rho + m)(K + B_0)}{F_K(K, N) - \delta - \rho}. \tag{13.20}$$

Hence, for $K \to \bar{K}$ from below we have, along the $\dot{C} = 0$ locus, $C \to \infty$. In addition, for $K \to 0$ from above, we have along the $\dot{C} = 0$ locus that $C \to 0$, in view of the lower Inada condition.

[6] In this setup, where there is neither population growth nor technical progress, the golden rule capital stock is that K which maximizes $C = F(K, N) - \delta K - \dot{K}$ subject to the steady state condition $\dot{K} = 0$.

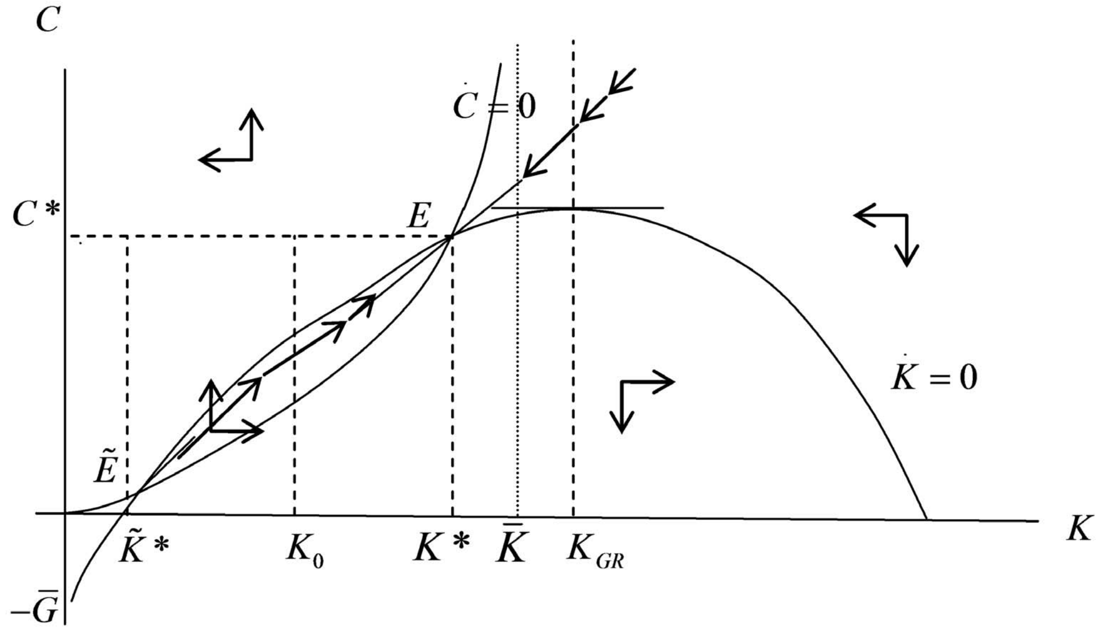

Figure 13.2: Phase diagram under a balanced budget.

Fig. 13.2 also shows the $\dot{C} = 0$ locus. We assume that $\bar{G}$ and B_0 are of "modest" size relative to the production potential of the economy for the given K_0 and N. Then the $\dot{C} = 0$ curve crosses the $\dot{K} = 0$ curve for *two* positive values of K, and the smallest of these is lower than K_0. Fig. 13.2 shows these steady states as the points E and Ẽ with coordinates (K^*, C^*) and $(\tilde{K}^*, \tilde{C}^*)$, respectively, where $\tilde{K}^* < K^* < \bar{K}$.

The direction of movement in the different regions of Fig. 13.2 are indicated by arrows determined by the differential equations (13.17) and (13.18). The steady state E is seen to be a saddle point, whereas Ẽ is a *source*.[7] We assume that $\bar{G}$ and B_0 are "modest" not only relative to the long-run production capacity of the economy but also relative to the given K_0. This means that $\tilde{K}^* < K_0$, as indicated in the figure.[8]

The capital stock is predetermined whereas consumption is a jump variable. Since the slope of the saddle path is not parallel to the C axis, it follows that the

[7] A steady state point with the property that all solution trajectories starting close to it move away from it is called a *source* or a *totally unstable* steady state.

[8] The opposite case, $\tilde{K}^* > K_0$, would reflect that G_0 and B_0 were very large relative to the initial production capacity of the economy, so large, indeed, that aggregate net saving would be chronically negative. Then a forever shrinking capital stock would be in prospect. The economy would in that case *not* converge towards the steady state E. This steady state would only be *locally* saddle-point stable, not globally saddle-point stable.

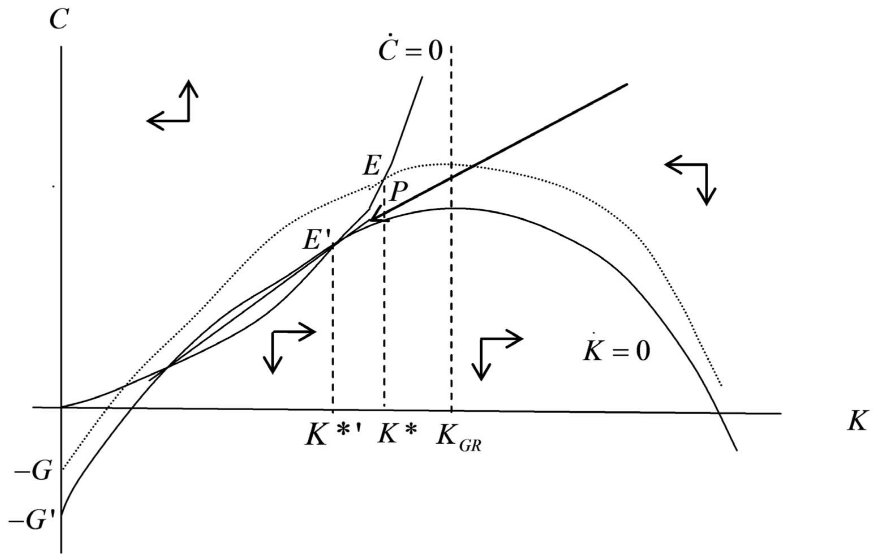

Figure 13.3: Tax-financed shift to higher public consumption.

system is saddle-point stable. The only trajectory consistent with *all* the conditions of general equilibrium (individual utility maximization for given expectations, profit maximization, continuous market clearing, fulfilled expectations) is the saddle path.[9] The other trajectories in the diagram violate the TVCs of the individual households. Hence, initial consumption, C_0, is determined as the ordinate to the point where the vertical line $K = K_0$ crosses the saddle path. Over time the economy moves along the saddle path, approaching the steady state point E with coordinates (K^*, C^*).

Although our main focus will be on effects of budget deficits and changes in the debt, we start with the simpler case of a tax-financed increase in $\bar{G}$.

Unexpected tax-financed shift to a higher level of public consumption Suppose that until time t_1 (> 0) the economy has been in the saddle-point stable steady state E. Hence, for $t < t_1$ we have zero net investment and $r = F_K(K^*, N) - \delta \equiv r^*$. Moreover, as $K^* < \bar{K}$, $r^* > \rho$ (≥ 0).

At time t_1 an unanticipated change in fiscal policy occurs. Public consumption shifts to a new constant level $\bar{G}' > \bar{G}$. Taxes are immediately increased by the same amount so that the budget stays balanced. We assume that everybody rightly expect the new policy to continue forever. The change to a higher G shifts the $\dot{K} = 0$ curve downwards as shown in Fig. 13.3, but leaves the $\dot{C} = 0$ curve

[9]By the same reasoning as in Appendix D of Chapter 12 it can be shown that when $\rho \geq 0$, the transversality conditions of the households will be satisfied in the steady state E, hence along paths converging towards E.

unaffected. At time t_1 when the policy shift occurs, private consumption jumps down to the level corresponding to the point P in Fig. 13.3. The explanation is that the net-of-tax human wealth, H_{t_1}, is immediately reduced as a result of the higher current and expected future taxes.

Owing to the upward-sloping new saddle path, cf. Fig. 13.3, the initial reduction in C is smaller than the increase in G (and T). Therefore net saving becomes negative and K decreases gradually until the new steady state, E', is "reached". To find the long-run effects on K and C we first equalize the right-hand sides of (13.19) and (13.20) and then use implicit differentiation w.r.t. $\bar{G}$ to get

$$\frac{\partial K^*}{\partial \bar{G}} = \frac{r^* - \rho}{C^* F_{KK}^* - (m + r^*)(\rho + m - r^*)} < 0;$$

next, from (13.19), by the chain rule we get

$$\frac{\partial C^*}{\partial \bar{G}} = \frac{\partial C^*}{\partial K^*}\frac{\partial K^*}{\partial \bar{G}} = r^* \frac{\partial K^*}{\partial \bar{G}} - 1 < -1,$$

where $r^* = F_K(K^*, N) - \delta$.[10] In the long run the decrease in C is *larger* than the increase in G because the economy ends up with a smaller capital stock.

Summing up That is, under full capacity utilization a tax-financed shift to higher G crowds out private consumption *and* investment. Private consumption is in the long run crowded out *more* than one to one due to reduced productive capacity. In this way the cost of the higher G falls relatively more on the younger and as yet unborn generations than on the currently elder generations.[11]

Higher public debt

To analyze the effect of a rise in public debt, let us first see how it might come about.

A tax cut Assume again that until time t_1 (> 0) the economy has had a balanced government budget and been in the saddle-point stable steady state E. The level of the public debt in this steady state is $B_0 > 0$ and tax revenue is, by (13.16),

$$T = (F_K(K^*, N) - \delta)B_0 + \bar{G} \equiv T^*,$$

a positive constant in view of $F_K(K^*, L) - \delta = r^* > \rho \geq 0$.

[10] For details, see Appendix B.

[11] This might be different if a part of G were public *investment* (in research and education, say), and this part were also increased.

At time t_1 the government unexpectedly cuts taxes to a lower constant level, $\bar{T}$, holding public consumption unchanged. At least for a while after time t_1 we thus have

$$T_t = \bar{T} < T^*. \tag{13.21}$$

As a result, $\dot{B}_t > 0$. The tax cut make current generations feel wealthier, hence they increase their consumption. They do so in spite of being forward-looking and anticipating that the current fiscal policy sooner or later must come to an end (because it is not sustainable, as we shall see). The prospect of higher taxes in the future dampens the increase in consumption, but does not prevent it, since part of the future taxes will fall on new generations entering the economy.

The rise in C, combined with unchanged G, implies negative net investment so that K begins to fall, implying a rising interest rate, r. For some time, *three* differential equations, determining changes in C, K, and B, are active. Moreover, while (13.10) and (13.12) still hold, (13.11) need not. This is because of the uncertainty about *when* and *how* a fiscal tightening will take place. Anyway, three-dimensional dynamics are complicated and cannot, of course, be illustrated in a two-dimensional phase diagram. Hence, for now we leave the phase diagram.

The fiscal policy $(\bar{G}, \bar{T})$ is not sustainable By definition a fiscal policy (G, T) is *sustainable* if the government stays solvent under this policy. We claim that the fiscal policy $(\bar{G}, \bar{T})$ is *not* sustainable. Relying on principles from Chapter 6, there are at least three different ways to prove this.

Approach 1: Sustained rise in the debt-income ratio. The negative net investment continues. And along with the falling K, we have a falling aggregate income, $Y_t = F(K_t, N)$. So we are in a situation where the interest rate remains larger than the long-run output growth rate which in the absence of growth in technology or labor force is clearly non-positive. The falling K implies falling Y.

The combination of a rising B and falling Y implies a forever rising debt-income ratio, B/Y. The private sector will understand that bankruptcy is threatening and nobody will buy government bonds except at a reduced price, which means a higher interest rate. The high interest rate only aggravates the problem. That is, the fiscal policy $(G, \bar{T})$ breaks down.

An alternative argument is:

Approach 2: The government NPG condition is violated. In view of $K^* < \bar{K} < K_{GR}$, we have $r^* = F_K(K^*, L) - \delta > F_K(\bar{K}, L) - \delta = \rho \geq 0$. After time t_1, K_t is falling, at least for a while. So $K_t < K^*$ and thus $r_t = F_K(K_t, N) - \delta > r^*$ > 0. Thereby the fiscal policy $(\bar{G}, \bar{T})$ implies an interest rate forever larger than the long-run output growth rate which in the absence of growth in technology or labor force is zero. From Chapter 6 we know that in this situation a sustainable fiscal policy must satisfy the NPG condition

$$\lim_{t\to\infty} B_t e^{-\int_{t_1}^{t} r_s ds} \leq 0. \tag{13.22}$$

With a forever positive debt, this requires that there exists an $\varepsilon > 0$ such that

$$\lim_{t\to\infty} \frac{\dot{B}_t}{B_t} < \lim_{t\to\infty} r_t - \varepsilon, \tag{13.23}$$

i.e., the long-run growth rate of the public debt should be less than the long-run interest rate.

The fiscal policy $(\bar{G}, \bar{T})$ violates this condition, however. Indeed, we have, for $t > t_1$,

$$\begin{aligned} \dot{B}_t &= r_t B_t + \bar{G} - \bar{T} \\ &> r^* B_0 + \bar{G} - \bar{T} > r^* B_0 + \bar{G} - T^* = 0, \end{aligned} \tag{13.24}$$

where the first inequality comes from $B_t > B_0 > 0$ and $r_t = F_K(K_t, L) - \delta > r^* = F_K(K^*, L) - \delta$, in view of $K_t < K^*$. This implies $B_t \to \infty$ for $t \to \infty$. Hence, dividing by B_t in (13.24) gives

$$\frac{\dot{B}_t}{B_t} = r_t + \frac{\bar{G} - \bar{T}}{B_t} \to r_t \qquad \text{for} \qquad t \to \infty, \tag{13.25}$$

which violates (13.23). So the fiscal policy $(\bar{G}, \bar{T})$ is not sustainable. The crux of the matter is that in the absence of economic growth, lasting budget deficits indicate an unsustainable fiscal policy.

Approach 3. Yet another way of showing absence of fiscal sustainability is to start out from the intertemporal government budget constraint and check whether the primary budget surplus, $\bar{T} - \bar{G}$, which rules after time t_1, satisfies

$$\int_{t_1}^{\infty} (\bar{T} - \bar{G}) e^{-\int_{t_0}^{t} r_s ds} dt \geq B_{t_1}, \tag{13.26}$$

where $B_{t_1} = B_0 > 0$. Obviously, if $\bar{T} - \bar{G} \leq 0$, (13.26) is not satisfied. Suppose $\bar{T} - \bar{G} > 0$. Then

$$\int_{t_1}^{\infty} (\bar{T} - \bar{G}) e^{-\int_{t_1}^{t} r_s ds} dt < \int_{t_1}^{\infty} (\bar{T} - \bar{G}) e^{-r^*(t-t_1)} dt = \frac{\bar{T} - \bar{G}}{r^*} < B_0 = B_{t_1},$$

where the first inequality comes from $r_t > r^*$, the first equality from carrying out the integration $\int_{t_1}^{\infty} e^{-r^*(t-t_1)} dt$, and, finally, the second inequality from the equality in the second row of (13.24) together with the fact that $\bar{T} < T^*$. So the intertemporal government budget constraint is not satisfied. The current fiscal policy is unsustainable.

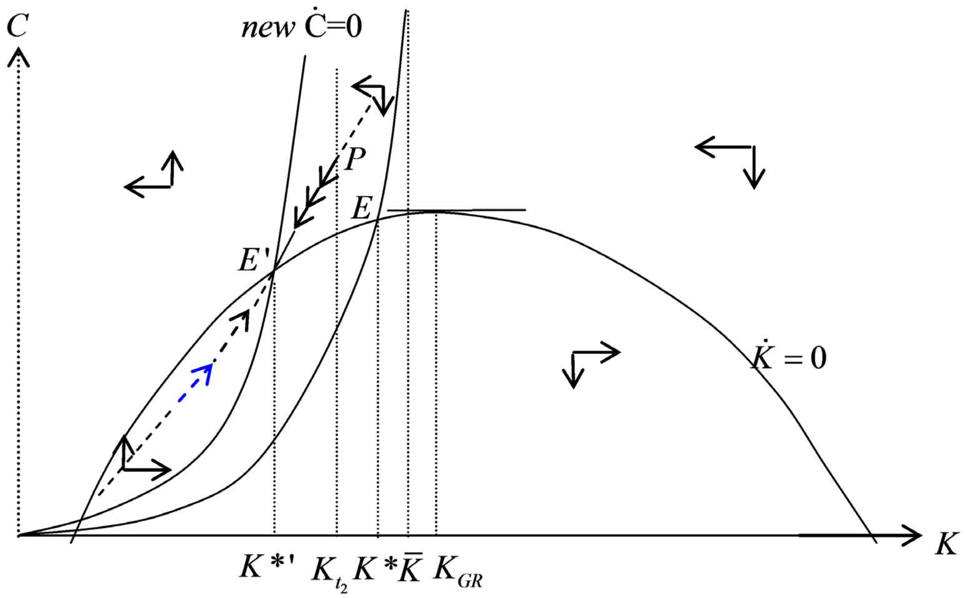

Figure 13.4: The adjustment after fiscal tightening at time t_2, presupposing $K^{*\prime} < K_{t_2}$.

Fiscal tightening and thereafter To avoid default on the debt, sooner or later the fiscal policy must change. This may take the form of lower of public consumption or higher taxes or both.[12] Suppose that the change occurs at time $t_2 > t_1$ in the form of a tax increase so that for $t \geq t_2$ there is again a balanced budget. This new policy is at time t_2 announced to be followed forever and we assume the market participants believe in this and that it holds true, at least "for a long time".

The balanced budget after time t_2 implies

$$T_t = (F_K(K_t, N) - \delta)B_{t_2} + \bar{G}. \tag{13.27}$$

The dynamics are therefore again governed by a two-dimensional system,

$$\dot{K}_t = F(K_t, N) - \delta K_t - C_t - \bar{G}. \tag{13.28}$$

$$\dot{C}_t = [F_K(K_t, N) - \delta - \rho]C_t - m(\rho + m)(K_t + B_{t_2}), \tag{13.29}$$

Consequently phase diagram analysis can again be used.

The phase diagram for $t \geq t_2$ is depicted in Fig. 13.4. The new initial K is K_{t_2}, which is smaller than the previous steady-state value K^* because of the negative net investment in the time interval $[t_1, t_2)$. Relative to Fig. 13.2, the $\dot{K} = 0$ locus is unchanged (since $\bar{G}$ is unchanged). But in view of the new constant debt level

[12] We still ignore financing by seigniorage.

B_{t_2} being higher than B_0, the $\dot{C} = 0$ locus has turned counter-clockwise. For any given $K \in (0, \bar{K})$, the value of C required for $\dot{C} = 0$ is higher than before, cf. (13.20). The intuition is that for every given K, private financial wealth is higher than before in view of the possession of government bonds being higher. For every given K, therefore, the generation replacement effect on the change in aggregate consumption is greater. Hence, so is the level of aggregate consumption that via the operation of the Keynes-Ramsey rule is required to offset the generation replacement effect and ensure $\dot{C} = 0$ (cf. Section 12.2 of the previous chapter).

The new saddle-point stable steady state is denoted E' in Fig. 13.4 and it has capital stock $K^{*\prime} < K^*$ and consumption level $C^{*\prime} < C^*$. As the figure is drawn, K_{t_2} is larger than $K^{*\prime}$. Intuitively, this case may arise if the tax cut at t_1 is "large" so that B_t rises fast in the time interval (t_1, t_2) and causes $K^{*\prime}$ to end up considerably below K^*. The level of consumption immediately after t_2, where the fiscal tightening sets in, is found where the vertical line $K = K_{t_2}$ crosses the new saddle path, i.e., the point P in Fig. 13.4. Immediately before t_2, consumption was at a higher level because the time of arrival of the tax increase was still uncertain. The movement of the economy after t_2 implies gradual lowering of the capital stock and consumption until the new steady state, E', is reached. (More details below in the section on time profiles.)

Alternatively, it can not be ruled out that K_{t_2} is smaller than $K^{*\prime}$ so that the new initial point, P, is to the left of the new steady state, E'. This case is illustrated in Fig. 13.5. Intuitively, this case may arise if the tax cut at t_1 is "small" so that B_t only rises slowly in the time interval (t_1, t_2), thereby causing $K^{*\prime}$ to end up not far below K^*. The low amount of capital at t_2 implies a high interest rate and the fiscal tightening must now be tough. This induces a low consumption level − so low that net investment becomes positive. Then the capital stock and output increase gradually during the adjustment to the steady state E'.[13]

Thus, in both cases the long-run effect of the transitory budget deficit is qualitatively the same, namely that the larger supply of government bonds crowds out physical capital in the private sector. Intuitively, a certain feasible time pro-

[13] A precise determination of conditions under which the case $K^{*\prime} < K_{t_2}$ versus the case $K^{*\prime} > K_{t_2}$ will occur is complicated. Three-dimensional dynamics is complex, and the uncertainty arising from t_2 not being fixed ex ante is a further complication. One might think that the longer the time interval (t_1, t_2) is, the more scope is there for the case $K_{t_2} < K^{*\prime}$ to arise. But the situation is less clear-cut than that because the longer the time interval (t_1, t_2) is, the larger is not only the fall in K but also the rise in B. Still, we might argue that there is a lower bound, $-\delta$, on the proportionate rate of change of the capital stock, whereas there is no comparable upper bound on how fast the government debt can increase. Hence, if the tax cut is substantial and the time interval (t_1, t_2) "small", it may seem likely that the fall in K is "dominated" by the rise in B as in Fig. 13.4. Anyway, numerical simulation and sensitivity analysis should be able to settle the matter but is not pursued here.

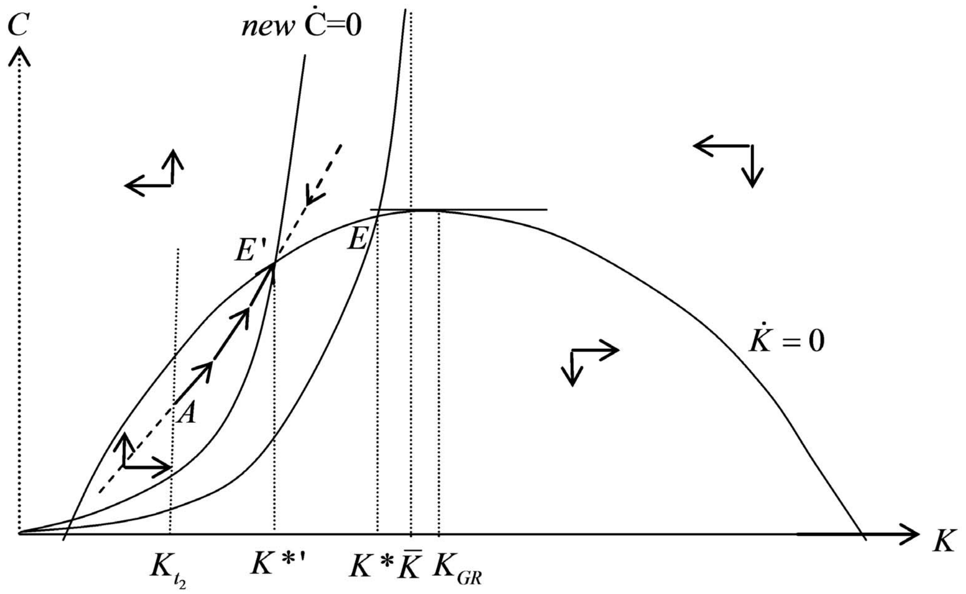

Figure 13.5: The adjustment after fiscal tightening at time t_2, presupposing $K^{*\prime} > K_{t_2}$.

file for financial wealth, $A = K + B$, is desired and the higher is B, the lower is the needed K. To this "stock" interpretation we may add a "flow" interpretation, saying that the budget deficit offers households a saving outlet which is an alternative to capital investment.

To be able to quantify the long-run effects of a change in the debt level on K and C we need the long-run multipliers. By equalizing the right-hand sides of (13.19) and (13.20), with B_0 replaced by $\bar{B}$, and using implicit differentiation w.r.t. $\bar{B}$, we get

$$\frac{\partial K^*}{\partial \bar{B}} = \frac{m\,(\rho + m)}{\mathcal{D}} < 0, \tag{13.30}$$

where $\mathcal{D} \equiv C^* F^*_{KK} - (r^* + m)(\rho + m - r^*) < 0$.[14] Next, by using the chain rule on $C^* = F(K^*, N) - \delta K^* - \bar{G}$ from (13.19), we get

$$\frac{\partial C^*}{\partial \bar{B}} = \frac{\partial C^*}{\partial K^*}\frac{\partial K^*}{\partial \bar{B}} = (F_K(K^*, N) - \delta)\frac{m\,(\rho + m)}{\mathcal{D}} = r^* \frac{m\,(\rho + m)}{\mathcal{D}} < 0.$$

The multiplier $\partial K^*/\partial \bar{B}$ tells us the approximate size of the long-run effect on the capital stock, when a temporary tax cut causes a unit increase in public debt. The resulting change in long-run output is approximately $\partial Y^*/\partial \bar{B} = (\partial Y^*/\partial K^*)(\partial K^*/\partial \bar{B}) = (r^* + \delta) m\,(\rho + m)\,/\mathcal{D} < 0$. The elasticity of long-run out-

[14] For details, see Appendix B.

put with respect to public debt is $(\bar{B}/Y^*)\partial Y^*/\partial \bar{B} = (\bar{B}/Y^*)(r^*+\delta)m\,(\rho+m)\,/\mathcal{D} < 0$.

Under full capacity utilization government deficits have a crowding-out effects because they compete with private investment for the allocation of saving.

These results of course hinge on the assumption of permanent full capacity utilization in the economy (no idle capital, no idle labor). When the economy is far below full resource utilization, allowing a budget deficit to arise helps stimulating aggregate demand, output, and income. The resulting increase in aggregate saving raises the flow of loanable funds and create *downward* pressure on the interest rate.

Time profiles It is also useful to consider the time paths of the variables.

Case 1: $K_{t_2} > K^{*\prime}$. Fig. 13.6 shows stylized aspects of the time profile of T and B, respectively. The upper panel visualizes that the increase in taxation at time t_2 is larger than the decrease at time t_1. As (13.27) shows, this is due to public expenses being larger after t_2 because both the government debt B_t and the interest rate, $F_K(K_t, N_t) - \delta$, are higher. The further gradual rise in T_t towards its new steady-state level is due to the rising interest service along with a rising interest rate, caused by the falling K.

The middle panel of Fig. 13.6 is self-explanatory.

As visualized by the lower panel of Fig. 13.6, the tax cut at time t_1 results in an upward jump in consumption. This implies negative net investment, so that K begins to fall. The size of the upward jump in consumption at time t_1 and the subsequent time path of consumption in the time interval $[t_1, t_2)$ can not be precisely pinned down. We can not even be sure that C will be gradually falling within this time interval. Therefore the downward-sloping time path of C in the lower panel of Fig. 13.6 in this time interval illustrates just one of the possibilities.

The ambiguity arises for the following reason. Though the current generations will immediately feel wealthier and increase their consumption as a result of the tax cut, they have rational expectations and are thereby aware that sooner or later fiscal policy will have to be changed again. As the households may have uncertain and different beliefs about *when* and *how* the fiscal sustainability problem will be remedied, we can not theoretically assign a specific value to the new after-tax human wealth, even less a constant value. What we can tell is that H_{t_1}, and therefore C_{t_1}, will be "somewhat" larger than immediately before time t_1. Also private saving will rise, however. This is because the rise in consumption at time t_1 will be less than the fall in taxes. To see this, imagine first that the households expect a constant level, T, to last for a long time during which also the real interest rate and the real wage remain approximately unchanged. Perceived

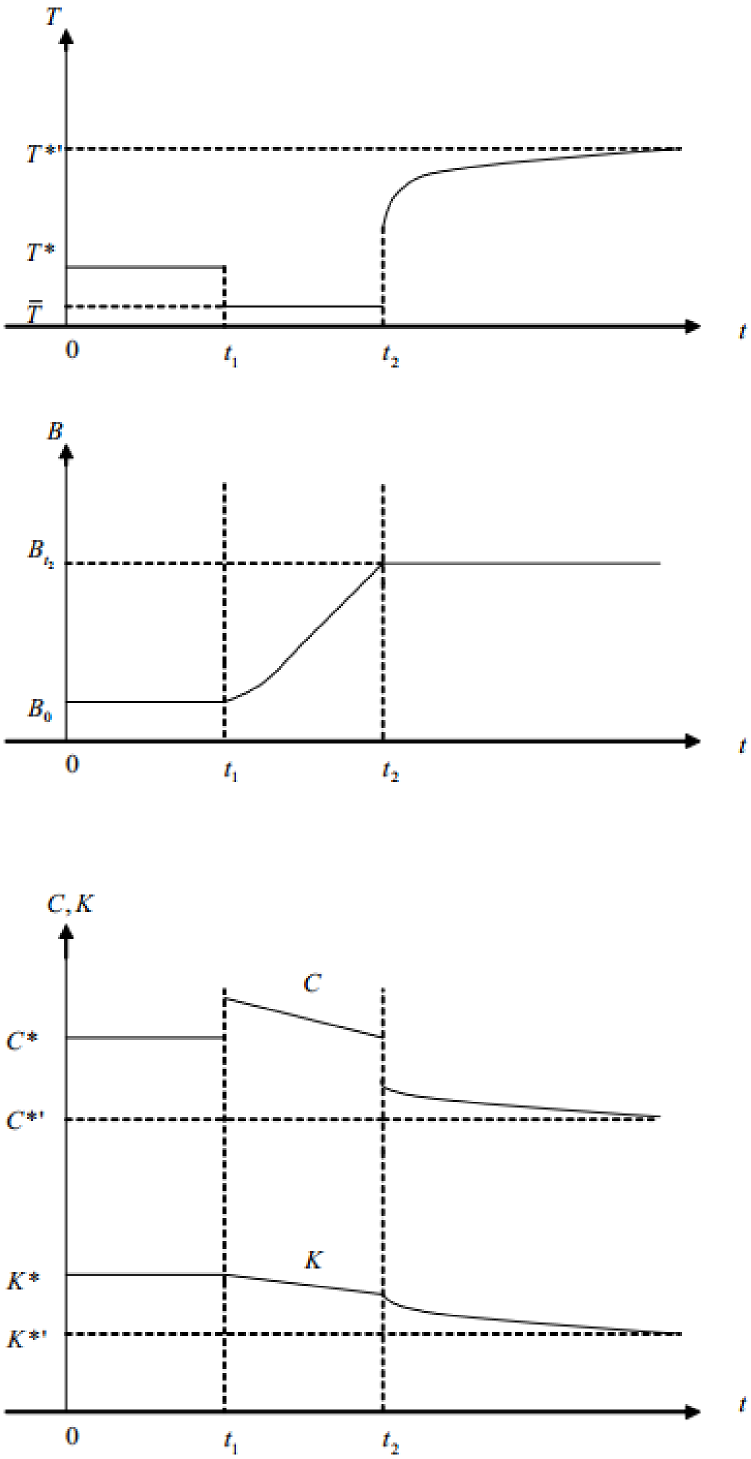

Figure 13.6: Case 1: $K_{t_2} > K^{*\prime}$. Regarding time path of C in the time interval (t_1, t_2) only one possibility shown.

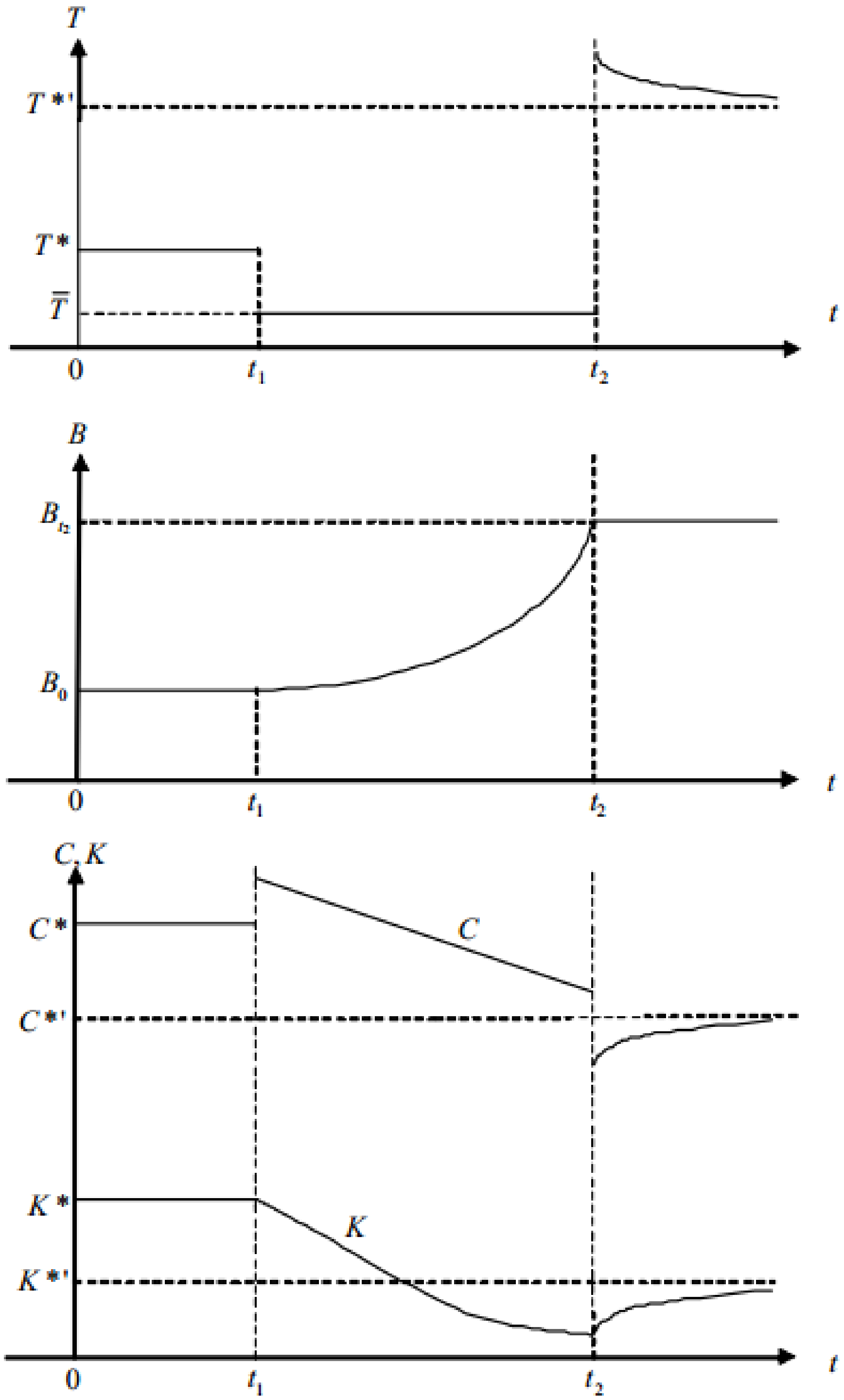

Figure 13.7: Case 2: $K_{t_2} < K^{*\prime}$. Regarding time path of C in the time interval (t_1, t_2) only one possibility shown.

human wealth would then be $H \approx (w^*N - T)/(r^* + m)$, from (13.15). By $C_t = (\rho + m)(A_t + H)$, we would have

$$\Delta C_t \approx dC_t = \frac{\partial C_t}{\partial T}dT = (\rho + m)\frac{\partial H}{\partial T}dT = -\frac{\rho + m}{r^* + m}dT < -dT, \qquad (13.31)$$

in view of $dT = \bar{T} - T^* < 0$ and $r^* > \rho$. To the extent that the households expect the new tax level $\bar{T}$ to last a *shorter* time, the boost to H, and therefore also to C, will be *less* than indicated by this equation. The boost to H and C is further dampened by the (correct) anticipation that the ongoing negative net investment will imply a falling K and thereby a falling real wage (due to the falling marginal productivity of labor) and a rising interest rate (due to the rising net marginal productivity of capital). So, at least for a while, there *will* be *positive private saving*, hence rising private financial wealth A. Meanwhile, H *will be falling* after t_1 due to the falling real wage, the rising interest rate, and the fact that the date of likely fiscal tightening is approaching, although uncertain.

So the two components of total wealth, A and H, move in opposite directions. Depending of which of these opposite movements is dominating, consumption will be rising or falling for a while after t_1 (Fig. 13.6 depicts the latter case). Anyway, because the exact time and form of the fiscal tightening is not anticipated, a sharp decrease in the present discounted value of after-tax labor income occurs at time t_2. This induces a downward jump in consumption as indicated in the lower panel of Fig. 13.6. Although the fall in consumption makes room for increased net investment, in Case 1 net investment remains negative so that the fall in K continues after t_2. Therefore, also the real wage continues to fall, implying continued fall in H, cf. (13.14) and (13.15), hence further fall in C, until the new steady-state level is reached.

If the time of the fiscal tightening were anticipated, consumption would not jump at time t_2. But the long-run result would be qualitatively similar.

Case 2: $K_{t_2} < K^{*\prime}$. In this case the tax revenue after t_2 has to exceed what is required in the new steady state. During the subsequent adjustment the taxation level will be gradually falling which reflects the gradual fall in the interest rate generated by the rising K, cf. Fig. 13.5. Private consumption will at time t_2 jump to a level *below* the new (in itself lower) steady state level, $C^{*\prime}$.

The above analysis is in a sense "biased" against budget deficits because it ignores economic growth. Thereby persistent budget deficits necessarily become incompatible with fiscal sustainability. With economic growth, persistent budget deficits are compatible with fiscal sustainability as long as the resulting government debt does not persistently grow faster than GDP. A further limitation of the analysis is its abstraction from the role of Keynesian aggregate demand factors in the process.

13.3 Public and foreign debt: a small open economy

Let the country considered be a small open economy (SOE). Suppose there is perfect substitutability and mobility of goods and financial capital across borders, but no mobility of labor. The main difference compared with the above analysis is then that the interest rate will not be affected by the public debt of the country (as long as its fiscal policy seems sound). Besides making the analysis simpler, this entails a *stronger* crowding out effect of public debt than in the closed economy. The lack of an offsetting increase in the interest rate means absence of the feedback which in a closed economy limits the fall in aggregate saving. In the open economy national wealth equals the stock of physical capital plus net foreign assets. And it is national wealth rather than the capital stock which is crowded out.

The model

The analytical framework is still Blanchard's OLG model with constant population. As above we concentrate on the simple case: $g = \lambda = 0$ and birth rate $=$ mortality rate $= m > 0$. The real interest rate is given from the world financial market and is a constant $r > 0$. Table 13.1 lists key variables for an open economy.

Table 13.1. New variable symbols

A_t^n	$= A_t - B_t = K_t + A_t^f =$ national wealth
$-B_t$	$= -$ government (net) debt $=$ government financial wealth
A_t^f	$=$ net foreign assets (the country's net financial claims on the rest of the world)
D_t	$= -A_t^f =$ net foreign debt
A_t	$= K_t + B_t + A_t^f =$ private financial wealth
$\dot{A}_t$	$= S_t^p =$ private net saving
$-\dot{B}_t$	$= S_t^g = T_t - G - rB_t =$ government net saving $=$ budget surplus
$\dot{A}_t^n$	$= \dot{A}_t - \dot{B}_t = S_t^p + S_t^g = S_t^n =$ aggregate net saving
NX_t	$=$ net exports
$\dot{A}_t^f$	$= \dot{A}_t - \dot{B}_t - \dot{K}_t = NX_t + rA_t^f = CAS_t =$ current account surplus
CAD_t	$= -CAS_t = rD_t - NX_t =$ current account deficit

In view of profit-maximization, the equilibrium capital stock, K^*, satisfies $F_K(K^*, N) = r + \delta$ and is thus a constant. The equilibrium real wage is $w^* = F_L(K^*, N)$. The increase per time unit in real private financial wealth is

$$\dot{A}_t = rA_t + w^*N - T_t - C_t = rA_t + (w^* - \tau_t)N - C_t, \tag{13.32}$$

where $\tau_t \equiv T_t/N$ is a per capita lump-sum tax. The corresponding differential equation for C_t reads $\dot{C}_t = (r-\rho)C_t - m(\rho+m)A_t$. To keep track of consumption in the SOE, however, it is easier to focus directly on the level of consumption:

$$C_t = (\rho + m)(A_t + H_t), \tag{13.33}$$

where H_t is (after-tax) human wealth, given by

$$H_t = N\int_t^\infty (w^* - \tau_s)e^{-(r+m)(s-t)}ds = \frac{Nw^*}{r+m} - N\int_t^\infty \tau_s e^{-(r+m)(s-t)}ds. \tag{13.34}$$

Suppose that from time 0 the government budget is balanced, so that B_t is constant at the level B_0 and $T_t = rB_0 + \bar{G} \equiv T^*$. Consequently,

$$\tau_t = \frac{T^*}{N} = \frac{rB_0 + \bar{G}}{N} \equiv \tau^*. \tag{13.35}$$

Under "normal" circumstances $\tau^* < w^*$, that is, B_0 and $\bar{G}$ are not so large as to leave non-positive after-tax earnings. Then, in view of the constant per capita tax, (13.34) gives

$$H_t = \frac{w^* - \tau^*}{r+m}N \equiv H^* > 0. \tag{13.36}$$

Consequently, (13.32) simplifies to

$$\begin{aligned} \dot{A}_t &= (r-\rho-m)A_t + (w^* - \tau^*)N - (\rho+m)\frac{w^*-\tau^*}{r+m}N \\ &= (r-\rho-m)A_t + \frac{r-\rho}{r+m}(w^*-\tau^*)N. \end{aligned} \tag{13.37}$$

Presupposing $r \neq \rho + m$, this linear differential equation has the solution

$$A_t = (A_0 - A^*)e^{(r-\rho-m)t} + A^*, \tag{13.38}$$

where A^* is the steady-state national wealth,

$$A^* = \frac{(r-\rho)(w^*-\tau^*)N}{(r+m)(\rho+m-r)}. \tag{13.39}$$

(For economic relevance of the solution (13.38) it is required that $A_0 > -H^*$, since otherwise C_0 would be zero or negative in view of (13.33).) Substitution into (13.33) gives steady-state consumption,

$$C^* = \frac{m(\rho+m)(w^*-\tau^*)N}{(r+m)(\rho+m-r)}. \tag{13.40}$$

By an argument similar to that in Appendix D of Chapter 12, it can be shown that the transversality conditions of the individual households are satisfied along the path (13.38).

By (13.37) we see that the steady state, A^*, is asymptotically stable if and only if

$$r < \rho + m. \tag{13.41}$$

Let us consider this case first. The phase diagram describing this case is shown in the upper panel of Fig. 13.8. The lower panel of the figure illustrates the movement of the economy in (A, C) space, given $A_0 < A^*$. The $\dot{A} = 0$ line represents the equation $C = rA + (w^* - \tau^*)N$, which in view of (13.32) must hold when $\dot{A} = 0$. Its slope is lower than that of the line representing the consumption function, $C = (\rho + m)(A + H^*)$. The economy is always at some point on this line.[15] A sub-case of (13.41) is the following case.

Medium impatience: $r - m < \rho < r$

As Fig. 13.8 is drawn, it is presupposed that $A^* > 0$, which, given (13.41), requires $r - m < \rho < r$. This is the case of "medium impatience".

A fiscal easing Imagine that until time $t_1 > 0$ the system has been in the steady state E. At time t_1 an unforeseen tax cut occurs so that at least for some spell of time after t_1 we have $T = \bar{T} < T^*$, hence $\tau = \bar{\tau} \equiv \bar{T}/N < \tau^*$. Since government spending remains unchanged, there is now a budget deficit and public debt begins to rise. We know from the partial equilibrium analysis of Section 13.1 that current generations will feel wealthier and increase their consumption. Like in the similar situation in the closed economy of Section 13.2, we can not assign a specific value to the new after-tax human wealth, even less a constant value. The phase diagram as in Fig. 13.8 is thus no longer applicable and for now we leave phase diagram analysis.

We claim that the rise in consumption at time t_1 will be less than the fall in taxes. This amounts to positive private saving and rising private financial wealth for a while. To see this provisional outcome, imagine first that the agents expect taxation to be at a constant level, T, *forever*. Perceived human wealth would then be $H = (w^*N - T)/(r + m)$, in analogy with (13.36). From $C_t = (\rho + m)(A_t + H)$ we would have

$$dC_t \approx \frac{\partial C_t}{\partial T} dT = (\rho + m)\frac{\partial H}{\partial T} dT = -\frac{\rho + m}{r + m} dT < -dT, \tag{13.42}$$

[15] If we (as for the closed economy) had based the analysis on *two* differential equations in A and C, then a saddle path would arise and this path would coincide with the $C = (\rho + m)(A + H^*)$ line in Fig. 13.8.

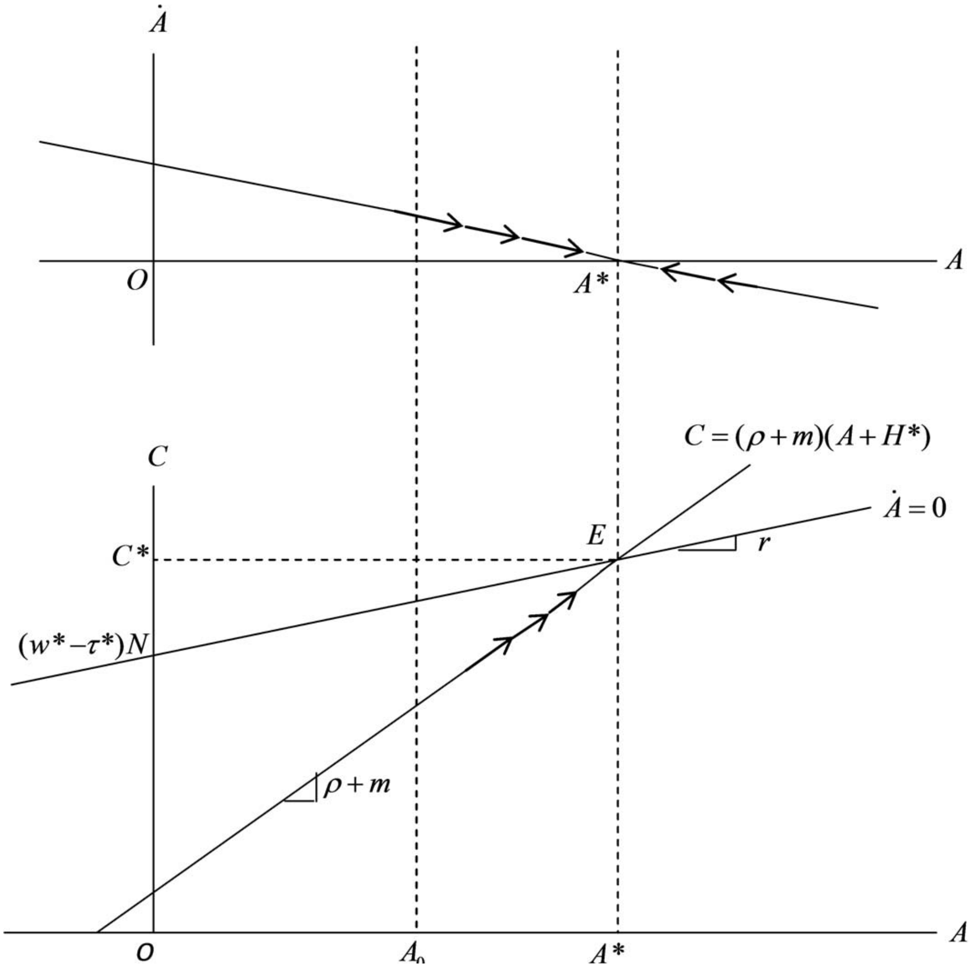

Figure 13.8: Dynamics of an SOE with *medium impatience*, i.e., $r - m < \rho < r$ (balanced budget).

in view of $dT = \bar{T} - T^* < 0$ and $r > \rho$. To the extent that the households expect the new tax level $\bar{T}$ to last a shorter time, the boost to H and C will be *less* than indicated by (13.42). This fortifies the rise in saving and the resulting growth in A.

Fiscal tightening at a higher debt level As hinted at, the fiscal policy $(\bar{G}, \bar{T})$ is not sustainable. It generates a growth rate of government debt which approaches r, whereas income and net exports are clearly bounded in the absence of economic growth.[16] To end the runaway debt spiral a fiscal tightening sooner or later is carried into effect. Suppose this happens at time $t_2 > t_1$. Let the fiscal tightening take the form of a return to a balanced budget with unchanged $\bar{G}$. That is, for $t \geq t_2$ the tax revenue is

$$T = rB_{t_2} + \bar{G} \equiv T^{*\prime} > T^*,$$

where the inequality is due to $B_{t_2} > B_0$. The corresponding per-capita tax is $\tau^{*\prime} \equiv T^{*\prime}/N > \tau^*$.

Since the budget is now balanced, a phase diagram of the same form as in Fig. 13.8 is again valid and is depicted in Fig. 13.9. Compared with Fig. 13.8 the $\dot{A} = 0$ line is shifted downwards because $w^* - \tau^{*\prime}$ is lower than before t_1. For the same reason the new level of human wealth, $H^{*\prime}$, is lower than the old, H^*. So the line representing the consumption function is also shifted down compared to the situation before t_1. Immediately after time t_2 the economy is at some point like P, where the vertical line $A = A_{t_2}$ $(> A^*)$ crosses the new line representing the consumption function. The economy then moves along that line and converges toward the new steady state, E'. At that point we have $A = A^{*\prime} < A^*$ and $C = C^{*\prime} < C^*$.

As a consequence *national wealth* goes down *more* than one to one with the increase in government debt when we are in the medium impatience case. Indeed, for a given level $\bar{B}$ of government debt, long-run national wealth is

$$A^{n*} \equiv A^* - \bar{B}. \tag{13.43}$$

An increase in government debt by $d\bar{B}$ increases national wealth by $\Delta A^{n*} \approx dA^{n*} = (\partial A^*/\partial \bar{B} - 1)d\bar{B} < -d\bar{B}$, since $\partial A^*/\partial \bar{B} < 0$ when $r - m < \rho < r$. The explanation follows from the analysis above. On top of the reduction of government wealth by $d\bar{B}$ there is a reduction of private financial wealth due to the private dissaving during the adjustment process. This dissaving occurs

[16] Indeed, as in the analogue situation for the closed economy, $\dot{B}_t/B_t = r + (\bar{G} - \bar{T})/B_t \to r$ for $t \to \infty$. Because we ignore economic growth, lasting budget deficits indicate an unsustainable fiscal policy.

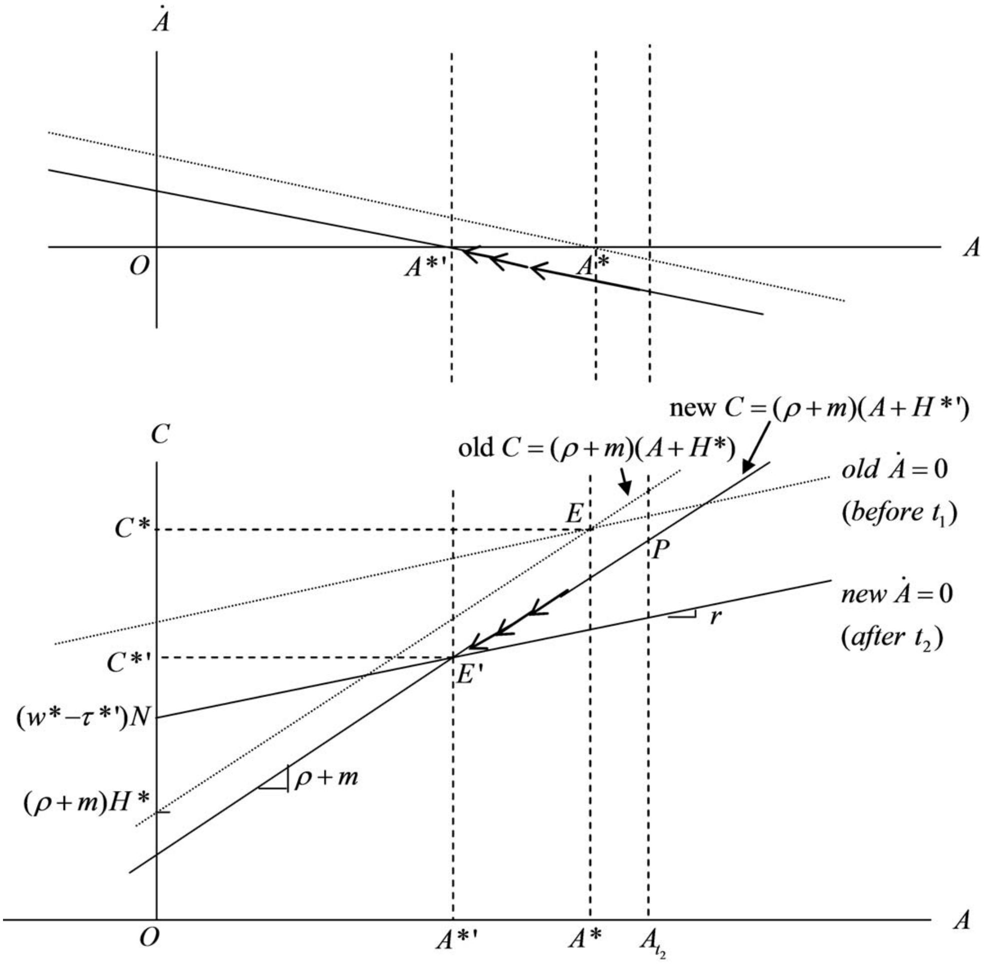

Figure 13.9: The adjustment after time t_2 showing the effect of a higher level of government debt.

because consumption responds less than one to one (in the opposite direction) when T is changed, cf. (13.42).

To find the exact long-run effect on national wealth of a rise in $\bar{B}$, in (13.35) replace B_0 by $\bar{B}$ and substitute into (13.39) to get

$$A^* = \frac{(r-\rho)(w^*N - r\bar{B} - \bar{G})}{(r+m)(\rho+m-r)}. \tag{13.44}$$

Inserting this into (13.43), we find the effect of public debt on national wealth in steady state to be

$$\frac{\partial A^{n*}}{\partial \bar{B}} = -\frac{(r-\rho)r}{(r+m)(\rho+m-r)} - 1. \tag{13.45}$$

This gives the size of the long-run effect on national wealth when a temporary tax cut causes a unit increase in long-run government debt. In our present medium impatience case, $r - m < \rho < r$ and so (13.45) implies $\partial A^{n*}/\partial \bar{B} < -1$.[17]

Very high impatience: $\rho > r$

Also this case with high impatience is a sub-case of (13.41). When $\rho > r$, (13.45) gives $-1 < \partial A^{n*}/\partial \bar{B} < 0$. This is because such an economy will have $0 < \partial A^*/\partial \bar{B} < 1$. In view of the high impatience, $A^* < 0$. That is, in the long run the SOE has *negative* private financial wealth reflecting that all physical capital in the country and some of the human wealth is essentially mortgaged to foreigners. This outcome is not plausible in practice. Owing to credit market imperfections there is likely to be difficulties of refinancing the debt in such a situation. In addition, politically motivated government intervention will presumably hinder such a development before national wealth is in any way close to zero.

Very low impatience: $\rho < r - m$

When $\rho < r - m$, an economically relevant steady state no longer exists since that would, by (13.40), require negative consumption. In the lower panel of Fig. 13.9 the slope of the $C = (\rho + m)(A + H^*)$ line will be smaller than that of the $\dot{A} = 0$ line and the two lines will never cross for a positive C.[18] With initial total wealth positive (i.e., $A_0 > -H^*$), the excess of r over $\rho + m$ results in sustained positive saving so as to keep A growing forever along the $C = (\rho + m)(A + H^*)$

[17] In the knife-edge case $\rho = r$, we get $A^* = 0$. In this case $\partial A^{n*}/\partial \bar{B} = -1$.

[18] In the upper panel of Fig. 13.9 the line representing $\dot{A}$ as a function of A will have positive slope. The stability condition (13.41) is no longer satisfied. There is still a "mathematical" steady-state value $A^* < 0$, but it can not be realized, because it requires negative consumption.

line. That is, the economy grows *large*. In the long run the interest rate in the world financial market can no longer be considered independent of this economy − the SOE framework ceases to fit.

As long as the country is still relatively small, however, we may use the model as an approximation. Though there is no steady state level of national wealth to focus at, we may still ask how the time path of national wealth, A_t^n, is affected by a rise in government debt caused by a temporary tax cut during the time interval $[t_1, t_2)$. We consider the situation after time t_2, where there is again a balanced government budget. For all $t \geq t_2$ we have $A_t^n = A_t - \bar{B}$, where $\bar{B} = B_{t_2}$ and, in analogy with (13.38),

$$A_t = (A_{t_2} - A^*)e^{(r-\rho-m)(t-t_2)} + A^*,$$

with A^* defined as in (13.44) (now a repelling state). For a given $A_{t_2} > -H^{*\prime}$ we find for $t > t_2$

$$\begin{aligned} \frac{\partial A_t^n}{\partial \bar{B}} &= \frac{\partial A_t}{\partial \bar{B}} - 1 = \left(1 - e^{(r-\rho-m)(t-t_2)}\right) \frac{\partial A^*}{\partial \bar{B}} - 1 \\ &= \left(1 - e^{(r-\rho-m)(t-t_2)}\right) \left(-\frac{(r-\rho)r}{(r+m)(\rho+m-r)}\right) - 1, \qquad (13.46) \end{aligned}$$

by (13.44).[19] Since $\rho < r - m$, the right-hand side of (13.46) is less than -1 and over time rising in absolute value. In spite of the lower private saving triggered by the higher taxation after time t_2, private saving remains positive due to the low rate of impatience. Financial wealth is thus still rising and so is private income. But the lower saving out of a rising income implies more and more "forgone future income". This explains the rising crowding out envisaged by (13.46).

Current account deficits and foreign debt

Do persistent current account deficits in the balance of payments signify future borrowing problems and threatening bankruptcy? To address this question we need a few new variables.

Let NX_t denote net exports (exports minus imports). Then, the output-expenditure identity reads

$$Y_t = C_t + I_t + G_t + NX_t. \qquad (13.47)$$

Net foreign assets are denoted A_t^f and equals minus net foreign debt, $-D_t =$

[19] The condition $A_{t_2} > -H^{*\prime}$ is needed for economic relevance since otherwise $C_{t_2} \leq 0$. The condition also ensures $A_{t_2} > A^*$, since $A^* < -H^{*\prime}$ when $\rho < r - m$.

$A_t - B_t - K_t$. Gross national income is $Y_t + rA_t^f = Y_t - rD_t$.[20] The current account surplus at time t is

$$\begin{aligned} CAS_t &= \dot{A}_t^f = \dot{A}_t - \dot{B}_t - \dot{K}_t = rA_t^f + NX_t \\ &= Y_t + rA_t^f - (C_t + I_t + G_t), \end{aligned} \quad (13.48)$$

by (13.47). The first line views CAS from the perspective of changes in assets and liabilities. The second line views it from an income-expenditure perspective, that is, the current account surplus is the excess of gross national income over and above home expenditure. Gross national saving, S_t, equals, by definition, gross national income minus the sum of private and public consumption, that is, $S_t = Y_t + rA_t^f - C_t - G_t$. Hence, the current account surplus can also be written as the excess of gross national saving over and above gross investment: $CAS_t = S_t - I_t$. Of course, the current account deficit is $CAD_t \equiv -CAS_t = I_t - S_t$.

In our SOE model above, with constant $r > 0$ and no economic growth, the capital stock is a constant, K^*. Then (13.47) gives net exports as a residual:

$$NX_t = F(K^*, N) - C_t - \delta K^* - \bar{G}, \quad (13.49)$$

where $C_t = (\rho + m)(A_t + H_t)$. We concentrate on the case where an economic steady state exists and is asymptotically stable, i.e., (13.41) holds. In the steady state being in force for $t < t_1$, $B_t = B_0$, $H_t = H^*$, and $A_t = A^*$, as given in (13.36) and (13.39), respectively. Thus, $A_t^f = A^* - B_0 - K^* \equiv A^{f*} \equiv -D^*$ so that $0 = \dot{A}_t^f = CAS_t \equiv -CAD_t$. Then, by (13.48),

$$NX_t = -rA^{f*} = rD^*. \quad (13.50)$$

This should also be the value of net exports we get from (13.49) in steady state. To check this, we consider

$$\begin{aligned} NX_t &= F(K^*, N) - C^* - \delta K^* - \bar{G} = F_K(K^*, N)K^* + F_L(K^*, N)N - C^* - \delta K^* - \bar{G} \\ &= (r + \delta)K^* + w^*N - C^* - \delta K^* - \bar{G}, \end{aligned}$$

where we have used Euler's theorem on a function homogeneous of degree one. Combining with (13.32) evaluated in steady state, we thus have

$$\begin{aligned} NX_t &= (r + \delta)K^* + w^*N - (rA^* + (w^* - \tau^*)N) - \delta K^* - \bar{G} \\ &= r(K^* - A^*) + \tau^*N - \bar{G} = r(K^* - A^* - B_0) = rD^*, \end{aligned}$$

where the third equality follows from the assumption of a balanced budget. Our accounting is thus coherent.

[20] In a more general setup also net foreign worker remittances, which we here ignore, should be added to GDP to calculate gross national income.

We see that permanent foreign debt is consistent with a steady state if net exports are sufficient to match the interest payments on the debt. That is, a steady state does not require trade balance, but a balanced *current account.* As we shall see in a moment, in an economy with economic *growth* not even the current account need be balanced. Before leaving the non-growing economy, however, a few remarks about the current account *out of steady state* are in place.

Emergence of twin deficits Consider again the fiscal easing regime ruling in the time interval $[t_1, t_2)$. The higher C_t resulting from the fiscal easing leads to a lower NX_t than before t_1, cf. (13.49). As a result, $CAD_t > 0$. So a current account deficit has emerged in response to the government budget deficit. This situation is known as the *twin deficits.* As we argued, the situation is not sustainable. Sooner or later, the incipient lack of solvency will manifest itself in difficulties with continued borrowing. Something must be changed.

From mere accounting we know that the current account deficit can also be written as the difference between aggregate net investment, I_t^n, and aggregate net saving, S_t^n. So

$$\begin{aligned} CAD_t &= I_t - S_t = I_t - \delta K_t - (S_t - \delta K_t) = I_t^n - S_t^n \\ &= I_t^n - (S_t^p + S_t^g) = I_t^n - S_t^p + \dot{B}_t, \end{aligned} \tag{13.51}$$

since public saving, S_t^g, equals $-\dot{B}_t$, the negative of the budget deficit. Now, starting from a balanced budget and balanced current account, whether a budget deficit tends to generate a current account deficit depends on how net investment and net private saving respond. In the present example we have $K_t = K^*$ and thereby $I_t^n = 0$ for all t. For $t < t_1$, also $S_t^p = rA^* + (w^* - \tau^*)N - C^* = 0$ and $\dot{B}_t = 0$. In the time interval $[t_1, t_2)$, we have $S_t^p > 0$ as well as $\dot{B}_t > 0$, but the budget deficit dominates and results in $CAD_t > 0$.

As before, suppose the government addresses the lack of fiscal sustainability by increasing taxation as of time t_2 so that the government budget is balanced for $t \geq t_2$. Then again $\dot{B}_t = 0$. Yet for a while $CAD_t > 0$ because now $S_t^p < 0$ as reflected in $\dot{A}_t < 0$, cf. Fig. 13.9. The deficit on the current account is, however, only temporary and not a signal of an impending default. It reflects that it takes time to complete the full downward adjustment of private consumption after the fiscal tightening.[21]

Let us consider a different scenario, namely one where the fiscal easing after time t_1 takes the form of a shift in government consumption to $\bar{G}' > \bar{G}$ without any change in taxation. Suppose the household sector expects that a fiscal tightening will not happen for a long time to come. Then, H_t and C_t are essentially unaffected, i.e., $C_t = C^*$ and $H_t = H^*$ as before t_1. So also A remains

[21] By construction of the model, households agents in the private sector are never insolvent.

at its steady-state value A^* from before t_1, given in (13.39). Owing to the absence of private saving, the government deficit must be fully financed by foreign borrowing. Indeed, by (13.51),

$$CAD_t = \dot{B}_t > 0$$

in this case. Here the two deficits exactly match each other. The situation is not sustainable, however. Government debt is mounting and if default is to be avoided, sooner or later fiscal policy must change.

It is the absence of Ricardian equivalence that suggests a positive relationship between budget and current account deficits. On the other hand, the course of events after t_2 in this example illustrates that a current account deficit *need not* coincide with a budget deficit. The empirical evidence on the relationship between budget and current account deficits is not entirely clear-cut. A cross-country regression analysis for 19 OECD countries with each country's data averaged over the 1981-86 period pointed to a positive relationship.[22] In fact, the attention to twin deficits derives from this period. Moreover, time series for the U.S. in the 1980s and first half of the 1990s also indicated a positive relationship. Nevertheless, other periods show no significant relationship. This mixed empirical evidence becomes more understandable when short-run mechanisms, with output determined from aggregate demand rather than supply, are taken into account.

The current account of a growing economy The above analysis ignored growth in GDP and therefore steady state required the current account to be balanced. It is different if we allow for economic growth. To see this, suppose there is Harrod-neutral technological progress at the constant rate g and that the labor force grows at the constant rate n. Then in steady state GDP grows at the rate $g+n$. From (13.48) follows, in analogy with the analysis of government debt in Chapter 6, that the law of movement of the foreign-debt/GDP ratio $d \equiv D/Y$ is

$$\dot{d} = (r - g - n)d - \frac{NX}{Y}. \qquad (13.52)$$

A necessary condition for the SOE to remain solvent is that circumstances are such that the foreign-debt/GDP ratio does not tend to explode. For brevity, assume NX/Y remains equal to a constant, $\bar{x}$. Then the linear differential equation (13.52) has the solution

$$d_t = (d_0 - d^*)e^{(r-g-n)t} + d^*,$$

where $d^* = \bar{x}/(r - g - n)$. If $r > g + n > 0$, the SOE will have an exploding foreign-debt/GDP ratio and become insolvent vis-a-vis the rest of the world unless

[22] See Obstfeld and Rogoff (1996, pp. 144-45).

$\bar{x} \geq (r - g - n)d_0$. The right-hand-side of this inequality is an increasing function of the initial foreign debt and the growth-corrected interest rate.

Suppose $d_0 > 0$ and $\bar{x} = (r - g - n)d_0$. Then d remains positive and constant. The SOE has a permanent current account deficit in that foreign debt, D, is permanently increasing. But net exports continue to match the growth-corrected interest payments on the debt, which then grows at the same constant rate as GDP. The conclusion is that, contrary to the presumption arising from the case with no GDP growth and prevalent in the media, a country *can* have a permanent current account deficit without this being a sign of economic disease and mounting solvency problems. In this example the permanent current account deficit merely reflects that the country for some historical reason has an initial foreign debt and at the same time a rate of time preference such that only part of the interest payment is financed by net exports, the remaining part being financed by allowing the foreign debt to grow at the same speed as production.

The required net exports-income ratio, $(r - g - n)d_0$, measures the burden that the foreign debt imposes on the country. And if the foreign debt directly or indirectly is *public* debt, the additional problem of levying sufficient taxation to service the debt arises. If we go a little outside the model and allow credit market imperfections, the higher the net exports-income ratio the greater the likelihood that the debtors will face financial troubles. As in Section 6.4.1, a vicious self-fulfilling expectations spiral may arise.

A worrying feature of the U.S. economy is that its foreign debt has been growing since the middle of the 1980s accompanied by a permanent *trade deficit*. The *triple deficits* characterizing the U.S. economy in the new millennium (government budget deficit, current account deficit, and trade deficit) looks like an unsustainable state of affairs.[23]

The debt crisis in Latin America in the 1980s From the mid-1970s there was an almost worldwide slowdown in economic growth. In the early 1980s, the real interest rate for Latin American countries rose sharply and net lending to corporations and governments in Latin America fell severely, as shown in Fig. 13.10. The solid line in the figure indicates the London Inter-Bank Offered Rate (LIBOR) deflated by the rate of change in export unit prices; the LIBOR is the short-term interest rate that the international banks charge each other for unsecured loans in the London wholesale money market. Interest rates charged on bank loans to Latin American countries were typically variable and based on

[23] How long time the role of the US dollar as the world's principal currency reserve can postpone a substantial depreciation of the dollar is an open question.

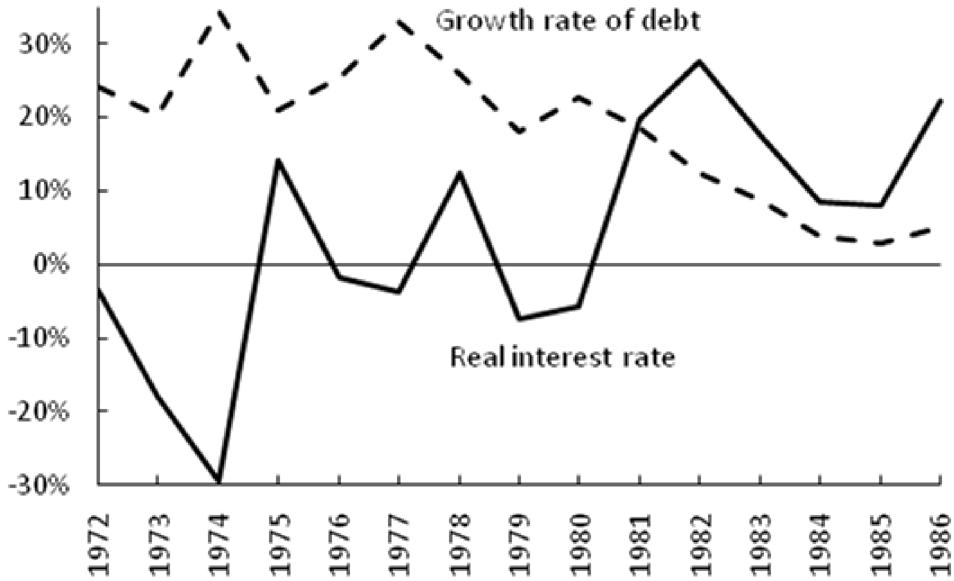

Figure 13.10

LIBOR.[24] A debt crisis ensued in the sense of mounting difficulties to refinance the debt. High interest rates and defaults resulted. Mexico suspended its payments in August 1982. By 1985, 15 countries were identified as requiring coordinated international assistance. The average debt-exports ratio (our d/x) peaked at 384 per cent in 1986 (Cline, 1995).

13.4 Government debt when taxes are distortionary*

So far we have, for simplicity, assumed that taxes are lump sum. Now we introduce a simple form of income taxation. We build on the same version of the Blanchard OLG model as was considered in Section 13.1. That is, the economy is closed, there is technological progress at the rate $g \geq 0$, and the population grows at the rate $n \geq 0$, whereas retirement is ignored (i.e., $\lambda = 0$). In addition to income taxation we bring in specific assumptions about government expenditure, namely that spending on goods and services as well as transfers grow at

[24]The correlation coefficient between the two variables in Fig. 13.10 is -0.615. The growth rate of total external debt is based on data for the following countries: Argentina, Bolivia, Brazil, Chile, Columbia, Costa Rica, Cuba, Dominican Republic, Ecuador, El Salvador, Guatemala, Haiti, Honduras, Mexico, Nicaragua, Panama, Paraguay, Peru, Uruguay, and Venezuela.

the rate $g+n$. The focus is on capital income taxation. Two main points of the analysis are that (a) capital income taxation results in lower capital intensity and consumption in the long run (if the economy is dynamically efficient); and (b) a higher level of government debt requires higher taxation and tends thereby to increase the excess burden of taxation.

Elements of the model

The household sector Assume there is a flat tax on the return on financial wealth at the rate τ_r. That is, an individual, born at time v and still alive at time $t \geq 0$, with financial wealth a_{vt} has to pay a tax equal to $\tau_r r_t a_{vt}$ per time unit, where τ_r is a given constant capital-income tax rate, $0 \leq \tau_r < 1$. The actuarial compensation is not taxed since it does not represent genuine income. There is symmetry in the sense that if $a_{vt} < 0$, then the tax acts as a subsidy (tax deductibility of interest payments). Labor income and transfers are taxed at a flat time-dependent rate, $\tau_{wt} < 1$. Only in steady state is the labor-income tax rate constant. Because labor supply is inelastic in the model, τ_{wt} acts like a lump-sum tax and is not of interest *per se.* Yet we include τ_{wt} in the analysis in order to have a simple tax instrument which can be adjusted to ensure a balanced budget when needed.

The dynamic accounting equation for the individual is

$$\dot{a}_{vt} = [(1-\tau_r)r_t + m]\, a_{vt} + (1-\tau_{wt})(w_t + x_t) - c_t, \qquad a_{v0} \text{ given},$$

where x_t is a lump-sum per-capita transfer. The No-Ponzi-Game condition, as seen from time $t_0 \geq v$, is

$$\lim_{t\to\infty} a_{vt} e^{-\int_{t_0}^{t}[(1-\tau_r)r_s+m]ds} \geq 0,$$

and the transversality condition requires that this holds with strict equality.

With logarithmic utility the Keynes-Ramsey rule takes the form

$$\frac{\dot{c}_{vt}}{c_{vt}} = (1-\tau_r)r_t + m - (\rho + m) = (1-\tau_r)r_t - \rho,$$

where $\rho \geq 0$ is the rate of time preference and $m > 0$ is the actuarial compensation, which equals the death rate. The consumption function is

$$c_{vt} = (\rho + m)(a_{vt} + h_t), \tag{13.53}$$

where

$$h_t = \int_t^{\infty} (1-\tau_{ws})(w_s + x_s) e^{-\int_t^s[(1-\tau_r)r_z+m]dz} ds. \tag{13.54}$$

At the aggregate level changes in financial wealth and consumption are:

$$\begin{aligned} \dot{A}_t &= (1-\tau_r)r_t A_t + (1-\tau_{wt})(w_t + x_t)N_t - C_t, \quad \text{and} \\ \dot{C}_t &= \left[(1-\tau_r)r_t - \rho + n\right] C_t - \beta(\rho + m)A_t, \end{aligned}$$

respectively, where β is the birth rate.

Production The description of production follows the standard one-sector neoclassical competitive setup. The representative firm has a neoclassical production function, $Y_t = F(K_t, \mathcal{T}_t L_t)$, with constant returns to scale, where $\mathcal{T}_t$ (to be distinguished from the tax revenue T) is the exogenous technology level, assumed to grow at the constant rate $g \geq 0$. In view of profit maximization under perfect competition we have

$$\frac{\partial Y_t}{\partial K_t} = f'(\tilde{k}_t) = r_t + \delta, \qquad \tilde{k}_t \equiv K_t/(\mathcal{T}_t L_t), \tag{13.55}$$

$$\frac{\partial Y_t}{\partial L_t} = \left[f(\tilde{k}_t) - \tilde{k}_t f'(\tilde{k}_t)\right] \mathcal{T}_t = w_t, \tag{13.56}$$

where $\delta > 0$ is the constant capital depreciation rate and f is the production function in intensive form, given by $\tilde{y} \equiv Y/(\mathcal{T}L) = F(\tilde{k}, 1) \equiv f(\tilde{k})$, $f' > 0, f'' < 0$. We assume f satisfies the Inada conditions. In equilibrium, $L_t = N_t$, so that $\tilde{k}_t = K_t/(\mathcal{T}_t N_t)$, a pre-determined variable.

The government sector Government spending on goods and services, G, and transfers, X, grow at the same rate as the work force measured in efficiency units. Thus,

$$G_t = \gamma \mathcal{T}_t N_t, \qquad X_t = \chi \mathcal{T}_t N_t, \qquad \gamma, \chi > 0. \tag{13.57}$$

Gross tax revenue, $\tilde{T}_t$, is given by

$$\tilde{T}_t = \tau_r r_t A_t + \tau_{wt}(w_t + x_t)N_t. \tag{13.58}$$

Budget deficits are financed by bond issue whereby

$$\begin{aligned} \dot{B}_t &= r_t B_t + G_t + X_t - \tilde{T}_t \qquad (13.59) \\ &= (1-\tau_r)r_t B_t + \gamma \mathcal{T}_t N_t + (1-\tau_{wt})\chi \mathcal{T}_t N_t - \tau_r r_t K_t - \tau_{wt} w_t N_t, \end{aligned}$$

where we have used (13.57) and the fact that in general equilibrium $A_t = K_t + B_t$. We assume parameters are such that in the long run the after-tax interest rate is higher than the output growth rate. Then government solvency requires the No-Ponzi-Game condition

$$\lim_{t\to\infty} B_t e^{-\int_0^t (1-\tau_r) r_s ds} \leq 0.$$

It is convenient to normalize the government debt by dividing with the effective labor force, $\mathcal{T}N$. Thus, we consider the ratio $\tilde{b}_t \equiv B_t/(\mathcal{T}_t N_t)$. By logarithmic differentiation w.r.t. t we find $\dot{\tilde{b}}_t/\tilde{b}_t = \dot{B}_t/B_t - (g+n)$, so that

$$\dot{\tilde{b}}_t = \frac{\dot{B}_t}{\mathcal{T}_t N_t} - (g+n)\tilde{b}_t = [(1-\tau_r)r_t - g - n]\,\tilde{b}_t + \gamma + (1-\tau_{wt})\chi - \tau_r r_t \tilde{k}_t - \tau_{wt}\tilde{w}_t,$$

where $\tilde{w}_t \equiv w_t/\mathcal{T}_t$. The tax τ_r redistributes income from the wealthy (here the old) to the poor (here the young), because the old have above-average financial wealth and the young have below-average wealth.

General equilibrium

Using that $n \equiv \beta - m$, we end up with three differential equations in $\tilde{k}$, $\tilde{c} \equiv C/(TN)$, and $\tilde{b}$:

$$\begin{aligned}
\dot{\tilde{k}}_t &= f(\tilde{k}_t) - \tilde{c}_t - \gamma - (\delta + g + \beta - m)\tilde{k}_t, && (13.60)\\
\dot{\tilde{c}}_t &= \Big[(1-\tau_r)(f'(\tilde{k}_t) - \delta) - \rho - g\Big]\,\tilde{c}_t - \beta(\rho+m)(\tilde{k}_t + \tilde{b}_t), && (13.61)\\
\dot{\tilde{b}}_t &= \Big[(1-\tau_r)(f'(\tilde{k}_t) - \delta) - g - (\beta - m)\Big]\,\tilde{b}_t + \gamma + (1-\tau_{wt})\chi \\
&\quad -\tau_r(f'(\tilde{k}_t) - \delta)\tilde{k}_t - \tau_{wt}\tilde{w}(\tilde{k}_t), && (13.62)
\end{aligned}$$

where $\tilde{w}(\tilde{k}_t) \equiv f(\tilde{k}_t) - \tilde{k}_t f'(\tilde{k}_t)$, cf. (13.56). Initial values of $\tilde{k}$ and $\tilde{b}$ are historically given and from the NPG condition of the government we get the terminal condition

$$\lim_{t\to\infty} \tilde{b}_t e^{-\int_0^t \left[(1-\tau_r)(f'(\tilde{k}_s)-\delta) - g - (\beta - m)\right] ds} = 0, \qquad (13.63)$$

assuming that the NPG condition is not "over-satisfied".

Suppose that for $t \geq 0$ the growth-corrected budget deficit is "structurally balanced" in the sense that the growth-corrected debt is constant. Thus, $\tilde{b}_t = \tilde{b}_0$ for all $t \geq 0$. This requires that the labor income tax τ_{wt} is continually adjusted so that, from (13.62),

$$\tau_{wt} = \frac{1}{\chi + \tilde{w}(\tilde{k}_t)} \left\{ \Big[(1-\tau_r)(f'(\tilde{k}_t) - \delta) - g - (\beta - m)\Big]\,\tilde{b}_0 + \gamma + \chi - \tau_r(f'(\tilde{k}_t) - \delta)\tilde{k}_t \right\}. \qquad (13.64)$$

Then (13.61) simplifies to

$$\dot{\tilde{c}}_t = \left[(1-\tau_r)(f'(\tilde{k}_t)-\delta)-\rho-g\right]\tilde{c}_t - \beta(\rho+m)(\tilde{k}_t+\tilde{b}_0),$$

which together with (13.60) constitutes an autonomous two-dimensional dynamic system. Only the capital income tax, τ_r, enters these dynamics. The labor income tax τ_{wt} does not. This is a trivial consequence of the model's simplifying assumption that labor supply is inelastic.

To construct the phase diagram for this system, note that

$$\dot{\tilde{k}} = 0 \text{ for } \tilde{c} = f(\tilde{k}) - \gamma - (\delta+g+\beta-m)\tilde{k}, \tag{13.65}$$

$$\dot{\tilde{c}} = 0 \text{ for } \tilde{c} = \frac{\beta(\rho+m)(\tilde{k}+\tilde{b}_0)}{(1-\tau_r)(f'(\tilde{k})-\delta)-\rho-g}. \tag{13.66}$$

There are two benchmark values of the effective capital-labor ratio, $\tilde{k}$. The first is the golden rule value, $\tilde{k}_{GR}$, given by $f'(\tilde{k}_{GR})-\delta = g+n$. The second is that value at which the denominator in (13.66) vanishes, that is, the value, $\bar{\tilde{k}}$, satisfying

$$(1-\tau_r)(f'(\bar{\tilde{k}})-\delta) = \rho+g.$$

The phase diagram is shown in Fig. 13.11. We assume $\tilde{b}_0 > 0$. But at the same time $\tilde{b}_0$ and γ are assumed to be "modest", given $\tilde{k}_0$, such that the economy initially is to the right of the totally unstable steady state close to the origin.

We impose the parameter restriction $\rho \geq n$, which implies $\bar{\tilde{k}} \leq k_{GR}$ for any $\tau_r \in [0,1)$, thus ensuring $\tilde{k}^* < k_{GR}$, in view of $\tilde{k}^* < \bar{\tilde{k}}$. That is,

$$f'(\tilde{k}^*)-\delta > f'(\bar{\tilde{k}})-\delta = \frac{\rho+g}{1-\tau_r} \geq \frac{g+n}{1-\tau_r} \geq g+n.$$

It follows that (13.63) holds at the steady state, E.[25] At time 0 the economy will be where the vertical line $\tilde{k} = \tilde{k}_0$ crosses the (stippled) saddle path. Over time the economy moves along this saddle path toward the steady state E with real interest rate equal to $r^* = f'(\tilde{k}^*)-\delta$. Further, in steady state the labor income tax rate is a constant,

$$\tau_w^* = \frac{\left[(1-\tau_r)(f'(\tilde{k}^*)-\delta)-g-n\right]\tilde{b}_0+\gamma+\chi-\tau_r(f'(\tilde{k}^*)-\delta)\tilde{k}^*}{\chi+\tilde{w}(\tilde{k}^*)}, \tag{13.67}$$

[25] And so do the transversality conditions of the households. The argument is the same as in Appendix D of Chapter 12.

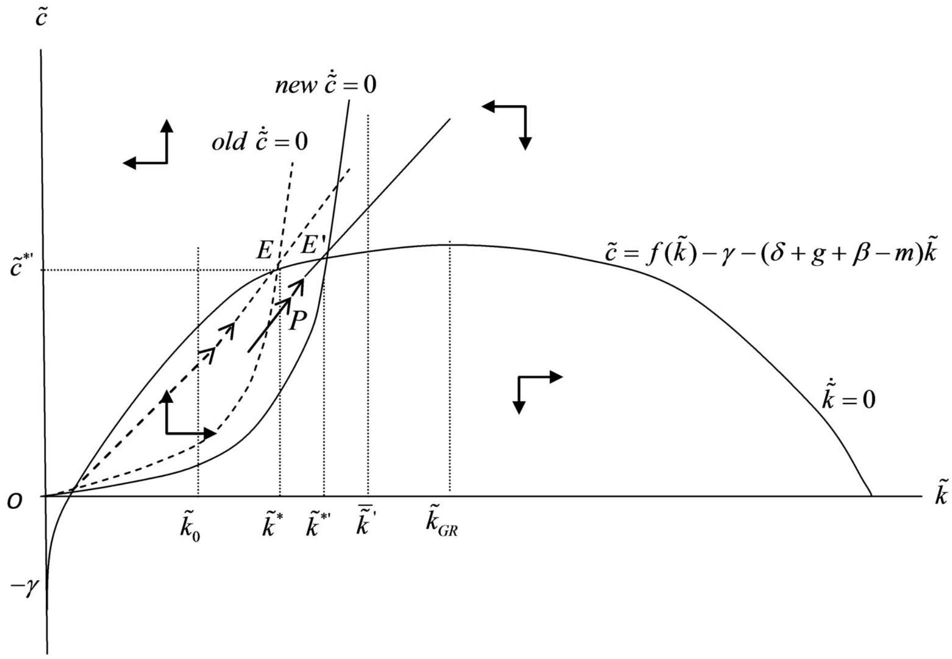

Figure 13.11: Phase diagram illustrating the effect of a fully financed reduction of capital income taxation.

from (13.64).

The capital income tax drives a wedge between the marginal transformation rate over time faced by the household, $(1-\tau_r)(f'(\tilde{k})-\delta)$, and that given by the production technology, $f'(\tilde{k})-\delta$. The implied efficiency loss is called the *excess burden* of the tax. A higher τ_r implies a greater wedge (higher excess burden) and for a given $\tilde{b}_0$, a lower $\tilde{k}^*$, cf. (13.66). Similarly, for a given τ_r, a higher level of debt, $\tilde{b}_0$, implies a lower $\tilde{k}^*$ and a higher r^* (and a corresponding adjustment of τ_w^*).[26] Finally, if for some reason (of a political nature, perhaps) τ_w^* is fixed, then a higher level of the debt may imply crowding out of $\tilde{k}^*$ for *two* reasons. First, there is the usual direct effect that higher debt decreases the scope for capital in households' portfolios. Second, there is the indirect effect, that higher debt may require a higher distortionary tax, τ_r, which further reduces capital accumulation and increases the excess burden.

We may reconsider the Ricardian equivalence issue from the perspective of both these effects. The Ricardian equivalence proposition says that when taxes are lump-sum, their timing does not affect aggregate consumption and saving. In the first section of this chapter we highlighted some of the reasons to doubt

[26] We can not say in what direction τ_w has to be adjusted. This is because it is theoretically ambiguous in what direction $(f'(\tilde{k}^*)-\delta)\tilde{k}^*$ moves when k^* goes down.

the validity of this proposition under "normal circumstances". Encompassing the fact that most taxes are *not* lump sum casts further doubt that debt neutrality should be a reliable guide for practical policy.

A fully financed reduction of capital income taxation

Now, suppose that until time t_1, the economy has been in its steady state E. Then, unexpectedly, the tax rate τ_r is *reduced* to a lower constant level, τ_r'. The tax rate is then expected by the public to remain at this lower level forever. The government budget remains "balanced" in the sense that taxation of labor income is immediately increased such that (13.64) holds for τ_r replaced by τ_r'. This shift in taxation policy does not affect the $\dot{\tilde{k}} = 0$ locus, but the $\dot{\tilde{c}} = 0$ locus is turned clockwise. At time t_1, when the shift in taxation policy occurs, the economy jumps to the point P and follows the new saddle path toward the new steady state with higher effective capital-labor ratio. (As noted at the end of the previous chapter, such adjustments may be quite slow.)

We see that the immediate effect on consumption is negative, whereas the long-run effect is positive (as long as everything takes place to the left of the golden rule capital intensity $\tilde{k}_{GR}$). The positive long-run effect on $\tilde{k}$ is due to the higher saving brought about by the initial fall in consumption. But what is the intuition behind this initial fall? Four effects are in play, a substitution effect, a pure income effect, a wealth effect, and a government budget effect. To understand these effects from a micro perspective, the intertemporal budget constraint as seen from time t_1 of an individual born at time $v \leq t_t$ is helpful:

$$\int_{t_1}^{\infty} c_{vt} e^{-\int_{t_1}^{t}[(1-\tau_r')r_s+m]ds} dt = a_{vt_1} + h_{t_1}. \tag{IBC}$$

The point of departure is that the *after-tax interest rate* immediately rises. As a result:

1) Future consumption becomes relatively cheaper as seen from time t_1. Hence there is a *negative substitution effect* on current consumption c_{vt_1}.

2) For given total wealth $a_{vt_1} + h_{t_1}$, it becomes possible to consume more at *any* time in the future (because the present discounted value of a given consumption plan has become smaller, see the left-hand side of (IBC)). This amounts to a *positive pure income effect* on current consumption.

3) At least for a while the after-tax interest rate, $(1 - \tau_r')r + m$, is higher than without the tax decrease. Everything else equal, this affects h_{t_1} negatively, which amounts to a *negative wealth effect.*

On top of these three "standard" effects comes the fact that:

4) At least initially, a rise in τ_w is necessitated by the lower capital income taxation if an unchanged $\tilde{b}$ is to be maintained, cf. (13.64). Everything else equal, this also affects h_{t_1} negatively and gives rise to a further *negative* effect on current consumption through what we may call the *government budget effect*.[27]

To sum up, the total effect on current individual consumption of a permanent decrease in the capital income tax rate and a concomitant rise in the tax on labor income and transfers consists of the following components:

$$\begin{aligned}&\text{substitution effect} + \text{pure income effect} + \text{wealth effect}\\&+ \text{effect through the change in the government budget} = \text{total effect.}\end{aligned}$$

From the consumption function $c_{vt} = (\rho + m)(a_{vt} + h_t)$, cf. (13.53), we see that the substitution and income effects exactly cancel each other out (due to the logarithmic specification of the utility function). This implies that the negative general equilibrium effect on current consumption, visible in the phase diagram, reflects the influence of the two remaining effects.

The conclusion is that whereas a tax on an inelastic factor (in this model labor) obviously does not affect its supply, a tax on capital or on capital income affects saving and thereby capital in the future. Yet such a tax may have intended effects on income *distribution*. The public finance literature studies, among other things, under what conditions such effects could be obtained by other means (see, e.g., Myles 1995).

13.5 Public debt policy

Main text for this section not yet available. See instead Elmendorf and Mankiw, Section 5 (Course Material).

13.6 Credibility problems due to time inconsistency

(incomplete)

When outcomes depend on expectations in the private sector, government policy may face a time-inconsistency problem.

[27] The proviso "everything else equal" both here and under 3) is due to the fact that at the aggregate level counteracting feedbacks in the form of higher future real wages and lower interest rates arise during the general equilibrium adjustment.

As an example consider the question: What is the stance taken by a government on negotiating with terrorists over the release of hostages? The official line, of course, is that the government will never negotiate. But ....

13.7 Literature notes

(incomplete)

Section 13.2 essentially builds on Blanchard and Fischer (1989).

For very readable surveys about how important – empirically – the departures from Ricardian equivalence are, see for example "Symposium on the Budget Deficit" in *Journal of Economic Perspectives*, vol. 3, 1989, Himarios (1995), and Elmendorf and Mankiw (1999).

In their analysis of 26 high public debt episodes in advanced economies 1800-2011 Reinhardt et al. (2012) find higher interest rate for 15 of the episodes. They find low economic growth in 23 of the episodes.

13.8 Appendix

A. A growth formula useful for debt arithmetic

Not yet available.

B. Long-run multipliers

We show here in detail how to calculate the long-run "crowding-out" effects of increases in government consumption and debt in the closed economy model of Section 13.2. In steady state we have $\dot{K}_t = \dot{C}_t = \dot{B}_t = \dot{T}_t = 0$, hence

$$F(K^*, N) - \delta K^* = C^* + \bar{G}, \tag{13.68}$$
$$(F_K(K^*, N) - \delta - \rho)C^* = m(\rho + m)(K^* + \bar{B}), \tag{13.69}$$
$$T^* = (F_K(K^*, N) - \delta)\bar{B} + \bar{G}. \tag{13.70}$$

We consider the level $\bar{B}$ of public debt as exogenous along with public consumption $\bar{G}$ and the labor force N. The tax revenue T^* in steady state is endogenous.

Assume (realistically) that $K^* + \bar{B} > 0$. Now, at zero order in the causal structure, (13.68) and (13.69) simultaneously determine K^* and C^* as implicit functions of $\bar{G}$ and $\bar{B}$, i.e., $K^* = K(\bar{G}, \bar{B})$ and $C^* = C(\bar{G}, \bar{B})$. Hereafter, (13.70) determines the required tax revenue T^* at first order as an implicit function of $\bar{G}$ and $\bar{B}$, i.e., $T^* = T(\bar{G}, \bar{B})$.

To calculate the partial derivatives of these implicit functions, insert $C^* = F(K^*, N) - \delta K^* - \bar{G}$ from (13.68) into (13.69) to get

$$(F_K^* - \delta - \rho)(F^* - \delta K^* - \bar{G}) = m(\rho + m)(K^* + B_0).$$

Next take the total differential on both sides:

$$(F_K^* - \delta - \rho)\left[(F_K^* - \delta)dK^* - d\bar{G}\right] + C^* F_{KK}^* dK^* = m(\rho + m)(dK^* + d\bar{B}), \quad \text{i.e.,}$$

$$\mathcal{D} \cdot dK^* = (F_K^* - \delta - \rho)d\bar{G} + m(\rho + m)d\bar{B}, \tag{13.71}$$

where

$$\mathcal{D} \equiv C^* F_{KK}^* + (F_K^* - \delta - \rho)(F_K^* - \delta) - m(\rho + m), \tag{13.72}$$

and the partial derivatives are evaluated in steady state.

We now show that in the interesting steady state we have $\mathcal{D} < 0$. As demonstrated in Section 13.2, normally there are *two* steady-state points in the (K, C) plane.[28] The lower steady-state point, that with $K = \tilde{K}^*$ in Fig. 13.2, is a "source" in the sense that all trajectories in its neighborhood points away from it. So the lower steady-state point is completely unstable. The upper steady-state point, that with $K = K^*$, is saddle-point stable. This is the interesting steady state (when $\bar{G}$ and $\bar{B}$ are of moderate size). In that state the $\dot{C} = 0$ locus crosses the $\dot{K} = 0$ locus from below. Hence

$$\begin{aligned}
\frac{\partial C}{\partial K}\Big|_{\dot{C}=0} &> F_K^* - \delta, \quad \text{i.e.,} \\
m(\rho + m)\frac{F_K^* - \delta - \rho - (K^* + \bar{B})F_{KK}^*}{(F_K^* - \delta - \rho)^2} &> F_K^* - \delta \Rightarrow \\
m(\rho + m) - m(\rho + m)\frac{(K^* + \bar{B})}{r^* - \rho}F_{KK}^* &> (r^* - \rho)r^* \Rightarrow \\
m(\rho + m) - C^* F_{KK}^* &> (r^* - \rho)r^* \Rightarrow \\
0 &> C^* F_{KK}^* + (r^* - \rho)r^* - m(\rho + m) = \mathcal{D},
\end{aligned} \tag{13.73}$$

where the first implication arrow follows from $F_K^* = F_K(K^*, N) - \delta = r^*$, the second from (13.69), and the third by rearranging. A perhaps more useful formula[29] for $\mathcal{D}$ is obtained by noting that

$$(r^* - \rho)r^* - m(\rho + m) = r^{*2} + mr^* - mr^* - \rho r^* - m(\rho + m) = (r^* + m)(r^* - (\rho + m)).$$

[28] This is so, unless $\bar{G}$ and $\bar{B}$ are so large that there is only one (a knife-edge case) or no steady state with $K > 0$.

[29] More useful in the sense of being more in line with analogue formulas for a small open economy, cf. Section 13.3.

Hence, by (13.73),

$$\mathcal{D} = C^* F^*_{KK} - (r^* + m)(\rho + m - r^*) < 0.$$

So the implicit function $K^* = K(\bar{G}, \bar{B})$ has the partial derivatives, also called the long-run or steady-state multipliers,

$$K_{\bar{G}} = \frac{\partial K^*}{\partial \bar{G}} = \frac{r^* - \rho}{\mathcal{D}} < 0, \tag{13.74}$$

$$K_{\bar{B}} = \frac{\partial K^*}{\partial \bar{B}} = \frac{m\,(\rho + m)}{\mathcal{D}} < 0, \tag{13.75}$$

using (13.71) and $r^* = F_K(K^*, N) - \delta > \rho$. As to the effect on K^* of balanced changes in $\bar{G}$, it follows that $\Delta K^* \approx dK^* = (\partial K^*/\partial \bar{G})d\bar{G} = (r^* - \rho)d\bar{G}/\mathcal{D} < 0$ for $d\bar{G} > 0$. This gives the size of the long-run effect on the capital stock, when public consumption is increased by $d\bar{G}$ ($d\bar{G}$ "small"), and at the same time taxation is increased so as to balance the budget and leave public debt unchanged in the indefinite future.

As to the effect on K^* of higher public debt, it follows that $\Delta K^* \approx dK^* = (\partial K^*/\partial \bar{B})d\bar{B} = m\,(\rho + m)\,d\bar{B}/\mathcal{D} < 0$ for $d\bar{B} > 0$. This formula tells us the size of the long-run effect on the capital stock, when a tax cut implies, for some time, a budget deficit and thereby a cumulative increase, $d\bar{B}$, in public debt; afterwards the government increases taxation to balance the budget forever.[30] Similarly, $\Delta r^* \approx dr^* = F_{KK}(K^*, N)dK^* \approx F_{KK}(K^*, N) \cdot (\partial K^*/\partial \bar{B})d\bar{B} > 0$, for $d\bar{B} > 0$.

The long-run or steady-state multipliers associated with the implicit function $C^* = C(\bar{G}, \bar{B})$ are now found by implicit differentiation in (13.68) w.r.t. $\bar{G}$ and $\bar{B}$, respectively. We get $\partial C^*/\partial \bar{G} = (F_K(K^*, N) - \delta)\partial K^*/\partial \bar{G} - 1 < -1$ and $\partial C^*/\partial \bar{B} = (F_K(K^*, N) - \delta)\partial K^*/\partial \bar{B} < 0$.

Similarly, from (13.70) we get $\partial T^*/\partial \bar{G} = F_{KK}(K^*, N)(\partial K^*/\partial \bar{G}) \cdot \bar{B} + 1 > 1$ and $\partial T^*/\partial \bar{B} = F_{KK}(K^*, N) \cdot (\partial K^*/\partial \bar{B})\bar{B} + F_K(K^*, N) - \delta > 0$ (since $F_{KK} < 0$).

13.9 Exercises

13.1

[30]We assume that $t_2 - t_1$, hence $d\bar{B}$, is not so large as to not allow existence of a saddle-point stable steady state with $K > 0$ after t_2.