Makroøkonomi 2 Note 1

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A glimpse of theory of the "level of interest rates"

This short note provides a brief sketch of what macroeconomics says about the general level around which rates of return fluctuate. We also give a "broad" summary of different circumstances that give rise to differences in rates of return on different assets.

In non-monetary models without uncertainty there is in equilibrium only one rate of return, r. If in addition there is a) perfect competition in all markets, b) the consumption good is physically indistinguishable from the capital good, and c) there are no capital adjustment costs, as in simple neoclassical models (like the Diamond OLG model and the Ramsey model), then the equilibrium real interest rate is at any time equal to the current net marginal productivity of capital evaluated at full employment ($r = \partial Y/\partial K - \delta$ in standard notation). Moreover, under conditions ensuring "well-behavedness" of these models, they predict that in the absence of disturbances, the technology-corrected capital-labor ratio, and thereby the marginal productivity of capital, adjusts over time to some long-run level (on which more below).

Different rates of return In simple neoclassical models with perfect competition and no uncertainty, the equilibrium short-term real interest rate is at any time equal to the net marginal productivity of capital ($r = \partial Y/\partial K - \delta$). In turn the marginal productivity of capital adjusts over time, via changes in the capital intensity, to some long-run level (on this more below). As we saw in Chapter 14, existence of convex *capital installation costs* loosens the link between r and $\partial Y/\partial K$. The convex adjustment costs create a wedge between the price of investment goods and the market value of the marginal unit of installed capital. Besides the marginal productivity of capital, the possible capital gain in the market value of installed capital as well as the effect of the marginal unit of installed capital on future installation costs enter as co-determinants of the current rate of return on capital.

	Arithmetic average	Standard deviation	Geometric average
	---------------- Percent	----------------	
Nominal values			
Small Company Stocks	17,3	33,2	12,5
Large Company Stocks	12,7	20,2	10,7
Long-Term Corporate Bonds	6,1	8,6	5,8
Long-Term Government Bonds	5,7	9,4	5,3
Intermediate-Term Government Bonds	5,5	5,7	5,3
U.S. Treasury Bills	3,9	3,2	3,8
Cash	0,0	0,0	0,0
Inflation rate	3,1	4,4	3,1
Real values			
Small Company Stocks	13,8	32,6	9,2
Large Company Stocks	9,4	20,4	7,4
Long-Term Corporate Bonds	3,1	9,9	2,6
Long-Term Government Bonds	2,7	10,6	2,2
Intermediate-Term Government Bonds	2,5	7,0	2,2
U.S. Treasury Bills	0,8	4,1	0,7
Cash	-2,9	4,2	-3,0

Table 1: Average annual rates of return on a range of U.S. asset portfolios, 1926-2001. Source: Stocks, Bonds, Bills, and Inflation: Yearbook 2002, Valuation Edition. Ibbotson Associates, Inc.

When *imperfect competition* in the output markets rules, prices are typically set as a mark-up on marginal cost. This implies a wedge between the net marginal productivity of capital and capital costs. And when *uncertainty* and limited opportunities for risk diversification are added to the model, a wide spectrum of expected rates of return on different financial assets and expected marginal productivities of capital in different production sectors arise, depending on the risk profiles of the different assets and production sectors. On top of this comes the presence of taxation which may complicate the picture because of different tax rates on different asset returns.

Nominal and real average annual rates of return on a range of U.S. asset portfolios for the period 1926–2001 are reported in Table 1. By a *portfolio* of n assets, $i = 1, 2, \ldots, n$ is meant a "basket", $(v_1, v_2, \ldots, v_n)$, of the n assets in value terms, that is, $v_i = p_i x_i$ is the value of the investment in asset i, the price of which is denoted p_i and the quantity of which is denoted x_i. The total investment in the basket is $V = \sum_{i=1}^{n} v_i$. If R_i denotes the gross rate of return on asset i, the overall gross rate of return on the portfolio is

$$R = \frac{\sum_i^n v_i R_i}{V} = \sum_{i=1}^{n} w_i R_i,$$

where $w_i \equiv v_i/V$ is the *weight* or *fraction* of asset i in the portfolio. Defining $R_i \equiv 1 + r_i$, where r_i is the net rate of return on asset i, the net rate of return on the portfolio can be written

$$r = R - 1 = \sum_{i=1}^{n} w_i(1 + r_i) - 1 = \sum_{i=1}^{n} w_i + \sum_{i=1}^{n} w_i r_i - 1 = \sum_{i=1}^{n} w_i r_i.$$

The net rate of return is often just called "the rate of return".

In Table 1 we see that the portfolio consisting of small company stocks throughout the period 1926-2001 had an average annual real rate of return of 13.8 per cent (the arithmetic average) or 9.2 per cent (the geometric average). This is more than the annual rate of return of any of the other considered portfolios. Small company stocks are also seen to be the most volatile. The standard deviation of the annual real rate of return of the portfolio of small company stocks is almost eight times higher than that of the portfolio of U.S. Treasury bills (government zero coupon bonds with 30 days to maturity), with an average annual real return of only 0.8 per cent (arithmetic average) or 0.7 per cent (geometric average) throughout the period. The displayed positive relation between high returns and high volatility is not without exceptions, however. The portfolio of long-term corporate bonds has performed better than the portfolio of long-term government bonds, although they have been slightly less volatile as here measured. The data is historical and expectations are not always met. Moreover, risk depends significantly on the *covariance* of asset returns within the total set of assets and specifically on the correlation of asset returns with the business cycle, a feature that can not be read off from Table 1. Share prices, for instance, are very sensitive to business cycle fluctuations.

The need for means of payment − money − is a further complicating factor. That is, besides dissimilarities in risk and expected return across different assets, also dissimilarities in their degree of liquidity are important, not least in times of financial crisis. The expected real rate of return on cash holding is minus the expected rate of inflation and is therefore negative in an economy with inflation, cf. the last row in Table 1. When agents nevertheless hold cash in their portfolios, it is because the low rate of return is compensated by the *liquidity* services of money. In the Sidrauski model of Chapter 17 this is modeled in a simple way, albeit ad hoc, by including real money holdings directly as an argument in the utility function. Another dimension along which the presence of money interferes with returns is through inflation. Real assets, like physical capital, land, houses, etc. are better protected against fluctuating inflation than are nominally denominated bonds (and money of course).

Without claiming too much we can say that investors facing such a spectrum of rates of return choose a composition of assets so as to balance the need for liquidity, the wish for a high expected return, and the wish for low risk. Finance theory teaches us that adjusted for differences in risk and liquidity, asset returns tend to be the same. This raises the question: at what level? This is where macroeconomics – as an empirically oriented theory about the economy as a whole – comes in.

Macroeconomic theory of the "average rate of return" The point of departure is that market forces by and large may be thought of as anchoring the rate of return of an average portfolio of interest-bearing assets to the net marginal productivity of capital in an aggregate production function, assuming a closed economy. Some popular phrases are:

- the net marginal productivity of capital acts as a centre of gravitation for the spectrum of asset returns; and
- movements of the rates of return are in the long run held in check by the net marginal productivity of capital.

Though such phrases seem to convey the right flavour, in themselves they are not very informative. The net marginal productivity of capital is not a given, but an endogenous variable which, via changes in the capital intensity, adjusts through time to more fundamental factors in the economy.

The different macroeconomic models we have encountered in previous chapters bring to mind different presumptions about what these fundamental factors are.

1. Solow's growth model The Solow growth model leads to the fundamental differential equation (standard notation)

$$\dot{\tilde{k}}_t = sf(\tilde{k}_t) - (\delta + g + n)\tilde{k}_t,$$

where s is an exogenous and constant aggregate saving-income ratio, $0 < s < 1$. In steady state

$$r^* = f'(\tilde{k}^*) - \delta, \tag{1}$$

where $\tilde{k}^*$ is the unique steady state value of the (effective) capital intensity, $\tilde{k}$, satisfying

$$sf(\tilde{k}^*) = (\delta + g + n)\tilde{k}^*. \tag{2}$$

In society there is a debate and a concern that changed demography and less growth in the source of new technical ideas, i.e., the stock of educated human beings, will in the future result in lower n and lower g, respectively, making financing social security more difficult. On the basis of the Solow model we find by implicit differentiation in (2) $\partial\tilde{k}^*/\partial n = \partial\tilde{k}^*/\partial g = -\tilde{k}^*\left[\delta + g + n - sf'(\tilde{k}^*)\right]^{-1}$, which is negative since $sf'(\tilde{k}^*) < sf(\tilde{k}^*)/\tilde{k}^* = \delta + g + n$. Hence, by (1),

$$\frac{\partial r^*}{\partial n} = \frac{\partial r^*}{\partial g} = \frac{\partial r^*}{\partial \tilde{k}^*}\frac{\partial \tilde{k}^*}{\partial n} = f''(\tilde{k}^*)\frac{-\tilde{k}^*}{\delta + g + n - sf'(\tilde{k}^*)} > 0,$$

since $f''(\tilde{k}^*) < 0$. It follows that

$$n\downarrow \text{ or } g\downarrow \Rightarrow r^*\downarrow. \tag{3}$$

A limitation of this theory is of course the exogeneity of the saving-income ratio, which is a key co-determinant of $\tilde{k}^*$, hence of r^*. The next models are examples of different ways of integrating a theory of saving into the story about the long-run rate of return.

2. The Diamond OLG model In the Diamond OLG model, based on a life-cycle theory of saving, we again arrive at the formula $r^* = f'(\tilde{k}^*) - \delta$. Like in the Solow model, the long-run rate of return thus depends on the aggregate production function and on $\tilde{k}^*$. But now there is a logically complete theory about how $\tilde{k}^*$ is determined. In the Diamond model $\tilde{k}^*$ depends in a complicated way on the lifetime utility function and the aggregate production function. The steady state of a well-behaved Diamond model will nevertheless have the same qualitative property as indicated in (3).

3. The Ramsey model Like the Solow and Diamond models, the Ramsey model implies that $r_t = f'(\tilde{k}_t) - \delta$ for all t. But unlike in the Solow and Diamond models, the net marginal productivity of capital now converges in the long run to a specific value given by the *modified golden rule* formula. In a continuous time framework this formula says:

$$r^* = \rho + \theta g, \tag{4}$$

where the new parameter, θ, is the (absolute) elasticity of marginal utility of consumption. Because the Ramsey model is a representative agent model, the Keynes-Ramsey rule holds not only at the individual level, but also at the aggregate level. This is what gives rise to this simple formula for r^*.

Here there is no role for n, only for g. On the other hand, there is an alternative specification of the Ramsey model, namely the "average utilitarianism" specification. In this version of the model, we get $r^* = f'(\tilde{k}^*) - \delta = \rho + n + \theta g$, so that not only a lower g, but also a lower n implies lower r^*.

Also the Sidrauski model, i.e., the monetary Ramsey model of Chapter 17, results in the *modified golden rule* formula.[1]

4. Blanchard's OLG model A continuous time OLG model with emphasis on life-cycle aspects is Blanchard's model, Blanchard (1985). In that model the net marginal productivity of capital adjusts to a value where, in addition to the production function, technology growth, and preference parameters, also demographic parameters, like birth rate, death rate, and retirement rate, play a role. One of the results is that when $\theta = 1$,

$$\rho + g - \lambda < r^* < \rho + g + b,$$

where λ is the retirement rate (reflecting how early in life the "average" person retire from the labor market) and b is the (crude) birth rate. The population growth rate is the difference between the birth rate, b, and the (crude) mortality rate, m, so that $n = b - m$. The qualitative property indicated in (3) becomes conditional. It still holds if the fall in n reflects a lower b, but not necessarily if it reflects a higher m.

5. What if technological change is embodied? The models in the list above assume a neoclassical aggregate production function with CRS and *disembodied* Harrod-neutral technological progress, that is,

$$Y_t = F(K_t, T_t L_t) \equiv T_t L_t f(\tilde{k}_t), \qquad f' > 0, f'' < 0. \tag{5}$$

This amounts to assuming that new technical knowledge advances the combined productivity of capital and labor *independently* of whether the workers operate old or new machines.

In contrast, we say that technological change is *embodied* if taking advantage of new technical knowledge requires construction of new investment goods. The newest technology is incorporated in the design of newly produced equipment; and this equipment will

[1] See Chapter 10, Section 10.5.

not participate in subsequent technological progress. Both intuition and empirics suggest that most technological progress is of this form. Indeed, Greenwood et al. (1997) estimate for the U.S. 1950-1990 that embodied technological change explains 60% of the growth in output per man hour.

So a theory of the rate of return should take this into account. Fortunately, this can be done with only minor modifications. We assume that the link between investment and capital accumulation takes the form

$$\dot{K}_t = Q_t I_t - \delta K_t, \tag{6}$$

where I_t is gross investment ($I = Y - C$) and Q_t measures the "quality" (efficiency) of newly produced investment goods. Suppose for instance that

$$Q_t = Q_0 e^{\gamma t}, \qquad \gamma > 0.$$

Then, even if no technological change directly appears in the production function, that is, even if (5) is replaced by

$$Y_t = F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha}, \qquad 0 < \alpha < 1,$$

the economy will still experience a rising standard of living.[2] A given level of gross investment will give rise to greater and greater additions to the capital stock K, measured in efficiency units. Since at time t, Q_t capital goods can be produced at the same cost as one consumption good, the price, p_t, of capital goods in terms of the consumption good must in competitive equilibrium equal the inverse of Q_t, that is, $p_t = 1/Q_t$. In this way embodied technological progress results in a steady decline in the relative price of capital equipment.

This prediction is confirmed by the data. Greenwood et al. (1997) find for the U.S. that the relative price of capital equipment has been declining at an average rate of 0.03 per year in the period 1950-1990, a trend that has seemingly been fortified in the wake of the computer revolution.

Along a balanced growth path the constant growth rate of K will now exceed that of Y, and Y/K thus be falling. The output-capital ratio in value terms, $Y/(pK)$, will be constant, however. Embedding these features in a Ramsey-style framework, we find the

[2] We specify F to be Cobb-Douglas, because otherwise a model with embodied technical progress in the form (6) will not be able to generate balanced growth and comply with Kaldor's stylized facts.

long-run rate of return to be[3]

$$r^* = \rho + \theta \frac{\alpha\gamma}{1-\alpha}.$$

This is of the same form as (4) since growth in output per unit of labor in steady state is exactly $g = \alpha\gamma/(1-\alpha)$.

Adding uncertainty and risk of bankruptcy Although absent from many simple macroeconomic models, uncertainty and risk of bankruptcy are significant features of reality. Bankruptcy risk may lead to a conflict of interest between share owners and managers. Managers may want less debt and more equity than the share owners because bankruptcy can be very costly to managers who loose a well-paid job and a promising carrier. So managers are unwilling to finance all new capital investment by new debt in spite of the associated lower capital cost (there is generally a lower rate of return on debt than on equity). In this way the excess of the rate of return on equity over that on debt, the equity premium, is sustained.

A rough behavioral theory of the equity premium goes as follows.[4] Firm managers prefer a payout structure with a fraction, s_f, going to equity and the remaining fraction, $1 - s_f$, to debt (corporate bonds). That is, out of each unit of expected operating profit, managers are unwilling to commit more than $1 - s_f$ to bond owners. This is to reduce the risk of a failing payment ability in case of a bad market outcome. And those who finance firms by loans definitely also want debtor firms to have some equity at stake.

We let households' preferred portfolio consist of a fraction s_h in equities and the remainder, $1 - s_h$, in bonds. In view of households' risk aversion and memory of historical stock market crashes, it is plausible to assume that $s_h < s_f$.

As a crude adaptation of for instance the Blanchard OLG model to these features, we interpret the model's r^* as an average rate of return across firms. Let time be discrete and let aggregate financial wealth be $A = pK$, where p is the price of capital equipment in terms of consumption goods. In the frameworks 1 to 4 above we have $p \equiv 1$, but in framework 5 the relative price p equals $1/Q$ and is falling over time. Anyway, given A at time t, the aggregate gross return or payout is $(1 + r^*)A$. Out of this, $(1 + r^*)As_f$ constitutes the gross return to the equity owners and $(1 + r^*)A(1 - s_f)$ the gross return

[3] See Exercise 18.??

[4] The following is inspired by Baker, DeLong, and Krugman (2005). These authors discuss the implied predictions for U.S. rates of return in the future and draw implications of relevance for the debate on social security reform.

to the bond owners. Let r_e denote the rate of return on equity and r_b the rate of return on bonds.

To find r_e and r_b we have

$$\begin{aligned}(1+r_e)As_h &= (1+r^*)As_f,\\ (1+r_b)A(1-s_h) &= (1+r^*)A(1-s_f).\end{aligned}$$

Thus,

$$\begin{aligned}1+r_e &= (1+r^*)\frac{s_f}{s_h} > 1+r^*,\\ 1+r_b &= (1+r^*)\frac{1-s_f}{1-s_h} < 1+r^*.\end{aligned}$$

We may define the *equity premium,* π, by $1+\pi \equiv (1+r_e)/(1+r_b)$. Then

$$\pi = \frac{s_f(1-s_h)}{s_h(1-s_f)} - 1 > 0.$$

Of course these formulas have their limitations. The key variables s_f and s_h will depend on a lot of economic circumstances and should be endogenous in an elaborate model. Yet, the formulas may be helpful as a way of organizing one's thoughts about rates of return in a world with asymmetric information and risk of bankruptcy.

There is evidence that in the last decades of the twentieth century the equity premium had become lower than in the long aftermath of the Great Depression in the 1930s.[5] A likely explanation is that s_h had gone up, along with rising confidence. The computer and the World Wide Web have made it much easier for individuals to invests in stocks of shares. On the other hand, the recent financial and economic crisis, known as the Great Recession 2007- , and the associated rise in mistrust seems to have halted and possibly reversed this tendency for some time (source ??).

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[5] Blanchard (2003, p. 333).