CONSUMPTION

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INTRODUCTION

So far: The average propensity to save has been exogenous

Albeit our analysis showed that difference in "s" matters to long-run prosperity, we so far as avoided asking *why* s might differ

So far: Only changes in current income matters to total savings.

This chapter: *Optimal* savings

–> Which factors determine savings if consumers are forward looking?

OVERVIEW OF THE CHAPTER

A. The intertemporal optimization problem of the consumer: **Private consumption**.

Business cycle fact: Consumption is less volatile than GDP. Why?

B. **Government consumption**

What is the impact of taxation on consumption/savings? Does it matter whether the government finances its spending via bonds or taxes?

A) PRIVATE CONSUMPTION

Assume households are equipped with *preferences* over consumption today (c_1), and tomorrow (c_2). Specifically

$$U(c_1, c_2) = u(c_1) + \frac{1}{1+\phi} u(c_2) = \begin{cases} \frac{\sigma}{\sigma-1} c_1^{\frac{\sigma-1}{\sigma}} + \frac{1}{1+\phi} \frac{\sigma}{\sigma-1} c_2^{\frac{\sigma-1}{\sigma}}, & \text{if } \sigma > 0, \neq 1 \\ \log(c_1) + \frac{1}{1+\phi} \log(c_2), & \text{if } \sigma = 1. \end{cases}$$

For per period felicity we assume $u'(c_i) > 0$, $u''(c_i) < 0$ for $i = 1, 2$. ϕ is the *rate of time preference*

So: (i) positive marginal utility from consumption today or tomorrow, (ii) diminishing marginal utility. Note: c_1 and c_2 are *normal* goods.

A) PRIVATE CONSUMPTION

Intertemporal utility function: $U(c_1, c_2) = u(c_1) + u(c_2)\frac{1}{1+\phi}$

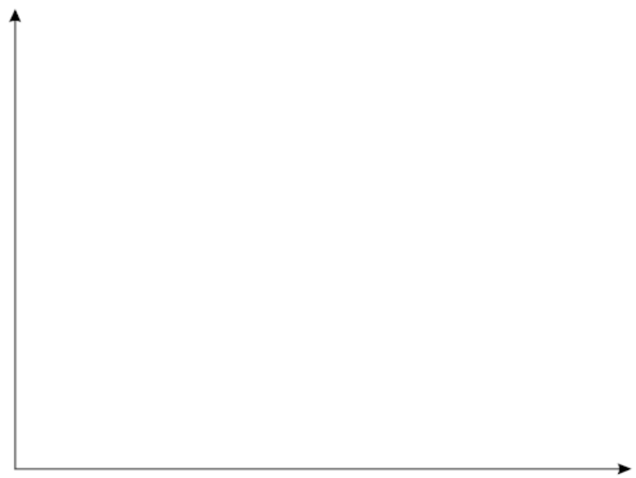


Figure 1:

Slope? MRS: Diff the utility function

$$u'(c_1)\,dc_1 + \frac{1}{1+\phi}u'(c_2)\,dc_2 = 0 \Leftrightarrow \frac{\partial c_2}{\partial c_1} = -(1+\phi)\frac{u'(c_1)}{u'(c_2)}$$

A) PRIVATE CONSUMPTION

Period 1 budget constraint. Income: Born with wealth V_1, work, Y_1^L, pay taxes: T_1. Expenditure: consumption, c_1, and savings, s. In sum

$$c_1 + s = V_1 + Y_1^L - T_1$$

In period 2 the consumer also works, Y_2^L, and pay taxes T_2. In addition, there might be income from savings

$$\underbrace{s}_{\text{Amount saved}} + \underbrace{rs}_{\text{interest earnings}} = (1+r)\, s \equiv \underset{\text{period 2 wealth}}{V_2}$$

Savings tranf c_1 into c_2; $(1+r)$ is therefore the marginal rate of transformation (MRT)

P**eriod 2** constraint is

$$c_2 = (1+r)\, s + Y_2^L - T_2$$

A) PRIVATE CONSUMPTION

Substitute for s, and we can consolidate the two constraints: **The intertemporal budget constraint**.

$$\underbrace{c_1 + \frac{c_2}{1+r}}_{\text{Lifetime consumption}} = \underbrace{V_1 + Y_1^L - T_1 + \frac{Y_2^L - T_2}{1+r}}_{\text{Life time income}} \equiv V_1 + \underbrace{H_1}_{\text{"human wealth"}}$$

Slope (MRT)? **E** is the endowment point. Lender vs. Borrower.

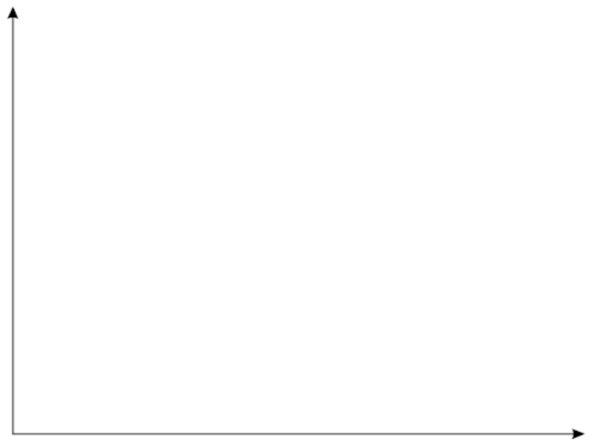


Figure 2:

A) PRIVATE CONSUMPTION

Optimal consumption. Choose highest attainable utility, given the intertemporal budget constant.

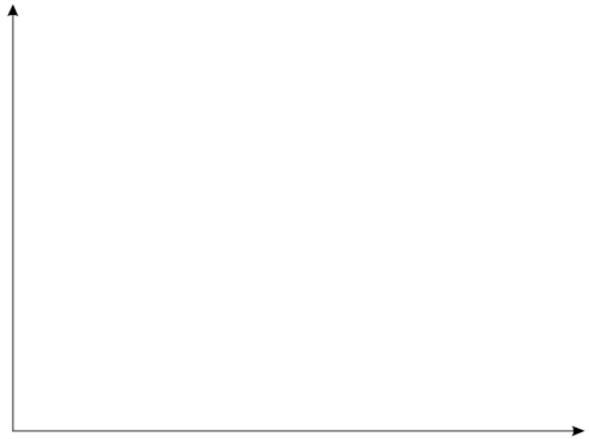


Figure 3:

Optimal consumption therefore implies MRS = MRT (also often referred to as "the Keynes-Ramsey rule")

$$-(1+\phi)\frac{u'(c_1)}{u'(c_2)} = -(1+r) \Leftrightarrow \frac{u'(c_1)}{u'(c_2)} = \frac{1+r}{1+\phi}.$$

A) PRIVATE CONSUMPTION

Experiment 1: *Temporary increase in income.* Period 1 income increases. What is the impact on c_1, c_2?

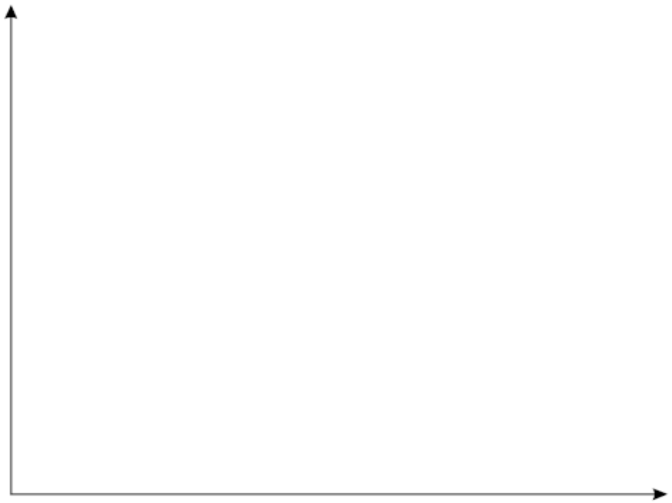
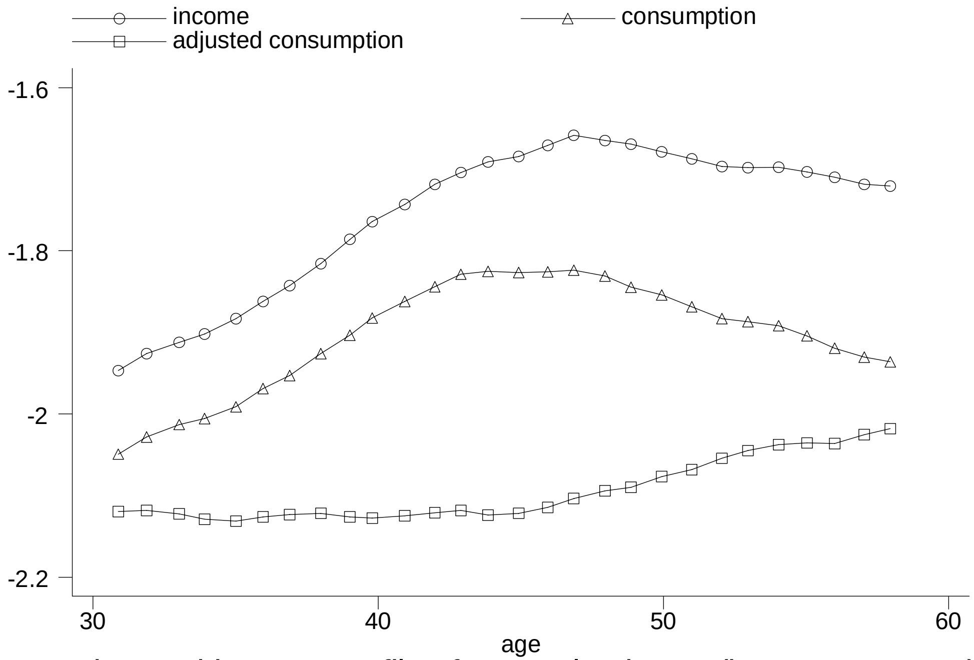


Figure 4:

Hence, since c_1, c_2 are normal goods -> consumption in period 1 always increases by *less* than income; some of the gain is passed on to tomorrow. "Consumption smoothing".

PRIVATE CONSUMPTION

Do people smooth consumption? That is, follow $\left(\frac{u'(c_1)}{u'(c_2)} = \frac{1+r}{1+\phi}\right)$



consumption and income profiles for couples in UK (born 1935-1939)

Figure 5:

A) PRIVATE CONSUMPTION

Experiment 2: *Permanent* increase in income. Period 1 *and* period 2 income rises. Suppose to same extent

$$dY_1^L = dY_2^L.$$

What is the impact on c_1, c_2?

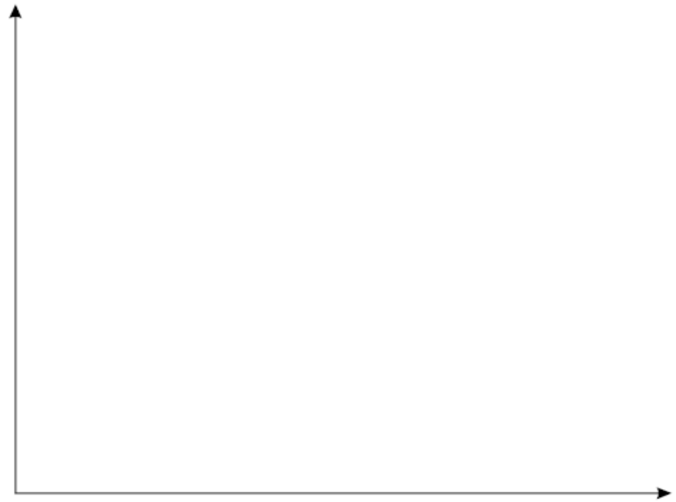


Figure 6:

SUMMARY OF INCOME CHANGES

Temporary changes in income (e.g., unemployment) -> less than proportional changes in consumption. Consumption smooting. The identification of the consumption smoothing effect is due to Modigliani - the life cycle theoryt of consumption.

Key explanation for low volatility of consumption relative to income (Cf BC facts).

Permanent changes in income has more dramatic effect on consumption. In theory we might find 1:1. The hypothesis that permanent income changes has a larger impact on consumption is due to Milton Friedman - The permanent income hypothesis).

NOTE: Income change can either be due to Y or T! Permanent tax cuts more important!

A) PRIVATE CONSUMPTION

Experiment 3: increasing r. Twisting the budget constraint in the endowment point

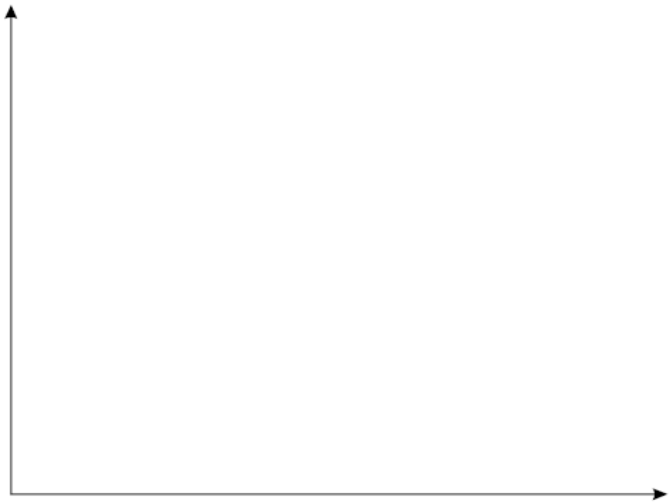


Figure 7:

Notice the difference between ex ante being a lender, or a borrower!

A) PRIVATE CONSUMPTION

Suppose the consumer is a lender. 2 effects from changing r.

1. Substitution effect. It becomes more expensive to consume today - alternative cost. $c_1 \downarrow$, $c_2 \uparrow$

2. Income effect. If you're a lender, $r \uparrow$ implies an increase in the budget. $c_1 \uparrow$, $c_2 \uparrow$ (normal goods).

Thus: Net impact on period 1 consumption, c_1, is ambigious.

Suppose the consumer is a borrower. The income effect works in reverse: $c_1 \downarrow$, $c_2 \downarrow$.

An increase in r will *always* lower consumption for borrowers.

A) PRIVATE CONSUMPTION

Geometry for the lending consumer

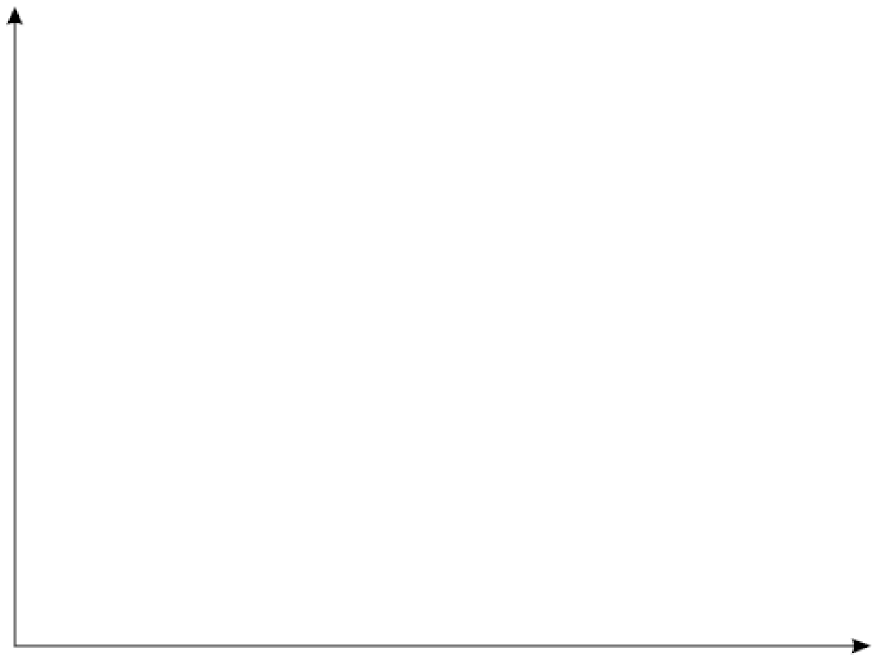


Figure 8:

A) PRIVATE CONSUMPTION

A few words about the *formal* analysis. The problem is to

$$\max_{c_1,c_2} \frac{\sigma}{\sigma-1} c_1^{\frac{\sigma-1}{\sigma}} + \frac{1}{1+\phi} \frac{\sigma}{\sigma-1} c_2^{\frac{\sigma-1}{\sigma}}$$

S.t.

$$c_1 + \frac{c_2}{1+r} = V_1 + H_1$$

Solve by substitution; maximize wrt c_1. The first order condition is MRS = MRT. Substitute for c_2 in the budget constraint. You find

$$c_1 = \theta \left(V_1 + H_1 \right)$$

where $\theta \equiv \left(1 + (1+r)^{\sigma-1}(1+\phi)\right)^{-1}$, and $H_1 \equiv Y_1^L - T_1 + \frac{Y_2^L - T_2}{1+r}$.

Observe that changes in r has an ambigious effect on c_1. $\sigma > 1$-> *substitution effect* dominates.

A) PRIVATE CONSUMPTION

Finally

$$c_1 = \theta \left(\frac{V_1 + H_1}{Y_1^L - T_1} \right) \left(Y_1^L - T_1 \right) \equiv \hat{\theta} Y_1^d$$

where

$$\hat{\theta} \equiv \theta \cdot \left[\frac{V_1}{Y_1^d} + 1 + \frac{1}{1+r} \frac{Y_2^d}{Y_1^d} \right]$$

Observe that keeping $\hat{\theta}$ constant $\frac{\partial c_1}{\partial Y_1^d} \frac{Y_1^d}{c_1} = 1$. BUT: $\frac{Y_2^d}{Y_1^d}$ constant -> *permanent change in income.*

Observe that changing Y_1^d only, leads to a smaller effect. *Temp. change.*

$$\frac{\partial c_1}{\partial Y_1^d} \frac{Y_1^d}{c_1} = \frac{Y_1^d}{V_1 + H_1} < 1.$$

PRIVATE CONSUMPTION

These predictions are rather general. The specific magnitudes, however, depends on the utility function. Hence, in general

$$C = C\left(\underset{(+)}{Y_1^d}, \underset{(+)}{g}, \underset{(\pm)}{r}, \underset{(+)}{V_1}\right)$$

where $g \equiv \frac{Y_2^d}{Y_1^d}$.

Consumption increases with income; less than 1 for 1 if only a temporary change. Growing income (g) will also increase consumption (higher period 2 income, for period 1 unaltered)

The effect of changes in the real rate of interest is ambigious

Higher financial wealth V_1, increases consumption.

PRIVATE CONSUMPTION: EMPIRICS

The association between $\hat{\theta}$ and V_1/Y_1^d:

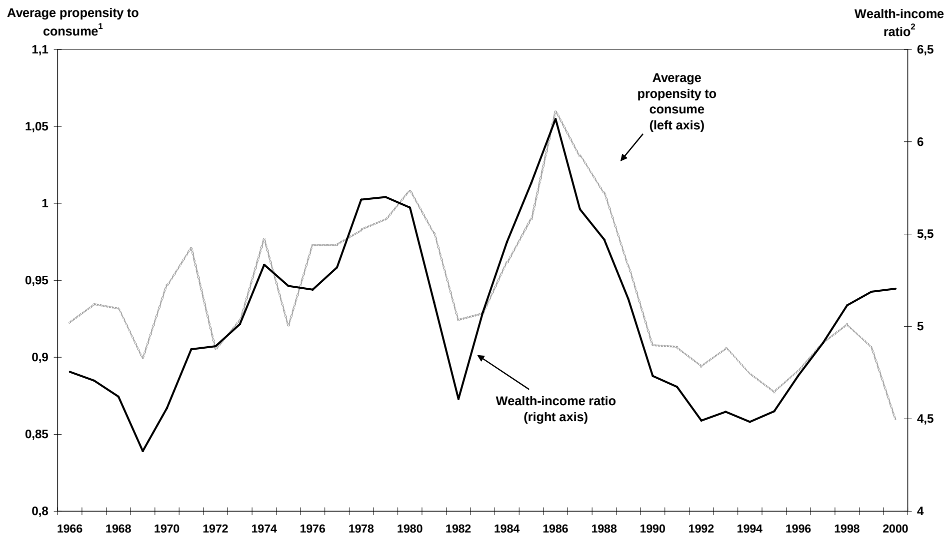


Figure 9: Private consumption and the wealth/income ratio in Denmark

PRIVATE CONSUMPTION: EMPIRICS

With roughtly constant growth in income Y_2^d / Y_1^d is about constant. Then we expect $c_1 / Y_1^d = \hat{\theta}$ which is constant for r constant and with V_1 / Y_1^d trendless (...remember Kaldor?). This seems to be true as well (for rich countries like US and DNK)

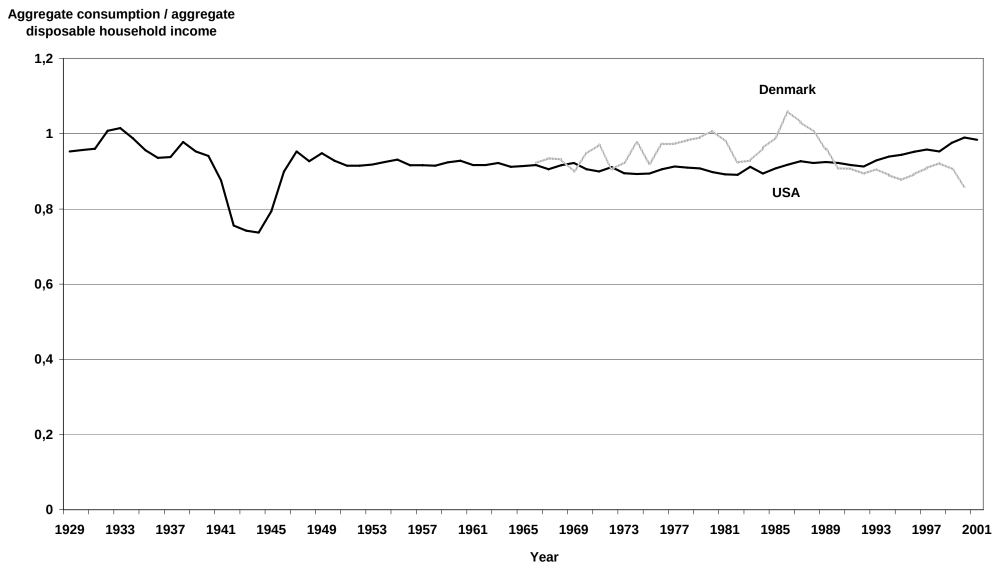


Figure 10: Average propensity to consume, Denmark and the US.

B. Government consumption: BACKGROUND

The public sector is rather large in a place like Denmark

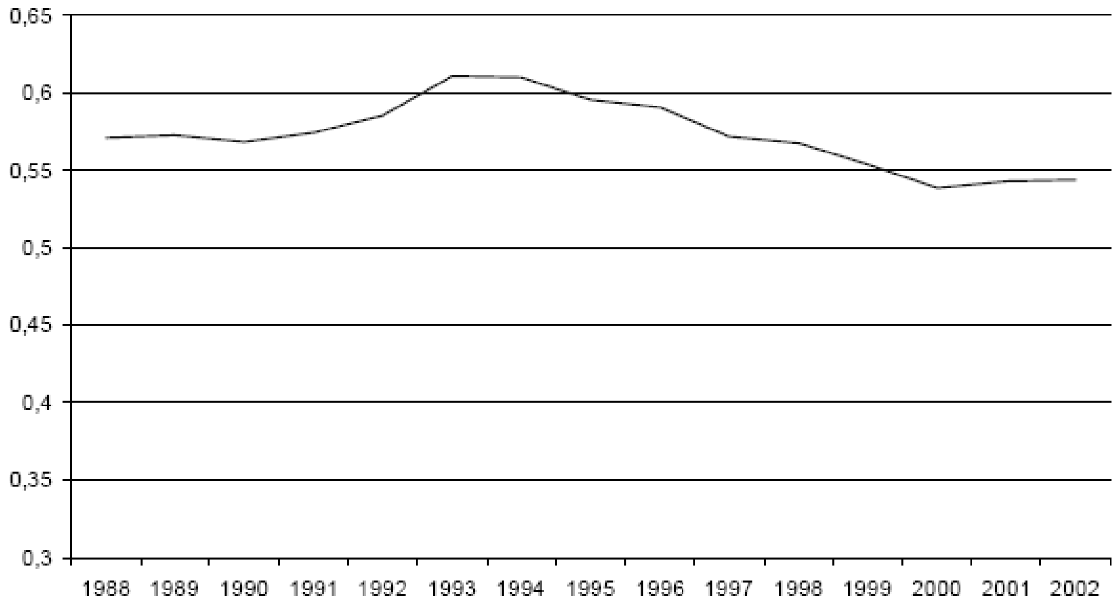


Figure 11: Government expenditures as a fraction of GDP, 1988-2002: Denmark.

B. Government consumption: BACKGROUND

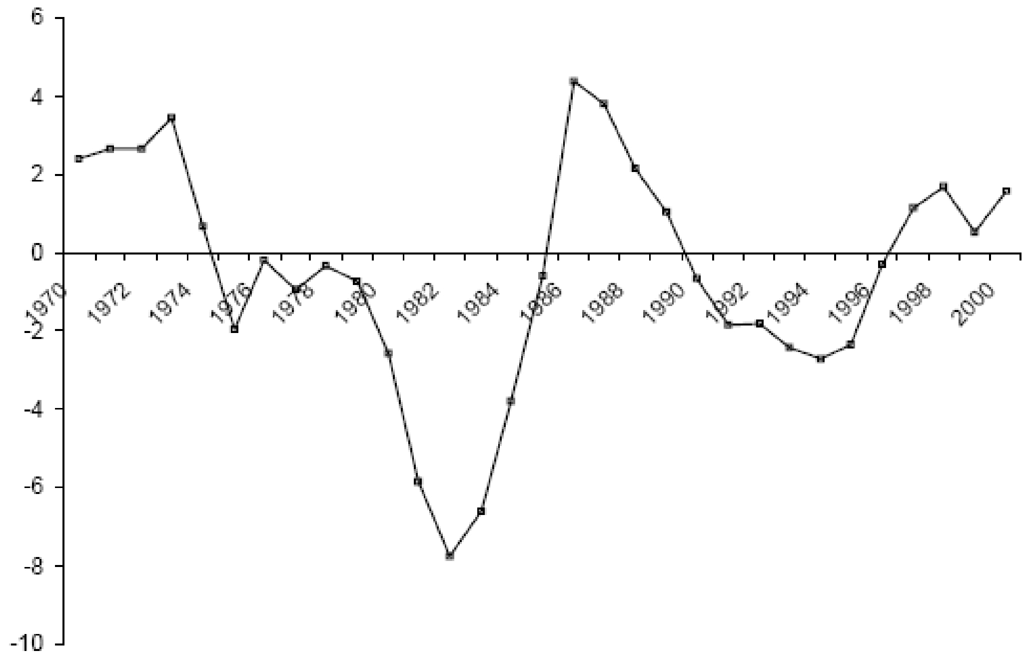


Figure 12: Public sector surplus as a fraction of GDP, 1970-2002: Denmark

.... and the government isn't always running surpluses... and the same is true in many other countries....

B. Government consumption: BACKGROUND

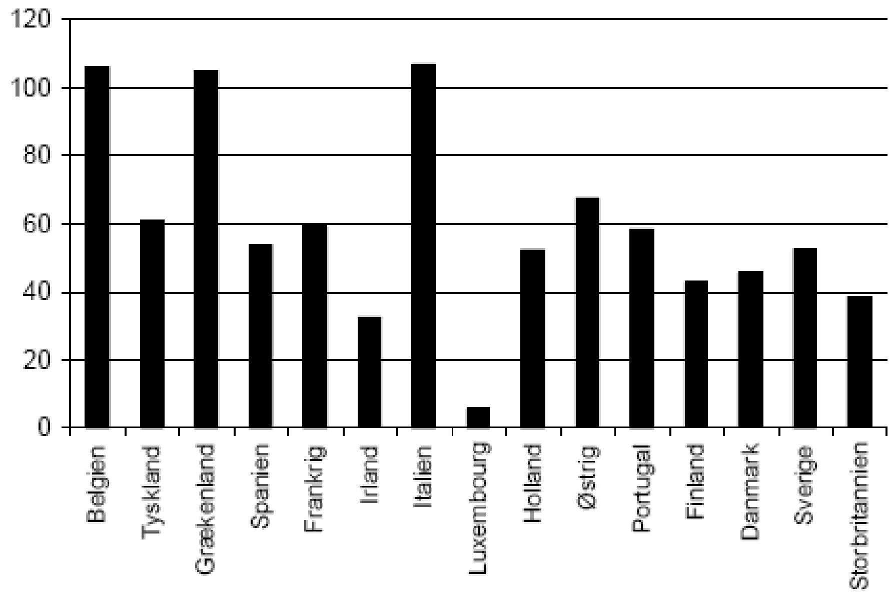


Figure 13: Public debt as a fraction of GDP: Selected countries in 2002.

B. AN INTERTEMPORAL PERSPECTIVE ON THE PUBLIC SECTOR

Suppose the government lives for two periods (like other agents).

Period 1:

$$G_1 = T_1 + B$$

where G_1 is public consumption, T_1 is tax revenue, and B represents net borrowing.

Period 2:

$$G_2 + (1 + r)\, B = T_2$$

There is no "the day after tomorrow"; all debt is "retired" in period 2.

The intertemporal budget constraint (substitute for B)

$$G_2 + (1 + r)(G_1 - T_1) = T_2 \Leftrightarrow \frac{G_2}{1 + r} + G_1 = \frac{T_2}{1 + r} + T_1$$

BUDGET DEFICITS: A PROBLEM?

It is often claimed that budget deficits are harmful

Question 1: Why?

Question 2: Will it always be harmful?

Structure of argument: (i) What does the government do? (ii) What does the households do in response? (iii) Aggregate assessment:

$$S = S^p + S^g = S_p + T - G = I$$

where the last equality represents a closed-economy assumption.

BUDGET DEFICITS: A PROBLEM?

(*re: i)* The current deficit

$$G = T + B$$

Experiment 1: T is decreased, and G kept constant.

$$-dT = dB, dT < 0$$

i.e., increased *debt*.

(*re: ii)* **Assume** households follow a simple rule of thumb

$$S^p = s\left(Y - T\right), s \in (0, 1)$$

As a result

$$dS^p = -sdT$$

BUDGET DEFICITS: A PROBLEM?

(*re: iii)* But what is the impact on *total savings* $S = S^p + T - G$?

$$dS = dS^p + dS^g = -sdT + dT < 0$$

Hence, if $I = S$ it is clear that total investment declines (bad in the long-run in particular).

Prediction: Increasing debt will imply lower investments.

Empirically, however, this is not easily found in the data ... for richer countries anyway. Why might that be?

BUDGET DEFICITS: A PROBLEM?

We now switch to our consumption theory based on optimizing behavior, and the intertemporal view of the government

(re: i) The current deficit. In period 1 we maintain $-dT = dB, dT < 0$. BUT, we also require the government to fulfill its intertemporal budget constraint

$$\frac{G_2}{1+r} + G_1 = \frac{T_2}{1+r} + T_1$$

This implies

$$\frac{dT_2}{1+r} + dT_1 = 0 \Leftrightarrow dT_2 = -(1+r)\,dT_1$$

Intuition: eventually the government will have to pay its debt (here: In period 2). Thus, inevitably (with unaltered G's) taxes will have to go up in the future

BUDGET DEFICITS: A PROBLEM?

(re: ii). Households. Will they change their optimal consumption choice? Yes, if the intertemporal budget constaint changes. Does it?

$$\underbrace{c_1 + \frac{c_2}{1+r}}_{\text{Lifetime consumption}} = \underbrace{V_1 + Y_1^L - T_1 + \frac{Y_2^L - T_2}{1+r}}_{\text{Life time income}} \equiv V_1 + H_1$$

$$dH_1 = -dT_1 - \frac{dT_2}{1+r}$$

Since $dT_2 = -(1+r)\,dT_1$

$$dH_1 = -dT_1 - \frac{(-(1+r)\,dT_1)}{1+r} = 0$$

Hence, consumers choose *exactly* the same consumption bundle (c_1, c_2) as before the tax cut!

BUDGET DEFICITS: A PROBLEM?

This is a somwhat famous result in economics; referred to as “ricardian equivalence”.

Definition The Ricadian Equivalence Theorem: If current and future government spending is held constant, then a change in current taxes with an equal and opposite change in the present value of future taxes leaves the consumption of individuals unchanged.

In this particular case, running a deficit does not affect the optimal consumption choice of individuals

BUDGET DEFICITS: A PROBLEM?

Doesn't anything change? Yes

$$c_1 + s = V_1 + Y_1 - T_1$$

so

$$ds = -dT_1$$

Hence, private savings increases 1:1 with the tax cut!

(re: iii). Total savings $S = S^p + T - G$?

$$dS = dS^p + dS^g = -dT_1 + dT_1 = 0$$

Hence, there is *no* effect on total savings (and thus total investments).

BUDGET DEFICITS

In the first case households do not take into account that they will have to pay taxes sooner or later

The theory developed in Ch 16 tell you that these intertemporal considerations may be important (with unaltered expenditures the government can chose to tax today, or tomorrow, but never ... never)

Consumption is determined by *lifetime income.* If the policy does not change it, there is no (in theory) impact on optimal consumption choices. Changes in savings ensure the old consumption plan can still be attained (i.e., you save the current tax reduction for the purpose of paying it back later)

This is the basic logic of the Ricardian equivalence theorem.

BUDGET DEFICITS

Observe that Ricardian equivalence does *not imply* that all budget deficits are "irrelevant".

Experiment 2: G_1 increases, G_2 unaltered and T_1 unaltered . Since

$$\frac{G_2}{1+r} + G_1 = \frac{T_2}{1+r} + T_1$$

it follows that dT_2 has to go up. If so the consumers are affected:

$$\underbrace{c_1 + \frac{c_2}{1+r}}_{\text{Lifetime consumption}} \qquad = V_1 + Y_1^L - T_1 + \frac{Y_2^L - T_2}{1+r}$$

(it's like a transitionary income reduction). Hence: The source of the deficit may matter.

LIMITATIONS OF THE RICARDIAN EQUIVALENCE RESULT

Finite lifetime

Imperfect credit markets (no longer possible to smooth consumption perfectly)

Symmetrical treatment of consumers.

Thought experiment: Two consumers. Situation is as above: tax reduction today, tax hike tomorrow; no change in G. BUT: suppose consumer 1 gets the entire tax cut, whereas they share the future tax hike.