CAPITAL ACCUMULATION AND GROWTH – THE SOLOW MODEL

Carl-Johan Dalgaard
Department of Economics
University of Copenhagen

AGENDA

1. THE BASIC FRAMWORK

(A) The basic assumptions and the equations of the model

(B) Solving the Model and observations about the steady state

2. STEADY STATE PROPERTIES AND EMPIRICAL IMPLICATIONS

(A) Can capital accumulation sustain economic growth in the long-run?

(B) Can the model explain persistent (+30 years) differences in growth rates?

(C) Can the model explain differences in GDP per worker of the magnitude observed in the data (1:35)?

(D) Convergence properties: Conditional convergence, Club Convergence, σ-Convergence.

(E) Some empirical tests of the model & growth accounting

1A. THE BASIC FRAMWORK

Closed economy

Time is discrete: t=0,1,2,...

No public sector

1 good economy. Output (Y) can either be consumed (C), or invested (I).

Price of output (thus consumption and investment) normalized to 1.

Perfectly competitive markets for output and factors of production (i.e., in particular: no unemployment)

1A. THE EQUATIONS OF THE MODEL

Consider the accounting identity

$$Y_t = C_t + I_t + G \ + NX$$

$\Rightarrow$

$$\ldots.. = \ldots.. = \ldots.$$

Next, assume that capital, K_t, changes over time in accordance with:

$$K_{t+1} = I_t + (1 - \delta) K_t, \ K_0 \text{ given},$$

where $\delta \in [0, 1]$. Taken together:

$$=$$

1A. THE EQUATIONS OF THE MODEL

The aggregate production function

$$Y_t = F\left(K_t, L_t; A\right)$$

where A is, for now, exogenous and constant.

At times we will employ a *specific* functional form: $F\left(.\right) = K^{\alpha}\left(AL\right)^{1-\alpha}$, with $\alpha \in (0,1)$.

A1. $F\left(.\right)$ is homogenous of degree one:

$$\lambda Y = F\left(\lambda K, \lambda L; A\right), \; \lambda > 0.$$

Motivation for the assumption: The replication argument.

Terminology: K, L are *rival* inputs. Technology, A, *non-rival*.

Implication of A1:

$$Y_t = L_t F\left(\frac{K_t}{L_t}, 1; A\right) \equiv L_t f\left(k_t, A\right), \; k_t \equiv K_t / L_t.$$

1A. THE EQUATIONS OF THE MODEL

A2. Capital is essential: $F(0, L; A) = 0$, and the production function exhibits diminishing returns to capital input

$$f'_k(k_t, A) \geq 0, f''_{kk}(k_t, A) < 0 \text{ for all } k \text{ (“diminishing returns”).}$$

Moreover

$$f'_A \geq 0, \;\; f''_{kA} \geq 0$$

$$\lim_{k\to\infty} f'_k(k_t, A) = 0, \lim_{k\to 0} f'_k(k_t, A) = \infty \text{ (the Inada conditions)}$$

[**INSERT Illustration of** f]

1A. THE EQUATIONS OF THE MODEL

A3. Savings behaviour

$$S_t = sY_t, \ s \in [0,1]$$

Some empirical justification offered by Mankiw and Cambell, 1989; Rule of Thumb behaviour.

A4. Population growth

$$L_{t+1} = (1+n)\,L_t, \ n \geq -1$$

Note: If all agents supply 1 unit of labor then, given competitive markets, $L =$ employment.

1B. SOLVING THE MODEL

$$K_{t+1} = S_t + (1-\delta) K_t \overset{A3}{=} sY_t + (1-\delta) K_t$$

Insert production function

$$K_{t+1} = sL_t f(k_t, A) + (1-\delta) K_t$$

Divide by L_t (and divide and multiply by L_{t+1} in the LHS)

$$\frac{K_{t+1}}{L_{t+1}} \left(\frac{L_{t+1}}{L_t} \right) = sf(k_t, A) + (1-\delta) k_t$$

using A4 we get the law of motion for capital

$$k_{t+1} = \frac{s}{1+n} f(k_t, A) + \frac{1-\delta}{1+n} k_t \equiv \Psi(k_t).$$

The model reduces to 1 (non-linear) first order difference equation.

1B. SOLVING THE MODEL

Definition The steady state of the model is a $k_{t+1} = k_t = k^*$ such that $k^* = \Psi(k^*)$.

To study the existence of a steady state, we express the law of motion for capital in terms of the *growth rate* of k (see textbook for alternative phasediagram)

$$\frac{k_{t+1}}{k_t} - 1 = \frac{1}{1+n}\left[s\frac{f(k_t, A)}{k_t} - (\delta + n)\right] \equiv G(k_t)$$

It is easy to prove that (i) $G'(k) < 0$ for all k, (ii) $\lim_{k\to 0} G(k) = \infty$ and (iii) $\lim_{k\to\infty} G(k) = -\frac{\delta+n}{1+n} < 0$ (the last two properties follow from applying L'Hopital's rule, and the Inada-conditions (cf. A2)

[**INSERT phasediagram**]

1B. SOME OBSERVATIONS ABOUT THE STEADY STATE

Unique (non-trivial) steady state, where

$$\frac{f(k^*, A)}{k^*} = \frac{\delta + n}{s} \Leftrightarrow k_{t+1} = k_t = k^*.$$

Globally stable. For any $k_0 > 0$ $\lim_{t\to\infty} k_t \to k^*$

k^* determined by structural charactaristics: s, A, n.

Specifically: $\partial k^*/\partial s > 0$, $\partial k^*/\partial n < 0$ and $\partial k^*/\partial A > 0$.

1B. SOME OBSERVATIONS ABOUT THE STEADY STATE

In the steady state we have the following properties:

A) Constant r. The marginal product of capital is $F'_K = f'(k; A)$, which is constant in steady state, where $k = k^*$ and constant.

B) Constant factor shares $(w/y, rk/y)$. To see this: Note $rk = f'(k; A)\, k$. By constant returns to K, L, there are no profits. Hence $w = f(k; A) - f'(k; A)\, k$. The shares, therefore:

$$\left(\frac{rk}{y}\right)^* = \frac{f'(k^*; A)\, k^*}{f(k^*; A)}, \left(\frac{w}{y}\right)^* = \frac{\left[f(k^*; A) - f'(k^*; A)\, k^*\right]}{f(k^*; A)}.$$

C) Therefore, constant capital-output ratio (since $(k/y)^* = k^*/f(k^*; A)$).

D) Growth in y?

2A. LONG-RUN GROWTH

Can capital accumulation sustain growth in GDP per capita (GDP per worker)?

No! Observe that $y_t = f(k_t; A)$. Why not?

What can we do?

Technological change (**insert phasediagram with discrete changes in** A)

2A. LONG-RUN GROWTH

The simplest extension is to allow for *exogenous* technological change (Chapter 5 in the textbook)

Assume, first

$$Y_t = F(K_t, A_t L_t) = A_t L_t f\left(\tilde{k}_t\right), \ \tilde{k}_t \equiv K_t / A_t L_t.$$

and second that $A_{t+1} = (1+g)\, A_t$. We now have

$$K_{t+1} = s A_t L_t f\left(\tilde{k}_t\right) + (1-\delta)\, K_t$$

which can be rewritten in terms of capital per *efficiency unit* of labor, $\tilde{k}_t$,

$$\tilde{k}_{t+1} = \frac{1}{(1+g)(1+n)} \left[s f\left(\tilde{k}_t\right) + (1-\delta)\, \tilde{k}_t \right].$$

The phasediagram is visually the same, with $\tilde{k}$ replacing k.

2A. LONG-RUN GROWTH

But now growth does not peter out, since GDP per worker is

$$\frac{Y_t}{L_t} = y_t = A_t f\left(\tilde{k}_t\right).$$

Hence, in the long run (i.e., in steady state)

$$\left(\frac{y_{t+1}}{y_t}\right)^* = \frac{A_{t+1} f\left(\tilde{k}^*\right)}{A_t f\left(\tilde{k}^*\right)} = 1 + g.$$

[**Insert st st path of y as predicted by the model with $g > 0$**].

With this addition the model is in full agreement with *the Kaldorian facts (KF)*, in so far as the steady state is concerned.

Bottom line: KF hold *in steady state*. Outside steady state: No. Might explain why KF hold in some places(/periods), not in other places (/periods)

2B. CAN THE MODEL MOTIVATE LONG-RUN GROWTH *DIFFERENCES*?

Is the extension involving g useful in terms of understanding growth *differences*? Yes; "g" differs from one country to the next!

Unattractive though: g is exogenous and cannot be directly observed. The statement that "g explains differences in $\frac{y_{t+1}}{y_t}$" is then pretty empty.

Hence, if the neoclassical growth model is to prove *useful* in thinking about growth differences, we should be able to motivate them without appealing to country specific g's.

Furthermore: There are reasons to expect g to be the same, in the long-run. Technology adption, otherwise: "Big bills left on the sidewalk".

Option 2? Yes: *Transitional Dynamics*.

2B. LONG-RUN GROWTH DIFFERENCES

From now on: $g = 0$ since it is not going to help us empirically anyway. How do we generate growth diff? Consider two countries. Country 1: High s, Country 2: low s. Initial conditions about the same (think Asia vs. Africa)

INSERT ILLUSTRATION

Qualitatively we can generate growth differences. But are they persistent enough, under plausible assumptions?

2B. LONG-RUN GROWTH DIFFERENCES

To be able to generate persistent (30 year +) growth differences countries need to be moving *slowly* towards the steady state

How fast or slow *are* they moving under the model?

Go back to our law of motion for capital

$$\frac{k_{t+1} - k_t}{k_t} = \frac{1}{1+n}\left[s\frac{f(k_t, A)}{k_t} - (\delta + n)\right] \equiv G(k_t)$$

Linearize **around steady state (**see lecture note for details on derivations; textbook Ch. 5 for case where $f = k^{\alpha}$).

2B. LONG-RUN GROWTH DIFFERENCES

When the "smoke clears" we are left with

$$\ln k_{t+1} - \ln k_t \approx -\left[1 - \alpha\left(k^*\right)\right] \frac{n+\delta}{1+n} \left(\ln k_t - \ln k^*\right),$$

where $\alpha\left(k^*\right) \equiv \left[f'\left(k^*; A\right) k^*\right] / f\left(k^*; A\right)$ is the share of capital in national accounts, in the steady state (i.e., a constant).

Solving this difference equation we can show

$$\frac{\ln k_t - \ln k^*}{\ln k_0 - \ln k^*} = \left[1 - \left[1 - \alpha\left(k^*\right)\right] \frac{n+\delta}{1+n}\right]^t.$$

Time to get half way

$$t_{1/2} = \frac{-\ln(2)}{\ln\left(\frac{(1+n)-(1-\alpha)(n+\delta)}{1+n}\right)}.$$

For $n = 0.01, \delta = 0.05$ and $\alpha = 0.4$, we get 19 years. Bottom line: *Lengthy transitions viable.*

2C. CAN THE MODEL MOTIVATE LONG-RUN GDP PER WORKER DIFFERENCES?

At this point we invoke the Cobb-Douglas production function. That is, $f(k;A) = Ak^{\alpha}$, where A is a constant.

In the steady state we have

$$\frac{f(k^*;A)}{k^*} = \frac{y^*}{k^*} = \frac{n+\delta}{s}.$$

Use the C-D technology

$$\frac{Y_t}{L_t} = y_t = Ak_t^{\alpha} \Leftrightarrow y_t = A^{\frac{1}{1-\alpha}} \left(\frac{k_t}{y_t}\right)^{\frac{\alpha}{1-\alpha}}$$

Hence, steady state GDP per worker

$$y^* = A^{\frac{1}{1-\alpha}} \left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

2C. CAN THE MODEL MOTIVATE LONG-RUN GDP PER WORKER DIFFERENCES?

We fix $\alpha = 0.4$ (why?)

Compare two countries which differ in terms of s only

$$\frac{y_1^*}{y_2^*} = \left(\frac{s_1}{s_2}\right)^{\frac{\alpha}{1-\alpha}}$$

Empirically, we observe differences in s by about 1:4. Hence, by way of s differences we can account for at most

$$\frac{y_1^*}{y_2^*} = (4)^{\frac{2}{3}} \approx 2.5$$

Allowing n to differ realistically does not matter. A is not a real candidate solution for this model (unobservable, exogneous). *Bottom line*: A long way from explaining observed differences in GDP per worker.

2D. CONVERGENCE PROPERTIES

The model clearly predicts that countries are converging to their own steady state depending on s, n.

In general, therefore, there is no reason why we should expect a negative correlation between initial conditions (y_0 or k_0) and transitional growth rates in y or k.

BUT: for countries with *similar* structural charactaristics this *is* what we should expect (cf. phasediagram)

This *explains* why we see "Gibrat's law" in the world at large, yet a clear negative association between growth and initial income in structually similar groups (you also see the latter pattern across US states, EU regions and Japanese prefectures for the same reason).

2D. CONVERGENCE PROPERTIES

Statistically, you can control for s and n. That is, ask the following question: *Conditional* on s and n, do we see a negative association between growth and initial income? Run the regression $g = \beta_0 + \beta_1 \log y_0 + \mathbf{z}'\boldsymbol{\gamma} + \varepsilon$, with $\mathbf{z}$ containing s, n, and ε being a noise term. We expect $\beta_1 < 0$; this prediction is confirmed:

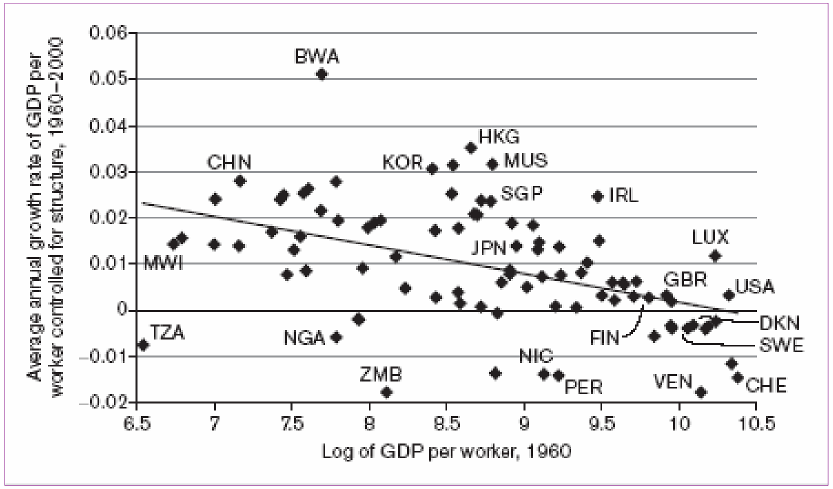

Figure 1: Source: Textbook, Ch. 2

2D. CONVERGENCE PROPERTIES

On this basis we will say that the standard Solow model predicts *conditional convergence.* Conditional convergence is defined in the following way

Definition Conditional convergence. Countries *with identical structural charactaristics* will converge in GDP per worker over time.

Observe: The central reason why you obtain this prediction is because the steady state is *unique;* this ensures that initial conditions (i.e., the position of k_0) does not matter for where you end up.

Plausible extensions/modifications of the model will lead to a very different prediction however.

2D. CONVERGENCE PROPERTIES

Geometrically the following scenario would not support conditional convergence [**Insert alternative phasediagram**]

Definition Club Convergence. Countries with identical structural characataristics *and initial conditions* will converge in GDP per worker over time.

What sort of mechanism's would generate this result?

Consider *subsistence consumption.* The idea is to postulate

$$s = \begin{cases} \bar{s} \text{ if } k > \bar{k} \\ \underline{\text{s}} \text{ otherwise} \end{cases}, \quad \bar{s} > \underline{\text{s}}$$

Alternatives include endogenous fertility (exercises)

2D. CONVERGENCE PROPERTIES

If we have the subsistence "story" the law of motion for capital becomes

$$\frac{k_{t+1}}{k_t} - 1 = \begin{cases} \frac{1}{1+n}\left[\bar{s}\frac{f(k_t,A)}{k_t} - (\delta + n)\right] & \text{if } k > \bar{k} \\ \frac{1}{1+n}\left[\underline{s}\frac{f(k_t,A)}{k_t} - (\delta + n)\right] & \text{otherwise} \end{cases}$$

Note: Different "regimes". Equally consistent with the data.

Does it matter which of the two convergence hypothesis is "correct"?

Consider the policy implications. Does a capital transfer matter in the long run if only *temporary* (e.g., foreign aid)?

Conditional convergence: ..... Club convergence: ......

2D. CONVERGENCE PROPERTIES

Statistically these two options (conditional vs. club) is very hard to tell apart

Club convergence is not inconsistent with a conditonal negative association between growth and initial conditions

Even if we can "prove" different regimes, this state of affairs may not be permanent ...

Insert phasediagram consistent with "stages of development"

2D. CONVERGENCE PROPERTIES

What about cross-country income *dispersion?*

Definition $\sigma-$convergence. $\sigma-$convergence is said to be present if the dispersion (suitably defined) of GDP per capita levels is declining over time.

Empirically, no sign of σ-convergence (note: when unit of analysis is the *country*). The model does *not* predict σ-convergence.

To see this, assume (counterfactually) that s and n does not matter for GDP per worker. That is, each country follows ($\beta_1 < 0$)

$\ln y_{i,t+1} - \ln y_{i,t} = \beta_0 + \beta_1 \ln y_{it} + u_{it},\ E(u_i) = 0,\ E(u_{it} \cdot y_{it}) = 0,$ $var(u_i) = \sigma_u$.If true, the economy converges to $E(y^*) = -\beta_0/\beta_1$. Hence, disp. should decline over time..?

2D. CONVERGENCE PROPERTIES

No nessesarily so. Take variance of last equation (recall $E\left(u_{it} \cdot y_{it}\right) = 0$)

$$\sigma_{y,t+1} = (1+\beta_1)^2 \sigma_{y,t} + \sigma_u \equiv \phi\left(\sigma_{y,t}; \sigma_u\right), \sigma_{y0} \text{ given}$$

INSERT PHASEDIAGRAM

2D. CONVERGENCE PROPERTIES

Conclusion: Even *if* structural charactaristics did not matter (i.e., "absolute convergence" prevail) we might see $\sigma - di$vergence in transition to long-run steady state. If, in addition, σ_y is affected by "**z**" even less reason to expect a declining tendency.

General remark: Even if you see a negative association between growth and initial GDP per worker this does *not imply nessesarily* that the dispersion is declining - that "inequality between nations" is declining.

SUMMARY OF BROAD EMPIRICAL IMPLICATiONS BEFORE TESTS

Conclusion 1: According to the Solow model, capital accumulation *cannot* sustain growth in GDP per worker

Conclusion 2: Persistent growth differences are, under the Solow model, due to *transitional dynamics.* Under plausible conditions, the transition to the steady state is lengthy –> transitional dynamics may make sense quantitatively

Conclusion 3: Long-run differences in labor productivity (y) are due to s and n differences (A and g unmeasurable, and exogenous: thus *not* key predictions). Quantitatively it seems to fall short of the target (1:35)... only much smaller differences can be motivated.

SUMMARY OF EMPIRICAL IMPLICATiONS BEFORE TESTS

Conclusion 4: The model predicts *conditional convergence.* Plausible extensions can, however, support *club convergence.* Conditional convergence implies that *temporary* changes in s, n etc only have *temporary* effects on GDP per worker. Club convergence: They may have *permanent* effects.

Conclusion 5: The Solow model does *not* predict $\sigma-$convergence.

SOME EMPIRICAL TESTS

At a finer level we have a rather strong prediction for the steady state.

Recall, from our discussion on differences in GDP per worker levels, *with a Cobb-Douglas production function* (i.e., $F(.) = K^{\alpha}(AL)^{1-\alpha}$):

$$\left(\frac{Y}{L}\right)_i^* = A_i \left(\frac{s_i}{n_i + \delta}\right)^{\frac{\alpha}{1-\alpha}},$$

where "i's" have been imposed to signify individual countries. In log terms

$$\ln(y_i) = \ln(A_i) + \frac{\alpha}{1-\alpha}\ln(s_i) - \frac{\alpha}{1-\alpha}\ln(n_i + \delta).$$

This is not quite a regression model yet, since there is no error term.

With an added assumption, this is remedied

SOME EMPIRICAL TESTS

Assume that

$$\ln(A_i) = \ln(A) + \epsilon_i,$$

where ϵ is $N(0, \sigma_\epsilon)$, and A is common for all.

This amounts to be saying that in *expected* terms all countries share the same level of sophistication. In practise, however, levels can differ, but only in a random fashion. An economic argument?

We now have

$$\ln(y_i) = \ln(A) + \frac{\alpha}{1-\alpha}\ln(s_i) - \frac{\alpha}{1-\alpha}\ln(n_i + \delta) + \epsilon_i,$$

which we can implement as a regression model.

SOME EMPIRICAL TESTS

We now have something which we can estimate by OLS (regression analysis)

Brief digression on regression analysis. We would like to estimate, say, $y_i = a + bx_i + e_i$, where e is $N\left(0, \sigma^2\right)$. The OLS estimator chooses a and b such that $\min \sum^N \left(y_i - a - bx_i\right)^2$ is attained. The solution for b (which is the sort of thing we usually are interested in, rather than the intercept a):

$$\hat{b} = \frac{\sum^N \left(y_i - \bar{y}\right)\left(x_i - \bar{x}\right)}{\sum^N \left(x_i - \bar{x}\right)^2} = b + \frac{\overbrace{\sum^N e_i \left(x_i - \bar{x}\right)}^{=cov(e_i,x_i)}}{\sum^N \left(x_i - \bar{x}\right)^2},$$

where $\bar{z}$ refers to the mean value of z. Note: *provided* $cov\left(e_i, x_i\right) = 0$, our OLS estimate $\hat{b} = b$ - i.e, the solution equals the "true value".

SOME EMPIRICAL TESTS

Specifically, we can try to estimate:

$$\ln(y_i) = \beta_0 + \beta_1 \ln(s_i) - \beta_2 \ln(n_i + \delta) + \epsilon_i,$$

where, structurally, $\beta_1 = -\beta_2 = \alpha/(1-\alpha)$.
If we are going to estimate this equation by Ordinary Least Squares we need to believe in a few things:

* $cov(s, \epsilon) = cov(n, \epsilon) = 0$. That is, no impact from A_i on either of the two key structural charactaristics. *Key* identifying assumption.

* α is the same in all countries

We *expect:* $\beta_1 > 0, \beta_2 < 0$; $\beta_1 = -\beta_2$, and $\beta_1 = 1/2$ if $\alpha = 1/3$; or $2/3$ if $\alpha = 0.4$.

SOME EMPIRICAL TESTS

TABLE I
ESTIMATION OF THE TEXTBOOK SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	5.48	5.36	7.97
	(1.59)	(1.55)	(2.48)
ln(I/GDP)	1.42	1.31	0.50
	(0.14)	(0.17)	(0.43)
$\ln(n + g + \delta)$	−1.97	−2.01	−0.76
	(0.56)	(0.53)	(0.84)
$\overline{R}^2$	0.59	0.59	0.01
s.e.e.	0.69	0.61	0.38
Restricted regression:			
CONSTANT	6.87	7.10	8.62
	(0.12)	(0.15)	(0.53)
$\ln(I/GDP) - \ln(n + g + \delta)$	1.48	1.43	0.56
	(0.12)	(0.14)	(0.36)
$\overline{R}^2$	0.59	0.59	0.06
s.e.e.	0.69	0.61	0.37
Test of restriction:			
p-value	0.38	0.26	0.79
Implied α	0.60	0.59	0.36
	(0.02)	(0.02)	(0.15)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

Figure 2: From Mankiw et al. (1992)

SOME EMPIRICAL TESTS

The good news:

Correct signs for β_1, β_2

Fairly high explanatory power: About 60% of variation can be motivated

The structure of the model is supported: $\beta_1 = -\beta_2$

The bad news:

Estimated size of α too large to be consistent with National accounts data for capital's share.
$\Rightarrow$ "All is not well with the Solow model"

ABOUT GROWTH ACCOUNTING

The fundamental objective of growth accounting is to provide an answer to the following question: Given reasonable assumptions, *how big a fraction of past growth can be attributed to capital accumulation, and growth in the labor force*?

The backbone of the methodology is the following assumptions:

$Y_t = F(K_t, L_t; A_t)$; the aggregate production function

Perfectly competitive markets

This is how it works ...

ABOUT GROWTH ACCOUNTING

The simplest case is when we assume *a priori* the production function is Cobb-Douglas, $Y = AK^{\alpha}L^{1-\alpha}$, and consider small changes (i.e., annual). Take logs

$$\ln(Y_t) = \alpha \ln(K_t) + (1-\alpha)\ln(L_t) + \ln(A_t)$$

Substract the lagged version $(t-1)$, and you get

$$\ln\left(\frac{Y_t}{Y_{t-1}}\right) = \alpha \ln\left(\frac{K_t}{K_{t-1}}\right) + (1-\alpha)\ln\left(\frac{L_t}{L_{t-1}}\right) + \ln\left(\frac{A_t}{A_{t-1}}\right)$$

or

$$g_Y = \alpha g_K + (1-\alpha) g_L + g_A \Leftrightarrow g_y = \alpha g_k + g_A,$$

where $g_z = g_Z - g_L$. Also: Recall, that $\alpha = F_K' K/Y = rK/Y$, *given competitive markets.*

ABOUT GROWTH ACCOUNTING

Provided we can measure Y (GDP), K (the capital stock), L (Labor force, or employment - preferably by hours), and α (capital's share in national accounts), g_A follows; "total factor productivity"

$$g_A = g_Y - \alpha g_K - (1 - \alpha) g_L$$

Note I: Measuring K is not unproblematic. Perpetual inventory method: Assume $K_{t+1} = I_t + (1 - \delta) K_t$ for all t. Pick δ (5%, say) and guess K_0. We have data on I_t from national accounts. If the period over which I is available is long, problems with initial guess "washes out".

Note I: *All* measurement errors ends up in "A". "A measure of our ignorance" (Abramovitz, 1956).

ABOUT GROWTH ACCOUNTING

	USA (%)					United Kingdom (%)			
Period	g^k	g^y	$g^k/3$	$g^B = g^y - g^k/3$	Period	g^k	g^y	$g^k/3$	$g^B = g^y - g^k/3$
1960–65	2.0	3.5	0.7	2.9	1960–65	–	3.0	–	–
1965–70	2.4	1.2	0.8	0.4	1965–70	–5.8	2.7	–1.9	4.6
1970–75	1.2	0.2	0.4	–0.3	1970–75	2.3	1.6	0.8	0.9
1975–80	1.0	1.0	0.3	0.7	1975–80	1.5	1.1	0.5	0.6
1980–85	1.7	1.6	0.6	1.0	1980–85	1.1	1.3	0.4	0.9
1985–90	0.8	1.5	0.3	1.2	1985–90	2.5	2.6	0.8	1.7
1990–95	1.0	1.3	0.3	1.0	1990–95	3.2	1.8	1.1	0.8
1995–00	2.2	2.5	0.7	1.7	1995–00	3.6	2.6	1.2	1.4
1960–00	1.5	1.6	0.5	1.1	1965–00	1.2	2.0	0.4	1.6

Figure 3: Example of growth accounting. US and UK. Source: the textbook

Typical OECD: g_A accounts for more growth than $g_K - g_L = g_k$. If g_A is "technology" then bad for the Solow model.

Growth accounting can be useful. E.g., the "Asian Tiger economies". As it turns out: Lion's share of growth due to *factor accumulation.* Not "miraculous"

The productivity slowdown: An unusual period (70s and 80s)

ABOUT GROWTH ACCOUNTING

Pitfalls:

After growth accounting someone says: "*In the absence of technological change growth would have been* $g_Y - g_A$*%*"

Consider the steady state of a Solow model with Technological change: $g_y^* = g_k^* = g_A$! Hence, even if g_A is the source of all growth, a growth accounting exercise would still say $\alpha \cdot g_k = \alpha \cdot g_A$ is "attributable" to capital. Growth accounting does not *explain* growth, and cannot be used for counterfactuals.

ABOUT GROWTH ACCOUNTING

After growth accounting someone says: "*In the future growth of the labor force will fall, which means we can expect growth to decline by* $(1-\alpha)$ (change in labor force growth).

Consider the steady state of a Solow model with Technological change. Now n changes. If n declines, $\tilde{k}^*$ rises however. As a result, for a while, $g_k > g_A$ due to technological change. Ultimately, therefore, growth will *not* decline with $(1-\alpha)$ (change in labor force growth) since growth in k picks up. *Growth accounting does not explain growth*, and can therefore not be used for predictions either.

FINAL REMARKS

Solow model provides the most basic framework for thinking about economic growth

A series of useful results emerge (cf summary)

In many respects it does remarkably well: Sign of key variables correct, the structure is supported and it can motivate a lot of the variance in the “world distribution of income”

Fails, however, in one particular dimension: estimated α is too high. This will provide us with motivation for further extensions of the model (Human capital, specifically: Ch. 6).

In large economies, like the US, g_A seems to matter “ a lot”. Anoying we have no theory for it –> Later chapters remidies this.