Written exam for the M.Sc. in Economics
University of Copenhagen
Institute of Economics
Course: Economic Growth (videregående vækstteori)
June 16, 2005
4 Hours, closed book (ingen hjælpemidler)
The exam consists of 3 assignments and 4 pages

General remarks on the exam:

The exam consists of 3 assignments. Assignment 1, labelled "shorter questions", may (but need not) be answered without the use of algebra. Assignments 2 and 3 requires the use of algebra. Make sure to read the assignment in full before you begin.

You should attempt to answer all the questions. If you can only provide a partial answer then this is (much) better than nothing.

Note also that the weights used to determine the final grade are indicated in parenthesis. Finally, while the questions are stated in English the exam may be answered either in Danish or English.

Assignment 1: Shorter questions (30%)

Suppose you run a regression of the form

$$g = \beta_0 + \beta_1 \ln y_0 + \beta_2 h_0 + X\gamma + u,$$

where g is the growth rate of GDP per capita, y_0 is the initial level of GDP per capita, h_0 is the initial school enrollment rate, whereas X is a vector containing other relevant controls (population growth etc), and γ is a vector of associated parameters. Finally, u is an error term. The parameters, $\beta_0, \beta_1, \beta_2$ and γ, are estimated by Ordinary Least Squares. Among the results are a significantly negative estimate for β_1, and a significantly positive estimate for β_2.

Question 1:

With respect to the association between g and $\ln y_0$. Consider the following statement:

> "Theoretically, the key element behind the conditional convergence result in neoclassical growth models is diminishing returns to reproducible capital."

Do you agree with this statement? Explain why or why not.

Question 2:

On the basis of the detected association between g and h_0: (a) *Give a short description of growth theories which could motivate a causal effect of human capital (accumulation) on growth.*

Consider next the following statement:

> "The results from the regression analysis prove that human capital accumulation causes faster growth."

(b) *Does the empirical methodology above allow you to make this inference? Explain why or why not.*

Question 3:

Explain what the so-called "Jones critique of AK models" is about.

Assignment 2: Productive Government Investments and Growth (40%)

Consider the following growth model for a closed economy with a government sector. Firm i uses the following technology to produce output:

$$Y_{it} = AK_{it}^{\alpha}L_{it}^{1-\alpha}\hat{G}_t^{\pi}. \tag{1}$$

A is a constant, K_{it} is the capital stock of firm i, L_{it} total labor input in firm i, whereas

$$\hat{G}_t = \frac{G_t}{K_t^{\phi}L_t^{\delta}},\ 0 < \phi < \alpha < 1,\ 0 < \delta < 1 \tag{2}$$

where G_t represents government investments in infrastructure, while K_t and L_t are the aggregate stock of capital and the total labor force in the economy, respectively. Let r_t denote the real rate of return, and w_t the real wage. All markets are competitive, and the price of output is normalized to 1. For simplicity it is assumed that capital does not depreciate. G_t is financed by a wealth tax, levied on the households. The government balances the budget at all points in time. Finally, the total size of the labor force is constant at all points in time: $L_t = L$.

Question 1: *Provide an interpretation of equation* (2).

Question 2: *Solve the profit maximization problem for firm* i*, and proceed to show that the aggregate production function can be written*

$$Y_t = AK_t^{\alpha} L^{1-\alpha} \hat{G}_t^{\pi}.$$

Question 3: *What would* π *need to fulfill in order for the model to exhibit endogenous growth?*

Assume the restriction derived above holds. The representative agent maximizes discounted utility from consumption. More specifically, the problem is

$$\max_{\{c_t\}_{t=0}^{\infty}} \int_0^{\infty} \ln c_t e^{-\rho t} dt$$

s.t. (i) $c_t \geq 0$, (ii) $\dot{k}_t = (r - \tau) k_t + w_t - c_t$, k_0 given, and (iii) $\lim_{t\to\infty} k_t e^{-\int_{s=0}^{t} r_t dt} \geq 0$, where τ is the (time constant) wealth tax rate. It has already been used that as the economy is closed the wealth of the representative agent equals the capital/labor ratio, $k_t \equiv K_t/L$.

Question 4. (i) *Solve the consumer's problem and derive the growth rate of GDP per capita.* (ii) *Explain why the tax rate,* τ*, is related to the growth rate in the manner suggested by the formula.*

Question 5. The growth rate depends on the size of the labor force. (i) *Why does the model contain the property?* (ii) *Is it possible to eliminate scale effects in the present model while preserving endogenous growth?*

Assignment 3: R&D and Growth (30%)

Consider a closed economy. Aggregate output, Y_t, is used for three purposes: Production of ideas, production of intermediate goods, and consumption. The price of final output is normalized to 1. In the final goods sector perfect competition prevails. Specifically, final output of firm i, Y_{it}, is produced using the following technology

$$Y_{it} = AL_{it}^{1-\alpha} \sum_{j=1}^{N_t} x_{ijt}^{\alpha}, \ A > 0.$$

A is a parameter, L_{it} is labor input and x_{ijt} is input of intermediate good j in firm i at time t. It is assumed that the labor force is constant through time, $L_t = L = \sum_i L_{it}$. The factor prices for labor and intermediate good j are

w_t and P_{jt}, respectively. Profit maximization by final goods firm's imply that demand for intermediate good j for firm i satisfies $P_{jt} = \alpha A L_{it}^{1-\alpha} x_{ijt}^{\alpha-1}$ for all j.

Moreover, aggregate demand for intermediate good j, $X_{jt} = \sum_i x_{ijt}$, is $P_{jt} = \alpha A L^{1-\alpha} X_{jt}^{\alpha-1}$.

The intermediate goods sector consists of $j = 1, ..., N_t$ firms. Each firm operates as monopolist in their market for intermediate good j, since they all hold a patent of infinite duration. Each firm uses a technology which involves spending one unit of output so as to produce one unit of intermediate good. Accordingly, profits for intermediate goods firm j are given by $\Pi_{jt} = P_{jt} X_{jt} - X_{jt}$.

Question 1: *Solve the profit maximization problem for firm j, and go on to show that aggregate output can be written $Y_t = A^{\frac{1}{1-\alpha}} L \alpha^{\frac{2\alpha}{1-\alpha}} N_t$.*

Assume consumption is determined along "Solowian lines": $C_t = (1-s) Y_t$, where s is an exogenous savings rate. Throughout it is assumed that $s > \alpha^2$. Finally, assume that the production of 1 idea requires the use of η units of output. The parameter η is, for now, a positive constant.

Question 2: *Write the resource constraint for the economy, and show that the stock of knowledge, N_t, (and thereby aggregate output) evolves in accordance with*

$$\frac{\dot{N}_t}{N_t} \equiv \gamma_{Nt} = \frac{s - \alpha^2}{\eta} A^{\frac{1}{1-\alpha}} L \alpha^{\frac{2\alpha}{1-\alpha}}.$$

Comment on this expression and its empirical relevance.

Suppose it becomes more costly to innovate as the stock of knowledge rises. In particular, suppose $\eta = \eta(N_t) \equiv \phi N_t^{\sigma}, \sigma < 1$. Otherwise the model is as above.

Question 3: (a) *Does this model allow for sustained exponential growth in GDP per capita? (b) What if the labor force is growing at the rate, n? (c) Discuss whether this model is more empirically meaningful than the version of the model from question 2.*

Solutions:

Assignment 1:

Question 1: One should disagree (Galor, 1996). The key element behind conditional convergence is not technology, but lies on savings behavior. In a Solow model, the assumption is that savings are given as a constant share of total output. If, in stead, savings fulfill

$$S_t = s^w w_t L_t + s^r r_t K_t$$

and in addition $s^w > s^r$ the possibility of muliple steady states, and therefore Club Convergence, arises even when per capita output is $y = f(k)$, where $f'(k) > 0$ for all k, $f''(k) < 0$ for all k.

Question 2: (a) The simplest theory is that of Mankiw, Romer and Weil (1992). Here human capital is viewed as another form of capital which can be build up by investing income. Eventually the model predicts that a higher investment rate in human capital will lead to higher long run productivity, and therefore in transition, faster growth. Another theory is due to Uzawa and Lucas (cf. Barro and Sala-i-Martin (2004, Ch. 5); Lucas (1993)). In the Uzawa/Lucas framework households invest time in education, for which they become compensated by a higher future wage. If the human capital production sector is sufficiently productive, human capital accumulation can lead to perpetual (endogenous) growth. In both theories "human capital" is thought of a "knowledge", which implies that the stock of human capital can rise for ever, even with constant educational effort (years of schooling, or a constant investment rate in the MRW case). Hence, there is a clear sense in which the authors envision rising "quality" of human capital over time as a key feature of the growth process.

(b) No, you cannot. In general we would expect the schooling decision to depend on (expected) growth (Bills and Klenow, 2000). Accordingly, growth matters for human capital and vice versa. We are dealing with endogenous regressors, which means that, in principle, a positive slope estimate in the OLS regression could be consistent with a (near) zero causal effect of schooling on growth. In stead the association could reflect the reverse: faster growth induces people to spend more time being educated. Either way, since there is a two way positive association between the two variables, we don't know whether the OLS estimate over-or-underestimates the causal impact from schooling on growth. Bills and Klenow attempts to sort out the relative strenght of the two lines of causation, and find that the one running from growth to schooling is the strongest.

A final observation is that enrolement rates only captures the quantity of schooling. In principle quality could matter as well, indeed it could be the most important thing (cf. the theories above). However, Hendricks (2002) – who examines the relative wage income of immigrants to native americans find little evidence of large quality differences in the data.

Question 3: Consider for simplicity a Solow model augmented by the assumption that $Y = AK$. In this model growth in income per capita, g, fulfills

$$g = sA - \delta,$$

where s is the savings/investment rate, and δ is the rate of depreciation. Accordingly, if the investment rate changes, growth should also change.

Jones (1995, QJE) points out that while investment/output ratios have risen in many countries over the post world war II period, output growth rates have stayed roughly constant or have fallen. More specifically, Jones runs a set of time series tests on the data for a group of OECD countries, looking for statistically significant trends in investment rates and growth rates. He finds positive trends in investment rates—especially producers' durable equipment rates—but *not* in growth rates. In light of the growth equation above, he concludes that the main prediction of AK-style models is not consistent with the data. This is the "Jones critique of AK models".

While not required, one can make the following observations. More recently, Ellen McGratten (1998) provides a defence for AK models. First, from an empirical perspective she extends the analysis to comprise data over longer periods of time, where she is able to detect a positive association between investment rates, and average growth rates; as the theory predicts. Second, McGratten shows theoretically that it is possible to construct examples where a short term deviation (post wwII period, say) in certain investment components (durables say) does not manifest itself in faster growth. Hence, a positive long-run correlation between investment rates and growth, is reconcilable with (Jones' finding of) a no clear "short term" association between growth and investments.

Assignment 2:

This assignment draws on material from Barro and Sala-i-Martin (2004, Ch. 4) along with various articles (cited below). Still, this precise model is new to the students.

Question 1. The formulation captures public goods that are subject to congestion (queues on the highway, overloaded phone networks etc). Hence, in order to obtain increasing productivity, G will need to rise relative to the "demand for use", assumed to be proportional to K and L. A special case is where $\phi = 1$ and $\delta = 0$. This formulation is identical to the one presented in Barro and Sala-i-Martin (2004, Ch. 4).

Question 2.

The problem is to

$$\{K_i, L_i\} = \arg\max \left\{ AK_i^{\alpha} L_i^{1-\alpha} \hat{G}_t^{\pi} - rK_i - wL_i \right\}$$

yielding the first order conditions (where firm's take $\hat{G}$ as given)

$$\begin{aligned} r &= \alpha \frac{Y_i}{K_i} = \alpha A k_i^{\alpha-1} \hat{G}_t^{\pi} \\ w &= (1-\alpha) \frac{Y_i}{L_i} = (1-\alpha) A k_i^{\alpha} \hat{G}_t^{\pi} \end{aligned}$$

from which it follows that

$$\frac{w}{r} = \frac{(1-\alpha)}{\alpha} k_i.$$

Accordingly, all firms choose the same factor intensity: $k_i = k$ for all i.

Aggregation proceeds as follows

$$\begin{aligned} Y &= \sum_i AK_i^{\alpha} L_i^{1-\alpha} \hat{G}_t^{\pi} = A\hat{G}_t^{\pi} \sum_i k_i^{\alpha} L_i \\ \overset{\text{sym. eq.}}{=} A\hat{G}_t^{\pi} k^{\alpha} \sum_i L_i &= A\hat{G}_t^{\pi} k^{\alpha} L = A\hat{G}_t^{\pi} K^{\alpha} L^{1-\alpha}, \end{aligned}$$

which is the aggregate production function.

Question 3: For endogenous growth we need constant returns to the reproducible factor of production: Capital. Since balanced budget implies $\tau K_t = G_t$ (this can be deduced from the budget constraint of the representative household along with the fact that the size of the labor force – and population (competitive markets) – is L), we may write the aggregate production function

$$\begin{aligned} Y &= A \left(\frac{\tau K}{K_t^{\phi} L_t^{\delta}} \right)^{\pi} K^{\alpha} L^{1-\alpha} \\ Y &= A\tau^{\pi} K^{(1-\phi)\pi+\alpha} L^{1-\alpha-\pi\delta} \end{aligned}$$

Hence, we require the restriction

$$(1-\phi)\pi + \alpha = 1$$

or

$$\pi = \frac{1-\alpha}{1-\phi}.$$

Imposing this restriction leads to the production function

$$Y = A\tau^{\frac{1-\alpha}{1-\phi}} L^{\frac{(1-\phi-\delta)(1-\alpha)}{1-\phi}} K \equiv \tilde{A}(\tau) K,$$

which is of the “AK - variety".

Question 4:

(i) The Hamiltonian

$$H(c, k, \lambda, t) = \ln c_t e^{-\rho t} + \lambda((r-\tau) k_t + w - c_t).$$

The first order conditions

$$\frac{\partial H}{\partial c} = 0 : \frac{1}{c} e^{-\rho t} = \lambda$$

$$\frac{\partial H}{\partial k} = -\dot{\lambda} : \lambda(r-\tau) = -\dot{\lambda}$$

Differentiating $\frac{\partial H}{\partial c}$ wrt time, and inserting into $\frac{\partial H}{\partial k}$, yields the Keynes-Ramsey rule

$$\frac{\dot{c}}{c} = r - \tau - \rho.$$

Now, since the model is of the AK-variety it follows that the model exhibits balanced growth. Accordingly, if $\frac{\dot{c}}{c} > 0$ (which will be assumed), then all endogenous variables grow at the same rate, γ.

Therefore, to obtain the growth rate of GDP per capita all we need to do is to substitute for the equilibrium real rate of return:

$$r = \alpha \frac{Y_i}{K_i} \overset{\text{sym. eq}}{=} \alpha \frac{Y}{K} = \alpha \tilde{A}(\tau).$$

Hence, the growth rate is simply

$$\gamma = \alpha \tilde{A}(\tau) - \tau - \rho = \alpha A \tau^{\frac{1-\alpha}{1-\phi}} L^{\frac{(1-\phi-\delta)(1-\alpha)}{1-\phi}} - \tau - \rho.$$

(ii) Since $\phi < \alpha$, it follows that $\frac{1-\alpha}{1-\phi} < 1$. Accordingly, the relationsship between γ and τ is *hump shaped*. Specifically, growth is maximized when τ is put equal to:

$$\tau^* = \arg\max \gamma$$

FOC:

$$\frac{1-\alpha}{1-\phi} \alpha A (\tau^*)^{\frac{1-\alpha}{1-\phi}-1} L^{\frac{(1-\phi-\delta)(1-\alpha)}{1-\phi}} = 1$$

(Obviously the SOC is satisfied). Rearrangements yield:

$$\tau^* = \left[\frac{1-\alpha}{1-\phi} \alpha A\right]^{\frac{1-\phi}{\alpha-\phi}} L^{\frac{(1-\phi-\delta)(1-\alpha)}{\alpha-\phi}}$$

The reason for the hump-shaped association is simple. On the one hand, when taxes are raised then this allows for more productive government investments, which increases the marginal productivity of capital, and therefore increases growth. On the other, increasing taxes will have an incentive distorting effect on the desire so save. This tends to lower growth. If $\tau < \tau^*$ the former effect dominates; whereas the latter dominates when $\tau > \tau^*$.

Question 5:

In general the impact from scale is ambigious; since it depends on whether $1 - \phi \gtreqless \delta$. Suppose $1 - \phi > \delta$.

In this case the model exhibits a positive scale effect. This happens for three reasons. (1) Given non-rival public goods, diminishing returns to capital is off set. Hence the marginal product of capital (MP_K) remains constant. (2) Capital and labor are complements in the production function, in the sense that $\partial MP_K / \partial L > 0$, (3) Savings are increasing in the rate of return on investments. To see the latter in the present model (However, it is fine if the student simply states this as a matter of fact) can easily be inferred, since:

$$\frac{\dot{K}}{K} = \gamma = \left(\frac{I}{Y} \frac{Y}{K} \right)^* = \left(\frac{S}{Y} \frac{Y}{K} \right)^*$$

Using $\gamma = \alpha \tilde{A}(\tau) - \tau - \rho$ it follows that

$$\begin{aligned} \left(\frac{S}{Y} \right)^* &= \frac{1}{\left(\frac{Y}{K} \right)^*} \left[\alpha \tilde{A}(\tau) - \tau - \rho \right] \\ &= \frac{1}{\tilde{A}(\tau)} \left[\alpha \tilde{A}(\tau) - \tau - \rho \right] \equiv \alpha \left[1 - \frac{\tau + \rho}{MP_K} \right], \end{aligned}$$

where $MP_K \equiv \alpha \tilde{A}(\tau)$. The association between savings and the return on capital investments, hold in any infinite horrizon model (but not, e.g. in a over-lapping generations context). In sum, we have the following chain of causality: $L \uparrow \Rightarrow MP_K \uparrow \Rightarrow \gamma \uparrow$. The first arrow follows from (1) and (2), whereas the latter follows from (3).

In order to eliminate scale effects, we need to either modify the savings behaviour of households, or, ensure that $\partial MP_K / \partial L = 0$. The former solution is not feasible under the present microfoundations for consumers' behavior (perfect altruism). But the latter holds (in the aggregate, but not for individual firms) if

$$1 - \phi - \delta = 0 \Leftrightarrow \delta = 1 - \phi.$$

Under this restriction, congestion from the size of population exactly works to off set the otherwise present tendency for MP_K to rise due to an expanding labor force. Indeed, if $\delta > 1-\phi$ the impact from scale could be negative, since the congestion effect dominates globally.

Assignment 3

This assignment draws on material from Barro and Sala-i-Martin (2004, ch. 7). In addition various articles (cited below).

Question 1.

The problem is to find

$$p_j = \arg\max \left(p_j - 1\right) X_j$$

s.t.

$$\alpha A L^{1-\alpha} x_j^{\alpha-1} = p_j$$

Total revenue product is

$$\alpha A L^{1-\alpha} x_j^{\alpha} = p_j x_j,$$

so the optimal quantity is simply (MR=MC)

$$\alpha^2 A L^{1-\alpha} x_j^{\alpha-1} = 1$$

or

$$\begin{aligned} x_j^{\alpha-1} &= \alpha^{-2} A^{-1} L^{-(1-\alpha)} \\ x_j &= \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L = \bar{x} \text{ for all.} \end{aligned}$$

Hence, all intermediate goods firm's choose to produce exactly the same amount of goods. Equilibrium price ($\bar{x}$ inserted into the demand equation)

$$\frac{\alpha A L^{1-\alpha}}{\left(\alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L.\right)^{1-\alpha}} = \frac{1}{\alpha} = p_j = p.$$

That is, price = a mark up over marginal costs (=1). Finally, profits

$$\pi_j = \bar{\pi} = \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L.$$

Since $x_{ij} = x_j = \bar{x}$, we may write aggregate output

$$\begin{aligned} Y &= A L^{1-\alpha} N \bar{x}^{\alpha} \\ &= A L^{1-\alpha} N \left(\alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L\right)^{\alpha} \\ Y &= A^{\frac{1}{1-\alpha}} L N \alpha^{\frac{2\alpha}{1-\alpha}}. \end{aligned}$$

Question 2:

This way of “closing" the model is new to the students. The resource constraint is simply

$$Y = C + xN + \eta \dot{N}.$$

Since $C = (1-s)\,Y$, and given the expression for Y, it follows

$$\begin{aligned}
\dot{N} &= \frac{sY - Nx}{\eta} \\
\frac{\dot{N}}{N} &\equiv \gamma_N = \frac{s}{\eta}\frac{Y}{N} - \frac{x}{\eta} \\
\gamma_N &= \frac{s}{\eta} A^{\frac{1}{1-\alpha}} L \alpha^{\frac{2\alpha}{1-\alpha}} - \frac{1}{\eta}\alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L \\
\gamma_N &= \left[s - \alpha^2\right] \frac{A^{\frac{1}{1-\alpha}} L \alpha^{\frac{2\alpha}{1-\alpha}}}{\eta},
\end{aligned}$$

where $s - \alpha^2 > 0$ by assumption.

Hence, the model exhibits endogenous growth. It is in many ways a convex combination of the Solow model, and a Romer-type model. We may observe that, since output is used to produc ideas, we can write

$$\dot{N} = \frac{\left[s - \alpha^2\right]}{\eta} A^{\frac{1}{1-\alpha}} L \alpha^{\frac{2\alpha}{1-\alpha}} N$$

where $A^{\frac{1}{1-\alpha}} L \alpha^{\frac{2\alpha}{1-\alpha}} N = Y$. If we define $\frac{\left[s-\alpha^2\right]}{\eta} \equiv s_R$ we can write the reduced form association between ideas and output

$$\dot{N} = s_R Y,$$

which is akin to the model of Nonneman and Vanhaut (1996) (NV). In NV, however, s_R is fully exogenous, as in a Solow model. NV find support for the predictions of an "R&D augmented Solow model", using data for the OECD. Another angle on the model is that

$$\gamma_N = s_R \frac{Y}{N},$$

where Y/N is constant, given the assumptions made. Hence, the model predicts that if the investment share in R&D is constant, so should the growth rate of "TFP" be. Ha and Howitt (2005) find support for this prediction.

Finally, however, one may argue that Y/N is not likely to be constant. In fact, if L is growing it should be trending upwards, leading to accelerating growth. This property has been critized by Jones (1995, JPE).

Question 3.

If we assume $\eta = N^{\sigma}$, our differential equation becomes

$$\gamma_N = \frac{\left[s - \alpha^2\right]}{N^{\sigma}} A^{\frac{1}{1-\alpha}} L \alpha^{\frac{2\alpha}{1-\alpha}}. \tag{3}$$

(a) Hence, obviously

$$\lim_{N \to \infty} \gamma_N = 0$$

so the answer is **no**. (Though, the model does allow for perpetual growth – the growth rate only asymptotes to zero).

(b) Yes, now it is possible. If γ_N is to remain constant, we need L/N^{σ} to remain constant, which it is if

$$\gamma_N = \frac{1}{\sigma} n.$$

Note that since $Y = A^{\frac{1}{1-\alpha}} L \alpha^{\frac{2\alpha}{1-\alpha}} N$, per capita GDP will, in this "semi-endogenous" growth model growth at the rate $\frac{1}{\sigma} n$ as well. In other words, perpetual growth is limited in the economy's ability to sustain growth in the labor force.

(c) Hard to say; both a problematic. If we think about the "semi-endogenous" growth model as decribing individual economies in the world, there is little evidence to suggest that labor force growth, or population growth, stimulates growth in GDP per capita. And the present model holds this prediction.

Still, one can choose to think about the model as one decribing only very large and economically important economies, like the US. For minor countries, growth can be regarded as essentially given from abroad (World R&D effort, say). In this case it is harder to test the model.

Nevertheless, in general we can easily charactarize the transitional dynamics of the growth rate in N (or income per capita) in this model. Differentiate equation (3) wrt time to get

$$\dot{\gamma}_N = \gamma_N \left[n - \sigma \gamma_N\right].$$

If we draw this relationsship in a $(\gamma_N, \dot{\gamma}_N)$ diagram, a hump shaped relationsship emerges, with a peak at $\gamma_N = \frac{1}{2}\sigma^{-1} n$, and where $\dot{\gamma}_N = 0$ if $\gamma_N = 0$ or $\gamma_N = \sigma^{-1} n$. The latter is the interesting steady state of course.

Now, if initially $\gamma_N < \sigma^{-1} n$ the growth rate is accelerating towards its steady state value $\sigma^{-1} n$ ($\dot{\gamma}_N > 0$). If, initially, γ_N is above $\sigma^{-1} n$ it is deacclerating. Moreover, if n is gradually declining over time, as Ha and Howitt (2005) document (i.e thinking of n as the growth of S&E) we would expect either a declining growth rate, or a pattern involving first increasing but eventually declining growth in N (if initially $\gamma_{Nt} < \sigma^{-1} n_t$, but at some point in time T, $\sigma^{-1} n_T < \gamma_{NT}$).

Ha and Howitt (2005) examines these predictions and show that it is inconsistent with US data for the post war period. Over the last several decades the growth rate of scientists and engineers (S&E) has been declining, and yet the empirical counterpart to “N", namely total factor productivity" (TFP) growth has remained stationary. This seems at variance with the predictions of the model, which would tend to suggest that we should have seen a productivity slowdown (or perhaps accelerating growth, and then followed by a slowdown).

However the model from question 2 ($\sigma = 0$) would correspondingly suggest that growth in TFP should have accelerated (albeit at a decreasing pace), when $n > 0$. This has not been the case either. Both versions of the model is therefore empirically problematic.