Exercises in Growth Theory and Empirics[*]

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1. Short Questions

Do you agree with statements A - C? Explain why or why not.

A. "A lower income tax will always increase the long-run growth rate in models of endogenous growth featuring infinitely lived households".

B. "In semi-endogenous growth models the share of capital in total income converges to 1 as the capital stock per worker tends to infinity".

The table below (labeled "Table II") shows results from reestimating "the convergence-equation" associated with the augmented Solow model on a "full sample" of 98 countries, and on two sub-samples of countries consisting of 42 country observations each. Column II refers to 42 "initially poor countries", while column III refers to the results from estimating the model on a sample of initially "rich" countries.

C. "The results provide conclusive evidence in favour of the Club-Convergence hypothesis".

Table II. Cross-section regressions: initial output and literacy-based sample breaks: dependent variable: $\ln(Y/L)_{i,1985} - \ln(Y/L)_{i,1960}$

	M–R–W	$(Y/L)_{i,1960} < 1950$ and $LR_{i,1960} < 54\%$	$1950 \leq (Y/L)_{i,1960}$ and $54\% \leq LR_{i,1960}$
Observations	98	42	42
		Unconstrained regressions	
Constant	3·04[a]	1·40	0·450
	(0·831)	(1·85)	(0·723)
$\ln(Y/L)_{i,1960}$	−0·289[a]	−0·444[a]	−0·434[a]
	(0·062)	(0·157)	(0·085)
$\ln(I/Y)_i$	0·524[a]	0·310[a]	0·689[a]
	(0·087)	(0·114)	(0·170)
$\ln(n+g+\delta)_i$	−0·505	−0·379	−0·545
	(0·288)	(0·468)	(0·283)
$\ln(SCHOOL)_i$	0·233[a]	0·209[a]	0·114
	(0·060)	(0·094)	(0·164)
$\bar{R}^2$	0·46	0·27	0·48
σ_ε	0·33	0·34	0·30
		Constrained regressions	
Θ	−2·56[a,b]	2·29	−0·395
	(1·14)	(1·17)	(1·24)
α	0·431[a]	0·275[a]	0·509[a]
	(0·061)	(0·097)	(0·098)
γ	0·241[a]	0·217[a]	0·108
	(0·046)	(0·061)	(0·094)
$\bar{R}^2$	0·42	0·28	0·50
σ_ε	0·34	0·34	0·29

[a] Significance at asymptotic 5% level.

Figure 1: Sample splitting and the “convergence equation". Source: Durlauf and Johnson, 1995.

2. Endogenous Growth through Factor Accumulation

Consider a closed economy where production in firm i is given by

$$Y_i(t) = K_i(t)^{\alpha}(A(t)L_i(t))^{1-\alpha},$$

Y_i is output in firm i, K_i and L_i is capital and labor hired by firm i, respectively. A is an index of technological knowledge which is common to all firms in the economy. Specifically it is assumed that

$$A(t) = \tilde{A}K(t), \tag{1}$$

where $K = \sum_i K_i$ and $\tilde{A} > 0$ but constant. It is assumed that $\sum_i L_i(t) \equiv 1$ and constant over time.

Let the factor price on capital be given by r, while the labor is paid the real wage w. There is no capital depreciation. All markets are competitive, and firm's maximize profits.

A. Comment on equation (1) and in particular its empirical relevance.

B. Show that all firms in the economy will choose the same capital-labor ratio and proceed to show that aggregate output $Y = \sum_i Y_i$ can be written

$$Y(t) = \tilde{A}^{1-\alpha}K(t),$$

which implies that

$$r = \alpha A^{1-\alpha},$$

and that

$$w(t) = (1-\alpha)\tilde{A}^{1-\alpha}K(t).$$

Consumers are maximizing utility from consumption. They are infinitely lived. Accordingly, the representative households objective is to

$$\max_{\{c\}_0^{\infty}} \int_0^{\infty} \frac{c^{1-\theta}-1}{1-\theta}e^{-\rho t}$$

$$
\begin{gathered}
s.t. \\
c_t \geq 0 \\
\dot{b}(t) = rb(t) + w(t) - c(t), \ b(0) \text{ given} \\
\lim_{t\to\infty} b(t) e^{-rt} \geq 0.
\end{gathered}
$$

$b(t)$ signifies the wealth of the representative households, c is consumption per individual in the household, and θ,ρ are positive parameters.

C. Solve the maximization problem and show that

$$\frac{\dot{c}}{c} \equiv \gamma_c = \frac{1}{\theta}(r-\rho).$$

Comment on this first order condition.

Assume that the parameters of the model is such that $r > \rho$. Moreover, since the economy is closed

$$K = b.$$

D. (i) Show that *if* the capital stock is to grow *at a constant rate* γ_K, then it will have to hold that $\gamma_K = \gamma_c = \gamma$ for all t. (*Hint*: Show that the market clearing condition $K = b$ along with the first order conditions for the firm implies $Y = \dot{K} + C$, i.e the market clearing condition for final goods. Use this equation, the Keynes Ramsey rule and the aggregate production function to answer the question). **(ii)** Would one expect an optimal path for K and c to fulfill $\gamma_K \neq \gamma_c$? (An intuitive argument is fine).

Accordingly, the growth rate of income per worker is given by

$$\gamma = \frac{1}{\theta}\left(\alpha A^{1-\alpha} - \rho\right).$$

E. (i) Is this growth rate socially optimal? If not, what policies could restore the social optimum? **(ii)** is the model consistent with "conditional convergence"? If not, then how could the model be modified so as to imply "conditional convergence" and endogenous growth?

Endogenous Growth through Investments in R&D

Consider an economy consisting of three sectors. A final good sector, an intermediate good sector and an R&D sector. In the final goods sector firm i uses the following production technology:

$$Y_i = AL_{iY}^{1-\alpha} \sum_{j}^{N} x_{ij}^{\alpha}, \ A > 0, \tag{2}$$

where L_{Yi} is the amount of labor allocated to final goods production in firm i. The price of 1 unit of labor is the real wage, w, which the firms takes as given. x_{ij} represents the amount of intermediate good j used by firm i. The price of x_j is P_j.

In the intermediate goods sector 1 unit of final output is used to produced 1 unit of good x_j for all j. It is assumed that each produce of intermediate goods holds a patent of infinite duration.

Finally, the R&D sector uses the following technology

$$\dot{N} = \frac{L_R N}{\eta}, \tag{3}$$

where L_R is the number of researchers in the economy. N is the number of intermediate foods in existence at a given point of time, and η is a positive constant. Moreover, clearing in the labor market implies

$$L = L_Y + L_R.$$

L is constant.

A. Show that aggregate demand for intermediate good j satisfies

$$P_j = \alpha A x_j^{\alpha-1} L_Y^{1-\alpha}$$

B. Solve the intermediate goods producers problem, and show thereby that

$$x_j = \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L_Y$$

$$p_j = 1/\alpha.$$

$$V^j(t) = \int_{v=t}^{\infty} \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L_Y e^{-\int_{\omega=t}^{v} r(\omega)d\omega} dv,$$

where V represents the value of a patent. Comment on these results.

Households are maximizing discounted utility from consumption

$$\int_0^{\infty} \frac{c^{1-\theta}-1}{1-\theta} L e^{-\rho t} dt$$

subject to the requirement that consumption is non-negative, that their total wealth, b, evolves in accordance with $\dot{b}(t) = rb(t) + w(t) - c(t)L$, and subject to $\lim_{t\to\infty} b(t) e^{-rt} \geq 0$. Standard computations leads to the first order condition: $\dot{c}/c = \frac{1}{\theta}(r-\rho)$.

In the market for research free entry prevails. Hence the value of a patent equals the costs of obtaining one idea. There are no transitional dynamics, and N, c and Y grows at identical rates.

C. Calculate the long run growth rate of income per capita, γ, and explain why and how the various parameters affect γ in the manner suggested by the formula you find. Comment on whether the models' predictions are empirically reasonable.