Endogenous growth through R&D

– Empirical issues and extensions

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Lecture notes

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A SIMPLE VERSION OF THE R&D MODEL

Consider the model from B&S Ch. 6 with a slight simplification to fascilitate the discussion.

That is, we have the following structure. First, equilibrium production of each variety:

$$\bar{x} = \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L,$$

Second, total output is, in symmetrical equilibrium:

$$Y = A\bar{x}^{\alpha} L^{1-\alpha} N = A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} NL$$

Third, we have the resource constaint of the economy

$$Y = C + N\bar{x} + \eta\dot{N},$$

A SIMPLE VERSION OF THE R&D MODEL

so new ideas are produced assuming a lab-equipment formulation

$$\dot{N} = \frac{Y_R}{\eta} \equiv \frac{s_R}{\eta} Y,$$

where s_R is endogenous. Finally, here is the simplification:

$$C = (1 - s)\, Y,$$

i.e. in stead of Ramsey-consumers, we have "Solowian" consumption behavior. Inserting the consumption function into the resource constaint, and rearrangeing, gives us

$$Y = (1 - s)\, Y + N\bar{x} + \eta\dot{N} \Leftrightarrow \dot{N} = \frac{\left[s - \frac{N\bar{x}}{Y}\right]}{\eta} Y$$

A SIMPLE VERSION OF THE R&D MODEL

Finally, inserting for $\bar{x}$ and Y leaves us with

$$\dot{N} = \left[\frac{s - \alpha^2}{\eta}\right] Y \equiv \frac{s_R^*}{\eta} Y.$$

Hence, in this version of the model the growth rate of the economy is simply

$$\frac{\dot{N}}{N} = \frac{s_R^*}{\eta}\left(\frac{Y}{N}\right)^* = s_R^* \frac{A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L}{\eta}$$

where obviously $g_N = g_Y$, and due to $C = (1 - s)\, Y$, it clearly follows that $g_C = g_N = g_Y$. That is, balanced growth prevails.

We are therefore left with a couple of testable predictions. (1) All other things remaining equal, a higher investment share in R&D should lead to faster growth. (2) A larger labor force should lead to faster growth.

INVESTMENTS IN R&D AND PROSPERITY

Given the lab-equipment R&D model we essentially have developed some microfoundations for an assumption like

$$\dot{N} = s_R Y.$$

That is, new ideas – or technology – are the result of investments in R&D. Nonneman and Vanhoudt (1996) use this formulation to provide a (further) augmentation of the Solow model, thus taking the empirical work of Mankiw, Romer and Weil (1992) slightly further. In their model s_R is exogenous.

Q: is higher s_R associated with faster growth? Also another motivation for re-visiting MRW: The human capital augmented solow model does not seem to do a particularly good job in explaining productivity differences in the OECD.

INVESTMENTS IN R&D AND PROSPERITY

TABLE V
TESTS FOR CONDITIONAL CONVERGENCE

Dependent variable: log difference GDP per working-age person 1960–1985

Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	3.04	3.69	2.81
	(0.83)	(0.91)	(1.19)
ln(Y60)	−0.289	−0.366	−0.398
	(0.062)	(0.067)	(0.070)
ln(I/GDP)	0.524	0.538	0.335
	(0.087)	(0.102)	(0.174)
$\ln(n + g + \delta)$	−0.505	−0.551	−0.844
	(0.288)	(0.288)	(0.334)
ln(SCHOOL)	0.233	0.271	0.223
	(0.060)	(0.081)	(0.144)
$\bar{R}^2$	0.46	0.43	0.65
s.e.e.	0.33	0.30	0.15
Implied λ	0.0137	0.0182	0.0203
	(0.0019)	(0.0020)	(0.0020)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

Figure 1: MRW p. 426.

The growth regression neither physical capital nor human capital are significant at 5%. In Levels-regressions, the fit becomes progressivly poorer as the sample is limited.

INVESTMENTS IN R&D AND PROSPERITY

Nonneman and Vanhoudt simply adds “knowledge capital" to the list of inputs $Y = K^{\alpha} H^{\beta} N^{\gamma} (AL)^{1-\alpha-\gamma-\beta}$. “A" is an exognous source of technological progress. Stricktly speaking, therefore, the model suggests that changes in s_R (for example) only leads to level effects. Growth effects are interpreted as transitional dynamics.

A property of the steady state is worth fleshing out. Since

$$\frac{\dot{N}}{N} = s_R \frac{Y}{N},$$

and assuming exogenous growth of x percent, labor force growth of n percent implies that, in steady state, $n+x = s_R \left(\frac{Y}{N}\right)^* \Leftrightarrow \left(\frac{Y}{N}\right)^* = \frac{n+x}{s_R}$.

Under competitive markets, the “Return to R&D" is $\gamma \frac{Y}{N}$. Hence

$$r_R^* = \gamma \frac{n+x}{s_R}.$$

INVESTMENTS IN R&D AND PROSPERITY

TABLE IV
LEAST SQUARES ESTIMATION RESULTS. DEPENDENT VARIABLE: $\ln Y_t/Y_0$)

Independent variable	Textbook Solow model $m = 1$	Augmented Solow model $m = 2$	Extended Solow model $m = 3$
Unrestricted			
$\ln Y_o$	−0.343	−0.384	−0.516
	(0.056)**	(0.059)**	(0.085)**
$\ln(S_k)$	+0.650	+0.579	+0.501
	(0.202)**	(0.196)**	(0.183)**
$\ln(S_h)$	—	+0.211	+0.180
		(0.121)*	(0.113)
$\ln(S_r)$	—	—	+0.118
			(0.059)*
$\ln(n + 0.05)$	−0.576	−0.657	−0.449
	(0.291)*	(0.279)**	(0.278)
Constant	+2.967	+3.536	+5.664
	(1.022)**	(1.024)**	(1.421)**
Adj. R^2	0.705	0.734	0.774
s.e.e.	0.133	0.126	0.116

Figure 2: Nonneman and Vanhoudt (1996), p. 950

* s_R does seem to be associated with faster growth (under the model – in transition to steady state).

* In general the augmentation improves the fit. α is estimated to about 1/3.

INVESTMENTS IN R&D AND PROSPERITY

* Compared with MRW the impact from HC is lowered ($\beta \approx .15$)

* Also, they find $\gamma \approx .085$. Plausible? Taking this finding seriously allows for a consistency check. Consider the US: $n = .025, x = 0.02$ and $s_R = 0.025$ gives $r_R^* = .085\frac{.045}{0.025} \approx 0.153$ (or about .20 if we also added a depreciation rate). Is this plausible?

* There is a literature which attempts to estimate r_R *directly*, using industry data. An idea that goes back to Grilliches (1979, Bell Journal of Economics).

To illustrate. First step, diff. the production tech wrt time

$$g_Y = \alpha g_K + \beta g_H + \gamma g_N + (1 - \alpha - \beta - \gamma)(x + n).$$

INVESTMENTS IN R&D AND PROSPERITY

Now suppose indeed $\dot{N}/N = g_N = s_R Y/N$.
Substitute back into the above equation

$$g_Y = \alpha g_K + \beta g_H + \gamma s_R Y/N + (1 - \alpha - \beta - \gamma)(x + n)$$

Since, in theory, $\gamma = r_R N/Y$ we now have

$$g_Y = \alpha g_K + \beta g_H + r_R s_R + (1 - \alpha - \beta - \gamma)(x + n),$$

where s_R are R&D investments in value added. We can think of r_R as a paramter to be estimated (not without problems).

Grilliches find r_R, for the US, to be around 20%; which is roughly consistent with Nonneman and Vanhoudts findings.

INVESTMENTS IN R&D, SCALE AND PROSPERITY

Going back to our expression for the growth rate:

$$g_Y = \frac{\dot{N}}{N} = \frac{s_R^*}{\eta}\left(\frac{Y}{N}\right)^* = s_R^* \frac{A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}L}{\eta}$$

N&V provide some evidence in favor of the prediction that $s_R^* \uparrow$ goes along with $\gamma_Y \uparrow$; at least in the OECD (and at least in transition).

But the model also suggest that $L \uparrow$ should lead to accelerating growth. In more "general R&D models", this would be "R&D labor", not just the labor force. Still, eventually the two would be proportional.

This implication is heavily criticized by Jones (1995), and started the "scale controversy".

INVESTMENTS IN R&D, SCALE AND PROSPERITY

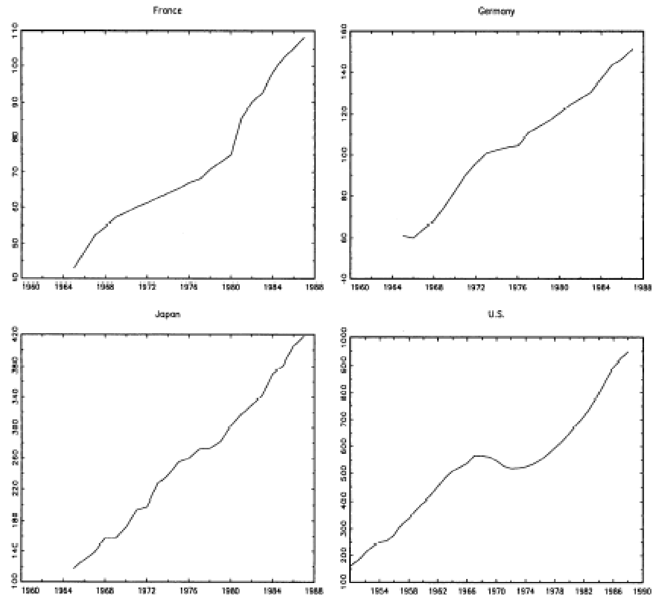

FIGURE IV
Scientists and Engineers Engaged in R&D (1000s)
Source. *NSF Science and Engineering Indicators 1989 and Bureau of the Census* (various).

Figure 3: Jones, 1995; p. 517.

Basic point: R&D labor has increased, but g_Y is US (and other places) have remained stationary. So can we come up with an alternative model?

A **"SEMI-ENDOGENOUS" GROWTH MODEL**

We keep basically the entire Romer framework, leading to**:**

$$g_Y = g_N = s_R^* \frac{A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L}{\eta}.$$

But, assume now that

$$\eta(N) = \phi N^\sigma, \phi > 0, \sigma > 0.$$

That is, suppose it progressively becomes more and more difficult to shift the frontier (to innovate), and therefore more costly to get the next good idea.[1]. We now have

$$g_N = s_R^* \frac{A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L}{\phi N^\sigma}.$$

[1] A version with Ramsey savings is discussed in B&S ch. 6.1.8.

A “SEMI-ENDOGENOUS" GROWTH MODEL

Is constant per capita growth feasible? Yes if L increases. Diff. the growth rate wrt time

$$\frac{\dot{g}_N}{g_N} = n - \sigma g_N = 0,$$

which therefore requires

$$g_N^* = \frac{1}{\sigma} n > n \text{ for } \sigma < 1.$$

Key implications:

1. Per capita growth rate is

$$g_Y^* - n = g_N^* - n = \left(\frac{1-\sigma}{\sigma} \right) n.$$

Hence constant *growth* in labor force (science input) is associated with constant growth in GDP per capita. n is exogenous, but tech. change endogenous (“semi")

A **“SEMI-ENDOGENOUS" GROWTH MODEL**

2. Policies (which matter for e.g. s_R) does not matter for the growth rate; but for the *level* of income per capita. To see this, note that on a balanced growth path

$$g_N^* = \frac{1}{\sigma} n = \frac{s_R^* A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}}}{\phi} \left(\frac{L}{N^\sigma} \right)^*$$

so the level of N

$$N^* = \left[\frac{s_R^* A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}}}{\phi \frac{1}{\sigma} n} \right]^{1/\sigma} L^{1/\sigma}.$$

so

$$\left(\frac{Y}{L} \right)^* = A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} N^* = A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} \left[\frac{s_R^* A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}}}{\phi \frac{1}{\sigma} n} \right]^{1/\sigma} L(0)^{1/\sigma} e^{(n/\sigma)t}.$$

A “SEMI-ENDOGENOUS" GROWTH MODEL

3. Scale still matters, but in a more subtle way: more L implies a higher level of GDP per worker.

4. worrisome prediction: If n declines (or fall), eventually growth in GDP per worker should move in the same direction, since $g_y^* = \left(\frac{1-\sigma}{\sigma}\right) n$.

As a matter of transitional dynamics:

If n *falls* the growth rate in N (thus Y/L) should be either montonically declining, or follow a hump shaped path (Phase diagram).

CONTRASTING EVIDENCE: HA AND HOWITT (2005)

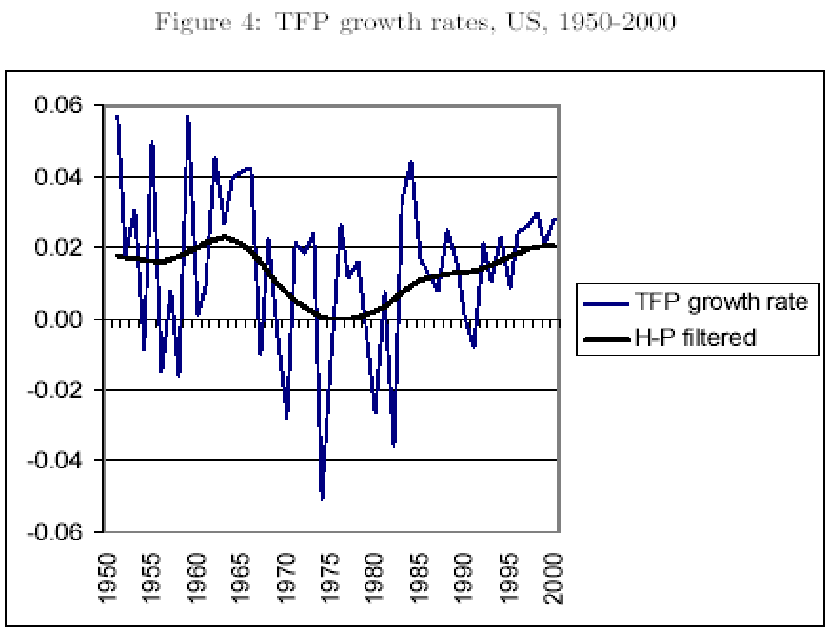

Figure 4: Ha and Howitt, 2005; p. 11

Observation 1: From 1950-2000 *TFP* growth has remained stationary.

CONTRASTING EVIDENCE: HA AND HOWITT (2005)

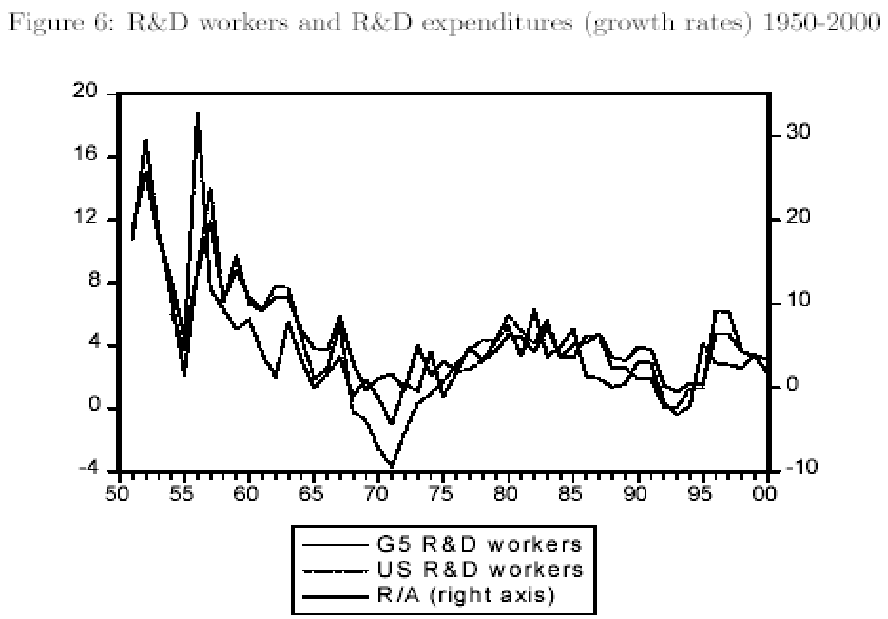

Figure 5: Ha and Howitt, 2005, p. 14

Observation 2: Growth in R&D input (however measured) has not remained constant.

CONTRASTING EVIDENCE: HA AND HOWITT (2005)

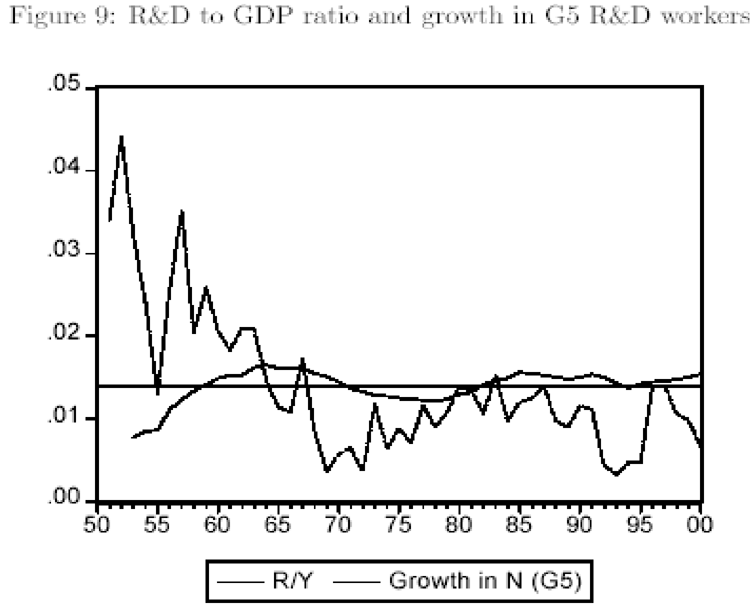

Figure 6: H & H (2005), p. 17

Observation 3: The share of R&D in GDP (i.e. s_R^*) has remained constant. Obs. 1 -3 hard to reconsile with semi-endogenous growth theory

A SIMPLE MODEL WHICH IS CONSISTENT WITH THE EVIDENCE (Dalgaard and Kreiner, 2001)

Final output, Y_t, is produced using human capital augmented labor input, $H_t = h_t L_t$, ideas, N_t, and some fixed factor of production denoted by Z

$$Y_t = H_t^{\alpha} Z^{1-\alpha} N_t^{\beta}, \quad Z \equiv 1, 0 < \alpha < 1, 0 < \beta \leq 1$$

Ideas are produced using units of final output (the 'lab-equipment' framework):

$$\dot{N}_t = s_R Y_t, \quad N_0 \text{ given}$$

The parameter s_R denotes the share of total output invested in R&D.

A SIMPLE MODEL WHICH IS CONSISTENT WITH THE EVIDENCE

Assume: (a) $H_t = h_t L_t$, $\dot{L}_t / L_t = n$, and h_t endogenously growing. (b) $\beta < 1$ and $\alpha + \beta = 1$.

Does perpetual human capital accumulation make sense?

- Human capital is not just quantity of information but also quality

- Complementarity between human capital and scientific knowledge

$\Rightarrow$ If science continues to progress, i.e. $\dot{N}/N > 0$, quality of knowledge may continue to expand.

To capture this in a simple fashion, we assume $\dot{H}_t = s_H Y_t$. Hence,

$$\dot{h}_t = \frac{s_H Y_t}{L_t} - n h_t, \quad h_0 \text{ given}$$

Important congestion effect: More pupils for a given amount of resources on education leads to lower quality growth.

The essentially reason why endogenous human capital formation does not entail new scale effects.

The growth rate of income per capita becomes

$$g_y = s_H^{\alpha} s_R^{1-\alpha} - n$$

Per capita income along the balanced growth path develops according to

$$y_t = \frac{N_t^{\alpha}(h_t L_t)^{1-\alpha}}{L_t} = \left(\frac{N_t}{h_t L_t}\right)^{\alpha} h_t = \left(\frac{s_R}{s_H}\right)^{\alpha} h_0 e^{g_y t}$$

Note: In the microfounded version of the model we can get $g_y = s_H^{\alpha} s_R^{1-\alpha}$. Changes in n leaves growth unaffected.

BOTTOM LINE

Evidence support the notion that R&D matters for GDP per capita growth. Size of the effect not pinned down (OLS). In principle, reverse causality might be lurking here as well

The scale effect prediction of basic R&D models, is not supported by empirical evidence for OECD.

The semi-endogenous growth model is superficially consistent with US evidence. But Ha and Howitt's analysis suggests there might be more to the story.

BOTTOM LINE

Properties of Dalgaard and Kreiner: (1) Policy matter for growth (via s_R, s_H), (2) growing quality of labor force (g_h) and quantity (n) consistent with constant growth – the latter potentially without any impact on g_y. (3) Scale (in the sence of $L(0)$) does not matter for y directly, only $h(0)$.

By now a number of models have been constructed which produce growth without scale effects (see Dalgaard and Kreiner, 2001 for a survey). Different mechanisms are purposed (increasing product proliferation is the leading “story" – see e.g. Howitt, 1999).