Some additional notes on the World distribution of Wealth and the Solow model*

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One of the major conclusions in lecture note 1 is that under the assumptions laid out the distribution of wealth becomes perfectly egalitarian in the limit, if the world economy is converging to the stable steady state k_h^*. It is well worth stressing, again, that this result hinges on us being willing to assume (among other things) the same marginal savings rate, s, identical population growth rates, n, and the same level of subsistence consumption, $\bar{s} < 0$. If any, or all, differ the long run distribution of wealth does not "collapse". Inequality persist; as the note also provides an example of. This result then, is simply "conditional convergence". Nothing more.

Nevertheless, in class one of you remarked, that it is not entirely obvious that "perfect equalization" arise, *even* under the strong set of assumptions made. The argument is the following. Consider the per capita savings of some country c: $s_{ct} = s(w_t + r_t a_{ct}) + \bar{s}$. Now, it clear that this need not be a positive number if (w_t, a_{ct}) are sufficiently small (and/or $\bar{s}$ sufficiently negative). Hence, one might think that some of the poorest countries never succeeds in mobilizing positive savings since initially their wealth is too low to support positive savings, making their stock of wealth even lower in the next period and so on. So the question is simply: do we need to assume "something" about the initial distribution of wealth and income, so as to obtain the "equalization" result? For example, would we need to assume that $s_{c0} > 0$ for all c?

This is a fine question which requires a full answer. In the end we will confirm that there is no need to place *any* restrictions on the initial wealth distribution so as to obtain the stated result. (So in brief, the answer to the question is "no"). Here's why.

Consider the evolution of wealth per capita in country c, evaluated when *the world economy* is in the stable steady state, k_h^* :

$$\frac{a_{ct+1}}{a_{ct}} = \frac{[1 + sr^*]}{1+n} + \frac{[sw^* + \bar{s}]}{1+n}\frac{1}{a_{ct}}.$$

Now, having the world economy in steady state does not imply that the distribution is not changing. But eventually it too will be in steady state. In particular, you may show that when $\frac{a_{ct+1}}{a_{ct}} = 1$ (steady state of wealth in country c)

$$a_c^* = \frac{sw^* + \bar{s}}{n - sr^*}, \tag{1}$$

which is clearly the same across all countries, when $s, \bar{s}$ and n are identical for all c, and given "full integration". For this to be meaningful we need to convince ourselves that $sw^* + \bar{s} > 0$. Consider Figure 1 below, which shows the phase diagram for the model.In

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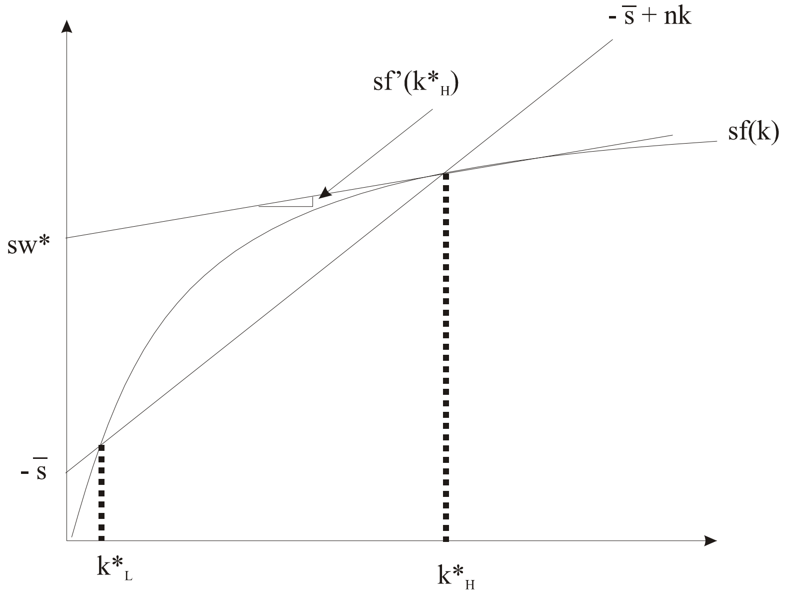

Figure 1: Some geometry of the steady state.

the figure we've drawn a tangent to the function $sf(k)$ in the steady state k_h^*. Formally, this is simply a straight line with slope $sf'(k_h^*)$. Since the line passes through the point $(sf(k_h^*), k_h^*)$, we can pin down the intersection point on the vertical axis as the solution to

$$sf(k_h^*) = sf'(k_h^*)\,k_h^* + x$$

or

$$sf(k_h^*) - sf'(k_h^*)\,k_h^* = x = sw^*$$

Visual inspection of Figure 1 show that $sw^* > -\bar{s}$. Hence, eventually per capita savings will be positive in all countries regardless of whether they were intitially, and all countries will end up (asymptotically) with a_c^*.

To be sure, this does *not* mean that for some country c, savings per capita cannot be negative *intitially*. That is, at $t = 0$ it may hold that $s(w_0 + r_0 a_{c0}) < -\bar{s}$ for some countries.[1] In this case a_{ct} could decline for a long time. But since a_c, even in this case, does not converge to zero in finite time – but only becomes "very small" – the day will come where savings turn positive (due to a rising wage), after which the initially very poor country will start to catch up.

[1]Of course, not for all. Our initial condition on the world average k_0 places some restrictions on the system in this respect.