The Augmented Solow Model Revisited*

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Abstract

This note briefly discusses some recent (re-)investigations of the (augmented) solow model, involving the use of panel data. Two issues are center for attention: (i) country specific effects, (ii) endogenous regressors.

1 Estimating the Solow Model – Again

In a previous note (#2), we derived the convergence equation, based on the Solow Model:

$$\begin{aligned}\ln\left(\frac{Y(t)}{L(t)}\right)-\ln\left(\frac{Y(0)}{L(0)}\right) &= \beta_0+\beta_1\ln\frac{Y(0)}{L(0)} \\ &\quad +\beta_2\ln\left(\frac{s}{n+\delta+x}\right)+\varepsilon\end{aligned}$$

where ε is assumed to be white noise, while the coefficients identify the rate of convergence, λ and capitals share, α:

$$\begin{aligned}\left[xt+\ln A(0)\left(1-e^{-\lambda t}\right)\right] &\equiv \beta_0 \\ -\left(1-e^{-\lambda t}\right) &\equiv \beta_1 \\ \left(1-e^{-\lambda t}\right)\frac{\alpha}{1-\alpha} &\equiv \beta_2,\end{aligned}$$

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MRW takes this equation to a data set consisting of cross-country observations, for the period 1960-85.

A crucial identifying assumption is that ε is uncorrelated with the right hand side variables (cf. Note #3). Recall that ε reflect the MRW assumption of “random" variation is levels of technology; Specifically, the authors assume that

$$\ln A^i(0) = \ln A(0) + \varepsilon^i.$$

Moreover, they present a theoretical argument as to why these disturbances should be independent of, for example, the investment rate: In models where households are equipped with CES preferences and thus endogenous savings, the savings rate turn out to be independent of the level of technology.[1] Hence, the level of technology should not affect s. They make a similar point about n.

The randomness can easily make (intuitive) sense if we think of, say, *climate* mattering for how effective countries are in converting capital and human input into output. Still, even interpreting A as being somehow related to climatic factors, this does not mean that the A's necessarily end up being uncorrelated with current variables. A series of recent papers have argued that geograhic features may in fact have explanatory power in understanding current-day institutions (protection of property right, efficiency of bureaucracy etc).[2] And arguably, the institutional framework may play a role as regards individuals' desire to save and invest.

Imagine, therefore, that $A(0)$ is country specific, and *non-random.* So the idea we now are entertaining is that that $A^i(0)$ captures slow-moving (approximately constant over, say, 30 years) structural charactaristics, like, for example, institutions. How would this view matter when estimating the above equation, while ignoring the fixed effect? The answer is that we have

[1] So in a "Ramsey" model where the felicity function is $u(c) = c^{1-\theta}/1-\theta$, this should be true. It is off course.

[2] Not because geography is crucial per se, but because climate arguably have mattered for the way colonial powers chose to act while being in control of various states. Important contributions include: Hall and Jones (1999) and Acemoglu et al (2001).

an omitted variables problem. In the MRW study, $\ln A^{i}(0)$ is omitted, and this will bias our results. In note #3 we looked at this in the context of a simple cross section regression. It was concluded that if the omitted variable and the right hand side variable exhibits a positive correlation, then the OLS estimator will be biased upward. The "convergence equation" is a dynamic equation - the lagged right hand side variable enters the right hand side. So how does the omitted variables situation play out here?

To see this clearly, let's state the convergence equation in a slightly different way from above:

$$\ln\left(\frac{Y(t)}{L(t)}\right)-\ln\left(\frac{Y(t-1)}{L(t-1)}\right)=\gamma_0+\gamma_1\ln\frac{Y(t-1)}{L(t-1)}+\gamma_2\ln\left(\frac{s}{n+\delta+x}\right)+u^{i} \tag{1}$$

where

$$u^{i}=\varepsilon_t^{i}+\eta^{i}.$$

The term ε_t^{i} is a purely random disturbance, a chock to income in "period t",[3] whereas η^{i} captures the country fixed effect, associated with the term $\ln A^{i}(0)\left(1-e^{-\lambda t}\right)$. Also, here we have specified the model in a slightly different form: In stead of looking at t period differences we are looking at differences of one period. Accordingly, γ_0 is now just "x" while $\gamma_1=-\left(1-e^{-\lambda}\right)$. Note, therefore, we are maintaining the assumption that x is constant – approximately at least – across countries. This ensures that γ_0 is constant.

On this basis, suppose we ignore the fact that we have a country "fixed effect", i.e. η^{i}, and proceed to estimate our cross-section model anyway. The OLS estimator for γ_1 is

$$\hat{\gamma}_1=\gamma_1+\frac{cov\left(\ln\frac{Y(t-1)}{L(t-1)},u^{i}\right)}{var\left(\ln\frac{Y(t-1)}{L(t-1)}\right)}=\gamma_1+\frac{cov\left(\ln\frac{Y(t-1)}{L(t-1)},\varepsilon^{i}+\eta^{i}\right)}{var\left(\ln\frac{Y(t-1)}{L(t-1)}\right)}.$$

[3] This is a little abusive of the framework. Technically we are in continuos time, so there is no such thing as "a period". If you prefer, think of the lag as being of increment Δt, and that we afterwards normalize Δt to one.

Note, however, that lagging equation (1) by one "period", we get

$$\ln\left(\frac{Y(t-1)}{L(t)}\right)-\ln\left(\frac{Y(t-2)}{L(t-2)}\right)=\gamma_0+\gamma_1\ln\frac{Y(t-2)}{L(t-2)}+\gamma_2\ln\left(\frac{s}{n+\delta+x}\right)+\varepsilon_{t-1}^i+\eta^i.$$

The key thing to note here, is that η^i enters into the determination of $\ln\left(\frac{Y(t-1)}{L(t-1)}\right)$. For this reason alone, it will be the case that *cov* $\left(\ln\frac{Y(t-1)}{L(t-1)},\varepsilon^i+\eta^i\right)\neq 0$, and our OLS estimate for γ_1 is biased. The omitted variable issue works exactly in the same way as in the pure cross-section context: A positive correlation between the lagged left hand side variable and the disturbance tends to bias the estimate upwards.

Does this matter? Yes, in that we get a biased estimate for the rate of convergence. To see this, recall that

$$-\gamma_1=1-e^{-\lambda},$$

so the estimated rate of convergence depends on the estimated size of γ_1, i.e. $\hat{\gamma}_1$, in the following way:

$$-\lambda=\ln\left(1+\hat{\gamma}_1\right).$$

The parameter estimate for γ_1 is negative. Consider the case where *cov* $\left(\ln\frac{Y(t-1)}{L(t-1)},\varepsilon^i+\eta^i\right)>0$. That is, *there is a positive correlation between the initial level of income per worker and* $A(0)$. Then our OLS estimate of γ_1 is biased *upward*, meaning that $\hat{\gamma}_1$ *becomes less negative.* The bottom line is that **if the coefficient for $\ln(Y(0)/L(0))$ is biased upward, the rate of convergence is biased downward!** This is visually obvious from Figure 1: The less negative $\hat{\gamma}_1$, the *slower the estimated rate of convergence.*

Question for review: Reconsider the MRW *levels-regression* for the **pure** Solow model, in the light of this discussion. Suppose that s is positively correlated with the omitted variable, $A(0)$. What would this imply for the estimate of α?

More recent investigations, invoking panel data (i.e. using both cross-country and time observations), have tried to adress this omitted variable

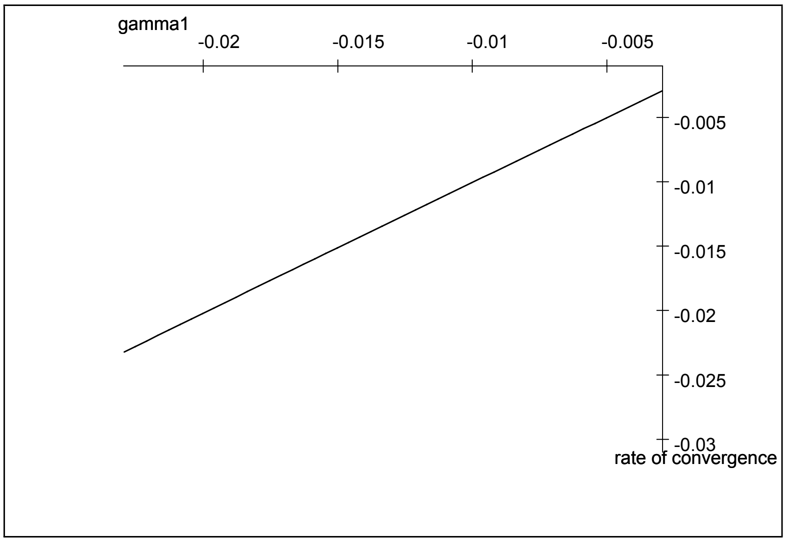

Figure 1:

problem. Accordingly, Islam (1995) provides estimates for the above mentioned fixed effects. Figure 2 shows a scatter plot of these estimates and the log of income per worker in 1960 and 1985, repectively. As is visually clear, log initial income and the estimated level of $\ln A(0)$ ($\ln\_a0$ in Islam's notation) are positively correlated. This supports the suspicion that $cov\left(\ln y(0), \varepsilon^i + \eta^i\right) > 0$. As a result, its not surpricing that Islam estimates the rate of convergence to be around 6 percent – much higher than the roughly two percent finding of MRW. The substantial result is that differences in the level of $A(0)$ are seemingly important determinants of growth and levels of income per worker. This finding, then, represents something of a problem for the "neoclassical view" (Recall: To be understood as featuring an assumption of **common growth and levels of technology**). If differences in $A(0)$ are instrumental in understanding cross-country income and growth differences, then the "neoclassical approach" has little to offer theoretically in ways of understanding these, and, consequently, is a less useful tool.

Another problem is that investments may not only affect growth, but

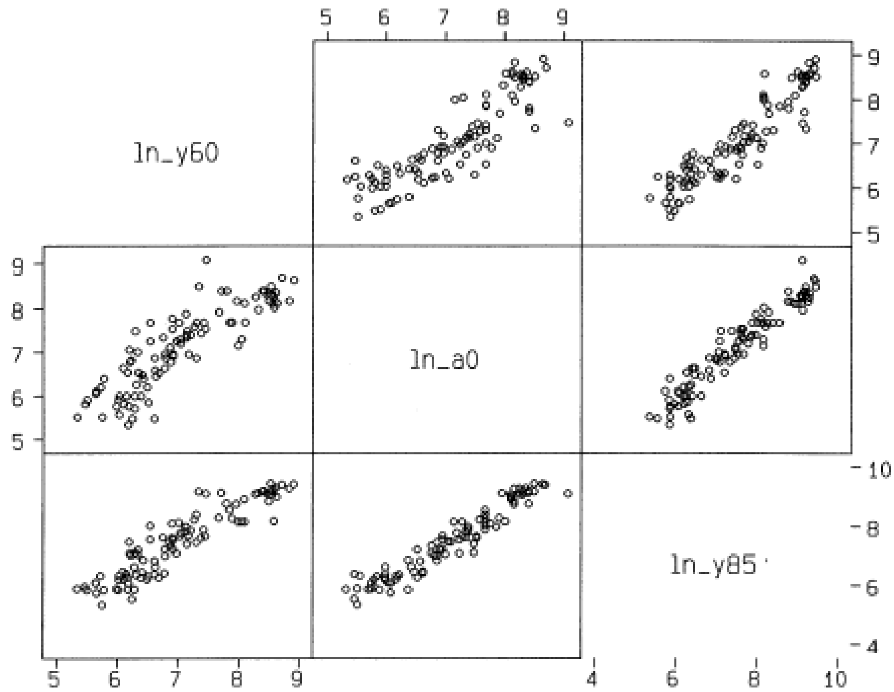

FIGURE I
Scatter Plot Matrix of ln $y60$, ln $a0$, and ln $y85$

Figure 2:

themselves being affected by real activity.[4] In that case, we also face a problem of *endogenous regressors*. As mentioned in lecture note #2, this too will bias our results. However, the size and direction is not *a priori* obvious. Caselli et al (1996) attempts to remedy this short comming (while at the same time adressing the omitted variables issue discussed above), using a more sophisticated estimation stategy (GMM). They reestimate the Solow model (and its augmented version). The key finding is that their estimated rate of convergence comes out in excess of 10 percent! Accordingly, the bias resulting from endogenous regressors, appears to be going in the same

[4]The same thing can be said about investments in human capital and changes in labor market participation (growth in labor force).

direction as the omitted variables bias. Taken at face value, this finding implies that the time it takes for the economy to move half way to its future steady state is

$$t_{1/2} = -\frac{\ln(1/2)}{0.1} \approx 7 \text{ years.}$$

The finding of a much faster rate of convergence makes it harder to appeal to transitional dynamics as being a major force behind long-lasting growth differences. Moreover, to repeat, these studies also indicate that the variation in $A(0)$ is important in understanding growth and income differences. Both of these conclusions makes one somewhat sceptical of the view that the neoclassical growth model is sufficient to understand the central questions of growth.

That said, it should also be noted that moving from pure cross sections to panels is not without potential drawbacks. In traditional cross-sections, income paths over 30 years or so are averaged over. In panel investigations, the length of a period may be as short as 5 years. This means that the results possibly are being influenced by business cycles. As a result, one may worry that, due to the mean-reversing nature of business cycles, shortening the period lenght could contribute to making the estimated rate of convergence faster. The extend to which this is true, however, is (to my knowledge at least) an unresolved issue.

References

[1] Acemoglu, D., S. Johnson and J. Robinson, 2001. The Colonial Origins of Comparative Development: An Empirical Investigation. American Economic Review, 91 , 1369-1401.

[2] Caselli, F, G. Esquivel and F. Lefort, 1996. Reopening the convergence debate: A new look at Cross-Country Growth Empirics. Journal of Economic Growth, 1, 363-89.

[3] Hall, R. and C. Jones, 1999. Why do some countries produce so much more output than others? Quarterly Journal of Economics, 114, 83-116

[4] Islam, N., 1995. Growth Empirics: A Panel Data Approach. Quarterly Journal of Economics, 110, 1127-1170.