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Economics Letters 87 (2005) 145–152

economics letters

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Capital utilization and the foundations of club convergence

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Received 26 May 2004; accepted 19 November 2004
Available online 26 January 2005

Abstract

Multiple steady states may arise in standard neoclassical growth models if the savings rate of wage income exceeds that of capital income. This paper shows that endogenous capital utilization may produce such savings behavior in an otherwise standard Solow model.

Keywords: Economic growth; Capital utilization; Club convergence

JEL classification: O10; O41

1. Introduction

The relative merits of the two competing convergence hypotheses—conditional convergence and (conditional) club convergence—has been a hotly debated issue for more than a decade. Theoretically, this debate is inescapably linked to the properties of the underlying dynamic system which is thought to be governing the evolution of GDP per worker. Whereas the hypothesis of conditional convergence rests on a vision of the growth process involving a unique (globally stable) steady state, the club convergence prediction emerges from models that allow for multiple (locally stable) steady states.

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0165-1765/$ - see front matter
doi:10.1016/j.econlet.2004.11.002

A startling recent paper by Canova (2004) reinvigorates this debate by showing that even within the OECD area one is able to detect distinct pools of attraction, consistent with the club convergence hypothesis.[1] This finding is in many ways rather surprising since most explanations for multiple steady states would seem to be more plausible in the context of far more heterogenous groupings of countries than the OECD sample.[2]

However, as pointed out in Galor (1996) multiple equilibria may arise in standard neoclassical growth models featuring exogenous fertility, fully competitive markets and homogenous technology. A necessary condition for this to occur is that the savings rate from wage income exceeds the corresponding savings rate out of capital income.[3]

The present paper contributes to the literature by providing microfoundations for this exact assumption within the boundaries of a Solow model. More specifically, we demonstrate that if a standard Solow model (where individuals save a constant fraction of current income) is augmented by endogenous capital utilization then not only will individuals, in effect, be saving different fractions of labor and gross capital income; the fraction they save from wage income will be strictly greater than the fraction they save from capital income. As a result, multiple equilibria may arise.

The next section provides the general analysis, whereas Section 3 provides a numerical example of multiplicity. Section 4 briefly concludes.

2. The model

Consider a continuous time version of the Solow model augmented by endogenous capital utilization.[4] Accordingly, the economy is closed, all markets are competitive, and consumers save a constant fraction, s, of their total income; the remaining part is consumed. The work force grows at a constant rate, n, and capital depreciates at the rate $\delta(t)$. There is no technological progress.[5] Output, $Y(t)$, is produced combining labor, $L(t)$, and capital services, $\beta(t)K(t)$

$$Y(t) = F[\beta(t)K(t), L(t)] = L(t)f[\beta(t)k(t)]. \tag{1}$$

$F(\cdot)$ satisfies all the neoclassical properties that assures the existence of an interior steady state. Capital utilization will be endogenously determined in a manner originally suggested by Taubman and Wilkinson (1970). Under this approach, the rate of capital utilization, $\beta(t)\in[0{:}1]$, is to be thought of as the intensity, or speed, at which capital is operated, per unit of time. The key assumption is that

[1] Durlauf and Johnson (1995), Quah (1996), and Kourtellos (2003) also find that the evidence suggests the existence of convergence clubs, but in much broader samples of countries.

[2] For example: Impatience traps, technology traps, traps related to fertility (Azariadis, 1996), or inequality in the presence of credit market imperfections (Galor and Zeira, 1993).

[3] As is well known, this sort of savings behavior is easily obtained within the context of a standard two-period overlapping generations model.

[4] Recent contributions examining the implications of endogenous capital utilization for the convergence process include Rumbos and Auernheimer (2001), Dalgaard (2003) and Chatterjee (2004). In contrast to these papers, however, the present analysis does not impose a specific functional form for the production technology. In this respect the analysis by Calvo (1975) is more closely related to the present one. But Calvo focuses on the existence of a golden rule steady state, when capital utilization is endogenous, and not on the issue of uniqueness of steady states.

[5] It is straightforward to augment the model with exogenous technical change, and nothing of what follows will be affected by this extension.

increasing utilization leads to accelerated capital depreciation and, as a result, to increased user costs of capital, due to the wear and tear on equipment.[6] Following Burnside and Eichenbaum (1996), this idea is formalized by assuming that the rate of depreciation is given by

$$\delta(t) = \delta[\beta(t)] \equiv d\beta(t)^{\phi}, \qquad \phi > 1, \quad d > 0. \tag{2}$$

Accordingly, it is assumed that depreciation is a convex, constant elasticity function of the rate of capital utilization.[7] The maximization problem, formulated in per worker terms, of the representative firm is to find

$$\{k(t), \beta(t)\} = \arg\max\{f[\beta(t)k(t)] - [r(t) + \delta(t)]k(t)\},$$

subject to Eq. (2). In an interior solution the first order conditions for $k(t)$ and $\beta(t)$ are respectively

$$f'[\beta(t)k(t)]\beta(t) = r(t) + \delta(t)$$

and

$$f'[\beta(t)k(t)] = \delta'[\beta(t)]. \tag{3}$$

For future reference, note that the first order condition with regard to $\beta(t)$, using Eq. (2), can be restated to yield

$$f'[\beta(t)k(t)]\beta(t) = \phi\delta(t). \tag{4}$$

Eq. (3) also implies that the following Lemma holds for $0<\beta(t)<1$, $\beta(t)k(t)>0$

Lemma. *(i) The utilization rate is declining in k(t). (ii) Capital services, β(t)k(t), is increasing in k(t).*

Proof. See Appendix A. □

For low values of $k(t)$, the optimal utilization rate is bounded from above by $\beta(t)=1$. The fact that the utilization rate is monotonically decreasing in $k(t)$ implies that there exists a threshold value, $\tilde{k}(t)$, such that the optimal utilization rate is unity for all values of $k(t)$ below $\tilde{k}(t)$. Intuitively, the marginal product of capital services is high when $k(t)$ is low, such that complete utilization of the capital stock becomes optimal.

The equation governing the evolution of capital per worker is

$$\dot{k}(t) = sf[\beta(t)k(t)] - \delta(t)k(t) - nk(t); \quad k(0) \text{ given.}$$

The key difference to the standard model is that $\delta(t)$ now is endogenous.[8] Specifically, from Eq. (4) it follows that $\delta k=[f'(\beta k)k\beta]/\phi$. Using this along with the fact that $sf(\beta k)=s[f'(\beta k)\beta k]+s[f(\beta k)-f'(\beta k)\beta k]$, we may restate the differential equation in the following way

$$\dot{k} \equiv s^{r} f'(\beta k)\beta k + s^{w}[f(\beta k) - f'(\beta k)\beta k] - nk,$$

[6] See Epstein and Denny (1980) and Johnson (1994) for corroborating evidence of this mechanism.

[7] Dalgaard (2003) surveys the evidence on ϕ and provides additional estimates. A plausible range seems to be $\phi \in [1.4;1.7]$.

[8] In the interest of brevity the time indexation is ignored from now on.

where $s^{\mathrm{r}} \equiv s - \frac{1}{\phi} < s^{\mathrm{w}} \equiv s$, and $[f(\beta k) - f'(\beta k)\beta k]$ is the marginal product of labor. Finally, by the Lemma, it follows that we may express this equation solely in terms of available capital

$$\dot{k} \equiv s^{\mathrm{r}} f'[\psi(k)]\psi(k) + s^{\mathrm{w}}\{f[\psi(k)] - f'[\psi(k)]\psi(k)\} - nk,$$

where $\psi(k) \equiv \beta k$ and $\psi'(k)>0$ for all $k>0$. As seen, endogenous capital utilization (on this form) implies that, in effect, people save a smaller fraction of capital income than of labor income. The reason is that total depreciation allowances, δk, increases with gross capital income, $f'(\beta k)\beta k$, but works so as to reduce the amount of capital available per worker. This is why $s^{\mathrm{r}} < s^{\mathrm{w}}$ even if the behavior of households is such that they save a constant fraction of current income. Observe that s^{r} may actually be negative (as in the two-period overlapping generations model).

In the end, the law of motion for k in the present model is clearly very similar to the equation derived by Galor (1996) for the standard Solow model augmented by the assumption $s^{\mathrm{w}} \neq s^{\mathrm{r}}$. Perhaps more importantly, the present microfoundations for the "Galor assumption" implies that the sign of $(s^{\mathrm{w}} - s^{\mathrm{r}})$ is known to be strictly positive for all $\phi>1$.

To investigate under which circumstances the present model may allow for multiple equilibria, we may restate the above differential equation in terms of the growth rate of k

$$\frac{\dot{k}}{k} = G(k) - n,$$

where

$$G(k) \equiv \left(s^{\mathrm{r}} - s^{\mathrm{w}}\right) f'[\psi(k)]\frac{\psi(k)}{k} + s^{w}\frac{f[\psi(k)]}{k}.$$

As is well known, in the standard Solow model the derivative of the equivalent of G would be negative for all k, thus implying a unique (interior) steady state where $\dot{k}=0$. Multiplicity, on the other hand, requires $G'(k)>0$ for some k. After some algebra (see Appendix B) we find the derivative

$$G'(k) = a\left\{\left(s^{\mathrm{r}} - s^{\mathrm{w}}\right)\phi + \frac{s^{\mathrm{w}}}{\alpha_K}[\sigma(k)(\phi - 1) + 1]\right\}, \tag{5}$$

where α_K is capital's share of total income, $\sigma(k)$ is the elasticity of substitution, and $a = \frac{1}{f'k^2}\frac{\psi'k}{(\phi-1)\psi} < 0$. In contrast to the standard Solow model this derivative may not be negative for all k. In fact, $G'(k)$ is positive if and only if

$$\sigma(k)(\phi - 1) < \frac{\alpha_K}{s^{\mathrm{w}}} - 1. \tag{6}$$

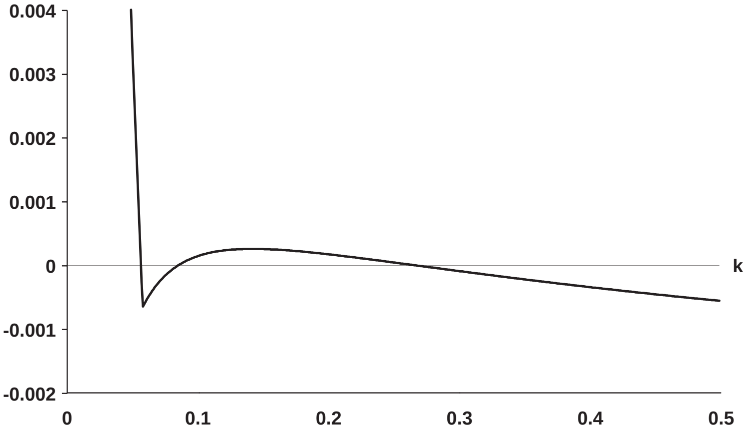

Fig. 1. Numerical example of multiple equilibria.

Depending on the behavior of $\sigma(k)$ and α_K, the above condition can be fulfilled for some k.[9] In other words we cannot exclude the possibility that several stable steady states exist. The next section provides a numerical example of a scenario where multiplicity indeed arises.

3. A numerical example

As can be seen from Eq. (6), multiplicity is possible if capital's share of total income is sufficiently high compared to the elasticity of substitution for some range of k. With a CES-production function α_K is declining in k if the elasticity of substitution is less than one, such that $G'(k)>0$ is possible for low values of k.

If we choose $f(\beta k)=[(\beta k)^{\gamma}+1]^{1/\gamma}$, the differential equation for the growth rate of k becomes

$$\frac{\dot{k}}{k} = [(\beta k)^{\gamma} + 1]^{\frac{1-\gamma}{\gamma}}\left[\left(s - \frac{1}{\phi}\right)\frac{(\beta k)^{\gamma}}{k} + \frac{s}{k}\right] - n,$$

where β is determined by Eq. (4) in all interior solutions. By choosing parameter values $\gamma=-0.25$, $\phi=1.7$, $d=0.08$, $s=0.41$, and $n=0.00375$, as well as numerically solving for the optimal β for each value of k, we obtain the transitional dynamics shown in Fig. 1.

In this case we have two stable equilibria at $k\approx0.057$ and $k\approx0.269$. There is a kink in capital services, $\psi(k)$, at $k\approx0.058$ as the representative firm switches from full capital utilization to a utilization rate less than one. This immediately enhances the growth rate such that it eventually becomes positive, and the possibility of a second stable equilibrium arises.

4. Conclusion

In growth models it is commonly assumed that the capital stock is fully utilized at all points in time. This assumption is clearly dubious from an empirical standpoint. Nevertheless, the simplification is

[9] A Cobb–Douglas production technology will not allow for multiple stable steady states but can allow for global contraction. See Dalgaard (2003) for a treatment of the Cobb–Douglas case.

justified if capital utilization can be said to be “relatively unimportant” to long-run issues. This paper shows that caution is warranted since endogenous capital utilization may contribute to the emergence of multiple stable steady states in an otherwise standard Solow model. From this perspective it seems that capital utilization may matter a great deal, even in a long-run context.

Acknowledgements

We would like to thank Oded Galor for valuable comments. The usual disclaimer applies. The activities of EPRU are financed by a grant from the Danish National Research Foundation.

Appendix A. Proof of Lemma

A very compact proof of this result can also be found in Calvo (1975). Fromthe first order condition (assuming an interior solution and suppressing the time index)

$$f'(\beta k) = \delta'(\beta).$$

Total differentiation

$$\begin{gathered} f''(\beta k)(\beta dk + k d\beta) = \delta''(\beta) d\beta \\ \Updownarrow \\ f''(\beta k)\beta dk + f''(\beta k) k d\beta = \delta''(\beta) d\beta \\ \Updownarrow \\ f''(\beta k)\beta dk = [\delta''(\beta) - f''(\beta k) k] d\beta \end{gathered}$$

Hence

$$\frac{\partial \beta}{\partial k} = \frac{f''(\beta k)\beta}{\delta''(\beta) - f''(\beta k)k} < 0.$$

To show the sign of $\partial(\beta k)/\partial k$

$$\frac{\partial \beta k}{\partial k} = \beta + k\frac{\partial \beta}{\partial k} = \beta\left(1 + \frac{f''(\beta k)k}{\delta'(\beta) - f''(\beta k)k}\right) > 0 \quad \text{for } \beta k > 0.$$

since $\dfrac{-f''(\beta k)k}{\delta''(\beta) - f''(\beta k)k} < 1.$ □

Appendix B. The derivative $G'(k)$

Starting from $G(k) \equiv (s^{\mathrm{r}} - s^{\mathrm{w}}) f'(\psi(k)) \dfrac{\psi(k)}{k} + s^{\mathrm{w}} \dfrac{f(\psi(k))}{k}$ straightforward differentiation yields

$$\begin{aligned} G'(k) &= (s^r - s^w)\left(f''\psi'\frac{\psi}{k} + f'\frac{\psi' k - \psi}{k^2}\right) + s^w \frac{f'\psi' k - f}{k^2} \\ &= \underbrace{(s^r - s^w)\left(\psi\frac{f''\psi' k - f'}{k^2}\right) + s^r \frac{f'\psi' k}{k^2}}_{>0} - s^w \frac{f}{k^2}. \end{aligned}$$

Hence, the sign of G' is generally ambiguous in this framework. However, when $\delta = d\beta^{\phi}$ we have

$$\psi' k = \left(\frac{(\phi - 1)f' - \psi f''}{\psi(\phi - 1)f'} \right)^{-1},$$

which can be substituted into the expression for $G'(k)$

$$\begin{aligned} G'(k) &= \frac{1}{k^2}\psi' k \left\{ \left(s^{\mathrm{r}} - s^{\mathrm{w}}\right)\left(f''\psi + f' - f'\psi\frac{1}{\psi' k}\right) + s^{\mathrm{w}}\left(f' - \frac{f}{\psi' k}\right)\right\} \\ &= \frac{1}{f'' k^2}\frac{\psi' k}{(\phi - 1)\psi}\left\{ (s^{\mathrm{r}} - s^{\mathrm{w}})\phi + s^{\mathrm{w}}\frac{f}{f'\psi}\left(-\frac{f'(f - f'\psi)}{\psi f f''}(\phi - 1) + 1\right)\right\}. \end{aligned}$$

The elasticity of substitution is defined as

$$\sigma(k) \equiv \frac{f'(\beta k)(f(\beta k) - \beta k f'(\beta k))}{\beta k f(\beta k) f''(\beta k)} = -\frac{f'(f - f'\psi)}{\psi f f''},$$

and the share of capital income is defined as

$$\alpha_K \equiv \frac{\beta k f'(\beta k)}{f(\beta k)} = \frac{\psi f'}{f}.$$

This leads us to our final expression

$$G'(k) = \frac{1}{f'' k^2}\frac{\psi' k}{(\phi - 1)\psi}\left\{ (s^{\mathrm{r}} - s^{\mathrm{w}})\phi + s^{\mathrm{w}}\frac{1}{\alpha_K}\left[\sigma(k)(\phi - 1) + 1\right]\right\}.$$

As $s^{\mathrm{r}} = s^{\mathrm{w}} - \frac{1}{\phi}$ the derivative $G'(k)$ is positive if and only if

$$\sigma(k) < \frac{\alpha_K}{s^{\mathrm{w}}(\phi - 1)} - \frac{1}{\phi - 1}.$$

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