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Life-cycle savings, bequest, and a diminishing impact of scale on growth

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ABSTRACT

The present paper shows that the savings motive critically affects the size and sign of scale effects in standard endogenous growth models. If the bequest motive dominates, the scale effect is positive. If the life-cycle motive dominates, the scale effect is ambiguous and may even be negative.

1. Introduction

An important difference between the two workhorse models in macroeconomics, the Diamond model and the Ramsey–Cass–Koopmans (RCK) model, is that they emphasize different motives for saving. In the Diamond model the bequest motive is absent, whereas life-cycle considerations play no role in the RCK framework. If we turn to data for present-day developed economies, the relative importance of bequest and life-cycle savings for capital accumulation remains unresolved (Dynan et al., 2002). Hence, from this perspective it is not obvious which framework is a better stylized representation of the process of capital accumulation. Moreover, the difference is far from trivial because it translates into different links between wage and capital income on the one hand, and the rate of capital accumulation on the other. In an RCK framework all wage income is consumed (along a steady state trajectory), whereas all capital income is consumed in the Diamond model (Bertola, 1993, 1996).

From this emanates radically different answers to questions of first-order importance. Consider the impact of taxes on growth, whereas a capital income tax reduces growth (or long-run income) in an RCK model, it can raise growth in the Diamond model (Uhlig and Yanagawa, 1996; Caballe, 1998). Likewise, the two models hold different predictions with respect to the prospect for cross country income equalization, whereas the steady state is unique in the RCK model, supporting the Conditional Convergence hypothesis, multiple steady states may arise in the Diamond model, supporting the Club Convergence hypothesis (Galor, 1996). Finally, whereas endogenous growth is feasible in convex RCK growth models (Jones and Manuelli, 1990), the same is not true in a Diamond environment (Jones and Manuelli, 1992).

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The present paper demonstrates that the relative importance of bequests and life-cycle savings is crucial for another important issue: the impact of scale on growth. Within a standard endogenous growth framework the scale effect is positive if the bequest motive for savings is paramount. But as we show, this conclusion is potentially overturned if life-cycle considerations influence savings decisions. Then the scale effect is ambiguous, and under plausible conditions it may be absent and even negative.

According to UN projections global population growth will continue to decline in the years to come. Indeed, according to some projections global de-population can be expected after 2040. What will be the implications for economic growth? Naturally, the answer is narrowly connected to how scale impacts on productivity growth. The analysis demonstrates that understanding the savings motive will help provide an answer; if savings mainly are based on life-cycle considerations the growth prospects for the future are far brighter than if savings mainly are undertaken in order to fund bequests.

The analysis employs a one sector overlapping generations model featuring endogenous growth. For ease of exposition the (baseline) analysis invokes an externality from the aggregate stock of capital yielding a simple "AK" production technology in reduced form.[1] People live for two periods. They derive utility from consumption in both periods, and from passing on bequest (i.e., "joy-of-giving"). This specification allows us to parameterize the strength of the bequest motive, relative to the life-cycle motive. Within this setup we demonstrate that unless the bequest motive is sufficiently strong, the impact from scale on growth is ambiguous. The intuition for this result is most readily explained in the special case of a Diamond model with Cobb–Douglas preferences. In the Diamond setting, bequest is absent by construction and savings' only role is to fund retirement (only the life-cycle motive is operative). As a result, the all-important driving force behind capital accumulation becomes wage income; the return to capital does not matter due to the absence of a bequest motive and because preferences are Cobb–Douglas.[2]

Increasing the labor force implies that more individuals are saving resources for old age which stimulates capital accumulation. At the same time, however, increasing the labor force entails diminishing marginal returns to labor input, which works so as to reduce the wage rate, savings, and capital accumulation. The impact on growth from an increasing population therefore depends on which of these two effects dominate. Formally, a larger labor force leaves growth *unaffected* if the elasticity of labor demand is equal to 1, or equivalently, if the elasticity of substitution between capital and labor equals capitals' share of income.

More generally, we provide numerical experiments which serves to quantify the significance of scale effects, when both motives for savings are operative. We find that under reasonable assumptions the scale effect is quantitatively small and diminishes as the size of the labor force increases. When bequest as well as life-cycle considerations influence aggregate savings, the impact from scale may change due to a changing factor distribution of income. If the elasticity of substitution is below one, a rising labor force will increase capital's share of national income and thus the (numerical value of the) elasticity of labor demand. When the elasticity of labor demand becomes sufficiently large, the scale effect changes sign. Hence, the scale effect may be positive when the labor force is "small" and turn negative when the labor force becomes sufficiently large.

It is important to stress that, in general, the correct measure of "scale" is not the labor force *per se*, but rather, the labor force in efficiency units. This is demonstrated in an extension to the baseline model, where growth is fueled by capital accumulation and (government financed) R&D effort. Importantly, the general points above turn out to be unaffected by such extensions.

This paper is related to the by now considerable literature on scale effects (see, e.g., Jones, 1995; Young, 1998; Dinopoulos and Thompson, 1998; Peretto, 1998; Howitt, 1999; Dalgaard and Kreiner, 2001; Peretto and Smulders, 2002; Strulik, 2005). A common feature of all of these contributions is that they are cast within an RCK framework, where the life-cycle motive for savings is absent. The importance of this modeling choice appears to be under-appreciated in the literature. In fact, it would appear that the scale issue has not been explored systematically within a framework where the life-cycle motive is operative. The present paper corrects this error of omission, and proceeds to show how the motive for savings have bearing on the issue at hand.

The paper is also related to another strand of literature, cited above, which highlights the importance of the savings motive for macroeconomic outcomes (Bertola, 1993, 1996; Caballe, 1998; Galor, 1996; Jones and Manuelli, 1992; Uhlig and Yanagawa, 1996). Whereas previous results have revealed its importance for growth and taxes, convergence, as well as the sustainability of growth through capital accumulation, the present paper demonstrates that it is also central to the scale effect property.

2. The model

Consider a closed economy where activity extends infinitely into the future, but where each individual lives for only two periods. Time is discrete, and denoted by $t = 1, 2, \ldots$. The economy produces a homogenous good that is either consumed or

[1] More generally, however, our argument pertains to a larger class of endogenous growth models that have the AK-structure as their ultimate form. For example, it is straightforward to show that a Romer (1987) model, featuring growth due to increasing specialization, can be reduced to an AK-model. See also the R&D driven endogenous growth model developed in Barro and Sala-i-Martin (1995, Chapter 6).

[2] With a unitary elasticity of intertemporal substitution (Cobb–Douglas preferences) the savings rate is independent of the real rate of interest.

saved/invested. The markets for output and factors of production, labor and capital, are competitive. The size of the population is assumed to be exogenously given and constant.

2.1. Firms

The representative firm produces output, Y_t, by combining capital, K_t, and labor, L:

$$Y_t = F(K_t, \bar{K}_t L), \tag{1}$$

$\bar{K}_t$ is an externality which will equal the aggregate stock of capital in equilibrium.[3] When optimizing producers take $\bar{K}$ as given. $F(\cdot)$ exhibits constant returns to rival inputs: K_t and L. In addition, we assume that F is twice differentiable in both arguments, and exhibits diminishing returns to capital and labor. If we define $F(1, \bar{K}_t L/K_t) \equiv f(K_t L_i/K_t)$ total output of the representative firm can be written

$$Y_t = K_t f\left(\frac{\bar{K}_t L}{K_t}\right).$$

The producers will acquire capital and hire labor until the (private) marginal products equals the user cost of capital, $r_t + \delta$, and the real wage, w_t:

$$r_t + \delta = f\left(\frac{\bar{K}_t L}{K_t}\right) - f'\left(\frac{\bar{K}_t L}{K_t}\right)\frac{\bar{K}_t}{K_t/L} = \partial Y/\partial K,$$

$$w_t = f'\left(\frac{\bar{K}_t L}{K_t}\right)\bar{K}_t = \partial Y/\partial L.$$

For the sake of brevity, we will assume that capital depreciates fully during a period: $\delta = 1$.

In equilibrium, where $K_t = \bar{K}_t$, it follows that

$$1 + r = f(L) - Lf'(L) \tag{2}$$

and

$$w_t = f'(L)K_t. \tag{3}$$

In addition, the aggregate production function simplifies to $Y_t = f(L)K_t$.

Notice the impact from scale on equilibrium factor prices. The real rate of interest is increasing in the size of the labor force. This follows from the fact that capital and labor are complements in the production function. In contrast, the wage is decreasing in L, for K_t given ($f''(L) < 0$). This follows from diminishing returns to labor input.

2.2. Consumers

In their first period of life, individuals supply one unit of labor in-elastically for which they receive a wage, w_t. They also receive bequest from their parent, b_t. On this basis the consumers divide their first period income between consumption today, c_t^1, and savings s_t. In the second period of life, individuals divide their capital income, $(1+r)s_t$, between consumption and bequest for the off spring. That is $c_{t+1}^2 = (1+r)s_t - b_{t+1}$.

We assume preferences are CES, and that individuals derive utility from own consumption during their life, and from passing on bequest:

$$U(c_t^1, c_{t+1}^2, b_{t+1}) = \frac{(c_t^1)^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\left[\frac{(c_{t+1}^2)^{1-\theta} - 1}{1-\theta} + \kappa\frac{b_{t+1}^{1-\theta} - 1}{1-\theta}\right]. \tag{4}$$

The parameters fulfill: $\theta > 0$, $\rho > 0$, and $\kappa \geqslant 0$.

An alternative to the above approach, would be to assume households are altruistic. That is, individuals care about the utility from own consumption, and about the *utility* of descendents. In this environment it can be shown that if the weight placed on future generations is sufficiently small bequests are zero. In this case the economy behaves as described by a Diamond model. However, if utility within and across generations are discounted at (say) the same rate, bequests will be passed on. In this case the economy behaves as described by the RCK model, and the influence from life-cycle considerations washes out (see Blanchard and Fisher, 1989, Chapter 3).

We adopt the joy-of-giving specification for two reasons. First, we wish to study the influence from the *relative* strength of the bequest motive on scale effects. We can accomplish this in a simple way by varying κ: the utility value from passing on bequest relative to own old-age consumption. Second, one may argue the joy-of-giving specification fits the data better than the altruistic household version (Altonji et al., 1997).

[3] See Barro and Sala-i-Martin (1995, Chapter 4) for an analysis involving this exact technology, though embedded in an RCK model.

Maximizing (4), with respect to savings and bequest and subject to the budget constraints, leads to the following two first order conditions:

$$b_{t+1} = \kappa^{1/\theta} c_{2t+1}$$

and

$$c_{2t+1} = \left(\frac{1+r_{t+1}}{1+\rho}\right)^{1/\theta} c_{1t}.$$

Straightforward computations lead to the following expression for the savings of the young:

$$s_t = \left[\frac{\left(\frac{1+r}{1+\rho}\right)^{1/\theta}(1+\kappa^{1/\theta})}{1+r+\left(\frac{1+r}{1+\rho}\right)^{1/\theta}(1+\kappa^{1/\theta})}\right](b_t + w_t) \tag{5}$$

and bequests

$$b_{t+1} = \kappa^{1/\theta}\left[\frac{(1+r)\left(\frac{1+r}{1+\rho}\right)^{1/\theta}}{1+r+\left(\frac{1+r}{1+\rho}\right)^{1/\theta}(1+\kappa^{1/\theta})}\right](b_t + w_t). \tag{6}$$

Eqs. (5) and (6) imply that

$$b_{t+1} = \frac{\kappa^{1/\theta}}{1+\kappa^{1/\theta}}(1+r)s_t. \tag{7}$$

Next, insert Eq. (7) (lagged) into Eq. (5), and employ the equilibrium condition $s_{t-1}L = K_t$. The result is the following expression for total savings:

$$S_t = s_t L = s^w w_t L + s^r (1+r) K_t, \tag{8}$$

where the marginal savings rates from labor income (s^w) and capital income (s^r) are given by

$$s^w \equiv \frac{(1+\kappa^{1/\theta})(1+r)^{(1-\theta)/\theta}}{(1+\rho)^{1/\theta} + (1+r)^{(1-\theta)/\theta}(1+\kappa^{1/\theta})}, \quad s^r \equiv \frac{\kappa^{1/\theta}}{1+\kappa^{1/\theta}} s^w. \tag{9}$$

It is easy to see that $\partial s^w/\partial r, \partial s^r/\partial r \gtreqless 0$ when $\theta \lesseqgtr 1$.

The key thing to observe from Eqs. (8) and (9) is the relationship between κ, factor incomes, and savings. The marginal propensity to save from the two sources of income is determined by κ; the utility weight on bequest. As κ rises, s^r/s^w increases, hence the bequest motive becomes stronger and capital income becomes a more important determinant of savings. This shows the close links between the motive for saving, the distribution of factor income, and aggregate savings.

2.3. Steady state growth and scale effects

The capital stock at time $t+1$ is given by the total savings of the young in period t. Thus, inserting Eqs. (2) and (3) (along with the assumption that $\delta = 1$) into Eq. (8), we obtain—after some rearrangements—the following expression for the growth rate of the capital stock (and therefore output):

$$g_Y = g_K = s^w\left[f(L)\frac{\kappa^{1/\theta}}{1+\kappa^{1/\theta}} + \frac{1}{1+\kappa^{1/\theta}} f'(L)L\right], \tag{10}$$

where $g_X \equiv X_{t+1}/X_t$.

Notice that whereas $f(L)$ is increasing in L, $f'(L)L$ is not unambiguously increasing in the labor force. Moreover, as κ is lowered, greater relative weight is attached to the second term. We have the following result:

Theorem 1 (*Scale and growth when households save for retirement and bequest*). *The effect on the long-run growth rate from an increase in the labor force will be positive, zero, or negative, depending on whether*

$$\varepsilon_{s,L}(\kappa^{1/\theta} + 1 - \alpha_K) + (1-\alpha_K)[(1+\kappa^{1/\theta}) - \alpha_K \sigma^{-1}] \gtreqless 0. \tag{11}$$

$\varepsilon_{s,L} \equiv (\partial s^w/\partial L)(L/s) \gtreqless 0$ for $\theta \lesseqgtr 1$, $\sigma \equiv -f'(L)(f(L) - Lf'(L))/f''(L)Lf(L)$ is the elasticity of substitution, while $\alpha_K \equiv (f(L) - Lf'(L))/f(L)$ is capital's share of total income.

Proof. Differentiation of Eq. (10) yields

$$\frac{\partial s^w}{\partial L}\left[f(L)\frac{\kappa^{1/\theta}}{1+\kappa^{1/\theta}} + \frac{1}{1+\kappa^{1/\theta}} f'(L)L\right] - s^w f'(L)\left[\frac{1}{1+\kappa^{1/\theta}}\frac{-f''(L)L}{f'(L)} - 1\right] \gtreqless 0,$$

which is equal to the condition

$$s^w f'(L)\left\{\frac{\partial s^w}{\partial L}\frac{L}{s^w}\left[\frac{f(L)}{f'(L)L}\frac{\kappa^{1/\theta}}{1+\kappa^{1/\theta}}+\frac{1}{1+\kappa^{1/\theta}}\right]-\left[\frac{1}{1+\kappa^{1/\theta}}\frac{-f''(L)L}{f'(L)}-1\right]\right\}\gtreqqless 0.$$

Now, notice that $f'/f''L=(f'(L)(f(L)-Lf'(L))/f''(L)Lf(L))(f(L)/(f(L)-Lf'(L)))=-\sigma/\alpha_K$, since $\sigma\equiv -f'(L)(f(L)-Lf'(L))/f''(L)Lf(L)$ and $\alpha_K\equiv rK/Y=f(L)/(f(L)-Lf'(L))$. Thus the condition can be restated to yield

$$\frac{s^w}{1+\kappa^{1/\theta}}\left\{\varepsilon_{s,L}\left[\frac{\kappa^{1/\theta}}{1-\alpha_K}+1\right]-\left[\frac{\alpha_K}{\sigma}-(1+\kappa^{1/\theta})\right]\right\}\gtreqqless 0,$$

from which the above stated condition is easily obtained. □

The following special case (Cobb–Douglas preferences) is a convenient starting point for an interpretation of the condition stated in the Theorem.

Corollary 2 (*Cobb–Douglas preferences*). *If $\theta=1$ the effect on the long-run growth rate from an increase in the labor force will be positive, zero, or negative, depending on whether*

$$1+\kappa\gtreqqless\frac{-f''L}{f'}=\frac{\alpha_K}{\sigma}. \tag{12}$$

As is apparent from the corollary, the scale effect is positive, *ceteris paribus*, if κ is sufficiently high. In the special case where the bequest motive is absent ($\kappa=0$), scale ceases to matter if $\alpha_K=\sigma$, which is the same as saying that the elasticity of labor demand equals 1. The intuition for this result is the following. When the bequest motive is absent, savings are based on the life-cycle motive alone. Consequently, savings are funded solely by wage income. While a larger labor force in itself spurs growth as the number of saving individuals rises, diminishing returns to labor input leads to lower wages and so has an opposing effect on savings and growth. In general, the sign of the impact from L on growth is ambiguous and depends on which of these two effects that dominates the other; when the elasticity of labor demand equals 1, the two effects precisely cancel each other out yielding *zero* impact from scale on growth. Notice that because the externality comes in the shape of the *aggregate* capital stock (Eq. (1)), the explained ambiguity is not in any way related to the well-known "trick" of assuming that the externality is a function of the capital–labor ratio (e.g., Barro and Sala-i-Martin, 1995, Chapter 4). Notice also that the ambiguity only arises when the elasticity of substitution is smaller than one. Hence, if the production function is assumed to be Cobb–Douglas, the scale effect will be unambiguously positive, even when bequests are absent.

When the bequest motive is operative ($\kappa>0$), savings are also influenced by capital income. Since the real rate of interest is increasing in L (due to capital–labor complementarity), an increase in L will through this channel have a positive effect on savings, hence growth. Accordingly, the previously explained negative effect due to the decreasing marginal product of labor must now content with two channels through which an increase in L increases savings: the bequest channel as well as the direct effect of an increasing work force on total wage income which works through the life-cycle motive. So is it not surprising that unity elasticity of labor no longer guarantees a zero scale effect. Instead, $-f''L/f'>1$ is required, or equivalently $(1+\kappa)\sigma=\alpha_K$.

When moving to the more general case where preferences are not assumed to be Cobb–Douglas ($\theta\neq 1$), the condition becomes more complex, since changes in L affects the savings *rate* via the real rate of interest. Since the real interest rate is increasing in the size of the labor force ($\partial r/\partial L=-f''(L)L>0$), the sign of $\partial s/\partial r$ will determine whether the first term in (11) works so as to promote or retard the scale effect. In general the scale effect is dampened when $\theta>1$; if $\theta>1$ the income effect is dominating the substitution effect, and so the savings rate falls when the interest rate increases.

While $\theta>1$ is often assumed in the literature on economic growth, available evidence suggest that changes in real rates of interest lead to only minor changes in the savings rate, if any at all.[4] The finding that $(\partial s/\partial r)r/s\approx 0$ is (in the present framework) consistent with Cobb–Douglas preferences: $\theta=1$. As a result, in what follows we shall focus on Cobb–Douglas preferences which leads to the conditions for a positive, zero, and negative scale effect stated in the corollary.

2.4. Numerical experiments

The discussion of the previous section establishes that scale may not matter to growth at all provided that the life-cycle motive is operative. In fact, the scale effect may even be negative, and its sign will in general depend on the *size* of the work force. But how quantitatively important is this effect? This is the question to which we now turn. As

[4] Modigliani (1986, p. 304. emphasis in original) states in his Nobel lecture that "... despite a hot debate, no convincing general evidence either way have been produced, *which leads me to the provisional view that s* [the savings rate] *is largely independent of the interest rate*." Arestis and Demetriades (1997) surveys the literature and reaches a similar conclusion.

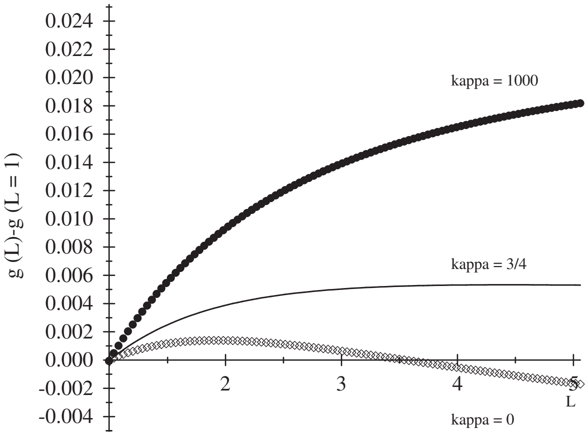

Fig. 1. The figure shows the impact on growth from varying the size of the labor force. *Notes*: The assumed production technology is CES. The solid line assumes $\kappa = 3/4$, the diamond line assumes $\kappa = 0$, whereas the dotted line assumes $\kappa = 1000$. The other parameter values are $\rho = 0.8$, $\sigma = 0.4$, $\beta = 0.2$, and $\theta = 1$. One generation is assumed to be 30 years.

motivated at the end of the previous section, we restrict attention to Cobb–Douglas preferences, which simplifies the analysis considerably.

In order to proceed we need to specify the production function, and choose a set of parameter values. In terms of technology, we assume output is produced by the following CES production function:

$$Y = F(K, \bar{K}L) = (\beta K^{(\sigma-1)/\sigma} + (1-\beta)(\bar{K}L)^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}.$$

Given this technology and $\theta = 1$ (Cobb–Douglas preferences), we get the following expression for the growth rate:

$$g_K = \frac{(1+\kappa)[\beta + (1-\beta)L^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)} - [\beta + (1-\beta)L^{(\sigma-1)/\sigma}]^{1/(\sigma-1)}\beta}{2+\rho+\kappa}.$$

This is the growth rate of capital (and output) from one generation to the next. In the numerical experiments below we will focus on annual growth rates. Accordingly, we need to pin down the length of a generation; we choose 30 years. In addition we need parameter values for κ, β, σ, and ρ.

Starting in reverse order, we pick 0.8 for ρ, which is equivalent to a 2 percent annual discount rate over a 30 year period. Based on the estimations of Chirinko et al. (2002) we put $\sigma = 0.4$. The weight parameter in the production function, β, is put at 0.2. This value is chosen so as to be able to generate reasonable values for capital's share of national accounts, which in the present case is given by $\alpha_K = \beta/[\beta + (1-\beta)L^{(\sigma-1)/\sigma}]$. This leaves κ. In order to pick a realistic value for the importance of the bequest motive, we rely on DeLong (2003, Fig. 2-1) who estimate that the contemporary share of bequest in total wealth is 43 percent. Using Eq. (7) and employing $s_{t-1}L = K_t$ one obtains

$$\frac{b_t}{k_t}\left(\frac{1}{1+r}\right) = \frac{\kappa}{1+\kappa},$$

where $k_t \equiv K_t/L$. Assuming $b_t/[(1+r)k_t]$ is 0.43, we get a value for κ of (roughly) 0.75.

Fig. 1 shows the impact on annual growth from varying the labor force by a factor of five, for three different values of κ: 0 (i.e., no bequest), 3/4 and 1000. The latter is intended to approximate "perfect altruism" where the bequest motive completely dominates the life-cycle motive.

Several features of the figure are worth noting. First, with a high value for κ the growth rate is monotonically rising in L. The effect is substantial: doubling the labor force (from one to two) increases the growth rate by 0.9 percentage points. Second, however, when the bequest motive is absent ($\kappa = 0$) the scale effect has a much smaller impact on growth. Indeed it is seen that in the Diamond environment, a doubling of the labor force from one to two only produces an increase in the growth rate of 1/5th of a percentage point. More substantially, the scale effect is seen to turn negative as L passes a certain threshold, namely $L = 1.84$. Finally, the "intermediate" case in the figure is where *both* motives for saving are present. The outcome is basically a convex combination of the two extremes we just described. For $\kappa = 3/4$ growth increases by 0.4 percentage points, when L rises from one to two. Increasing the labor force to four, however, only raises growth by additional 2/10th of a percentage point. Increasing L further still, to eight, *lowers* the growth rate by 0.1 percentage points. Hence, the scale effect peters out, as the size of the labor force rises.

To fully understand why the size of the scale effect changes with L, note from Eq. (12) (which is the critical condition in the present case) that the inequality depends on the factor distribution of income, which is affected by L. Mechanically, as L

raises capital's share in national income, and thus the elasticity of labor demand, the scale-effect limiting "diminishing returns channel" becomes increasingly powerful. Hence, when α_K is "small" scale effects are positive, but if the share becomes sufficiently large the scale effect may turn negative.[5] The key point is that the scale effect may go from positive to negative as the labor force expands.[6]

In addition to the previous endogenous channels, the scale effect will of course become progressively less important if the savings motive changes. In the present model, however, this can only be captured by an *ad hoc* change in the parameter κ (and specifically, a decrease in κ corresponds to a dampening of the bequest motive). In terms of the previous figure we can think of this as a situation where the economy starts at, say, the $\kappa = 1000$ schedule and moves down toward the $\kappa = 0$ schedule (as a consequence of some or other mechanism); at any given level of L the slope of the schedule relating L to growth is declining as κ falls.[7]

How sensitive are the results to changes in the underlying parameter values? The assumption $\theta = 1$ is clearly not driving the results of a modest scale effect. On the contrary, as explained in the previous section: when $\theta > 1$ (the realistic alternative) the scale effect is further dampened. So if we believe that $\theta > 1$, Fig. 1 actually *overestimates* the impact from scale on growth.

The assumption that $\sigma < 1$ is important for the quantitative nature of the results, but not for qualitative point that the life-cycle motive dampens the scale effect. Suppose, for example, that we chose $\sigma = 1.2$ (which is already a bit extreme from an empirical standpoint, where estimates for σ generally tend to be smaller than 1). Of course when $\sigma > 1$, the scale always increases growth in the model developed above (cf. the corollary, noting $\alpha_K < 1$). When $\kappa = 3/4$, an increase in L from one to two will now induce the growth rate to increase by 0.7 percentage points; nearly twice as large as that obtained for $\sigma = 0.4$. However, with $\kappa = 1000$ the same increase in L would instigate a growth acceleration of 1.1 percent per year. Accordingly, it remains true that the life-cycle motive dampens the size of the scale effect, regardless of the value chosen for σ.

In sum, even though the model admits scale to influence the growth rate, the size of the effect is small under reasonable parameter values. That is, if both the bequest and life-cycle motive are operative. If the bequest motive is the sole force behind savings the situation is rather different. Scale spurs growth, and the impact is substantial.

There are two objections to the model investigated so far that may warrant extensions. First, the notion of scale (the size of the labor force L) is overly simplistic. In a more general framework where growth is not *only* driven by capital accumulation, the appropriate measure of "scale" is modified. Second, the existing literature on scale effects have focused on R&D driven growth. It is therefore worthwhile to introduce endogenous R&D, so as to examine the robustness of the results obtained above. In the model developed in the next section growth is therefore fueled by capital accumulation and (government funded) R&D. As will become clear, all of our general conclusions are unaffected by such a change of framework.

3. Endogenous R&D

In this section we maintain the assumption that consumers derive utility from consumption during youth, old age, and from bequest. Moreover, we also maintain $\theta = 1$; Cobb–Douglas preferences. As a result, savings of the young—determined by wages *and* bequest received—fuel capital accumulation. In the absence of technological progress, however, growth will eventually cease. To sustain growth, technology therefore needs to progress.

Accordingly, the first new element to the model above is that we modify the production side of the economy. Specifically, we assume the production function of the representative firm is

$$Y_t = F(K_t, A_t L).$$

Hence, the externality is replaced by A, technological knowledge. When maximizing profits the firm takes A_t as given. This leaves us with two factor demand equations:

$$r_t = f(x_t L) - x_t L f'(x_t L),$$

$$w_t = f'(x_t L) A_t,$$

where $x_t \equiv A_t / K_t$.

[5] In the experiment above, capital's share is 0.2, 0.4 and 2/3 for $L = 1,\ 2$, and 4, respectively. These numbers are not completely outlandish. Judged from cross-country data capital's share varies quite a bit. While this variation is not systematically related to income per capita it is quite substantial, easily ranging from 0.2 to 0.5 (see, e.g., Aiyar and Dalgaard, 2008).

[6] Note that the process whereby the scale effect is dissipated through changing relative factor prices, would plausibly be very slow. Suppose the growth rate of the labor force is 1 percent per year. If so it would take about 160 years to increase the labor force fivefold.

[7] Interestingly, DeLong (2003) calculates that in preindustrial Eurasia bequest likely accounted for 90 percent of total wealth, compared with 43 percent today. While the exact numbers may be debated, it would in fact seem reasonable to assert a diminished role of bequest in the very long run, which could crudely be captured by a declining κ. As a consequence the impact from scale should diminish. Superficially at least, this could reconcile empirical evidence suggesting scale effects at the country level in pre-industrial times (e.g., Simon and Sullivan, 1989; Baker, 2007) with the contemporary lack of evidence in favor of the same (e.g., Jones, 1995; Rose, 2006).

The second new element relates to the evolution of A_t over time. Following Antinolfi et al. (2001), we assume that A_t expands as the result of public investments in R&D.[8] Antinolfi et al. (2001) assume that these investments are funded by a tax on the income of the young. In their model this is equivalent to a tax on wage income. In our model, in contrast, period 1 income also comprises bequests. As a result, budget balance then implies that

$$A_{t+1} = I_{A,t} = \tau(w_t + b_t)L,$$

where $\tau > 0$ is the (constant) tax rate on total period 1 income. This specification is consistent with a "fishing out" view of the research process; a given relative increase in the stock of knowledge gradually becomes more expensive as the stock of knowledge, A_t, expands. This can readily be seen by dividing through by A_t in the technology above. Unless $I_{A,t}$ rises growth in A_t will come to a halt.[9]

Given Cobb–Douglas preferences, and the presence of a tax on first period income, aggregate savings (and thereby capital in period $t+1$) is given by

$$K_{t+1} = S_t = s_t L = \left(\frac{1+\kappa}{2+\kappa+\rho}\right)(1-\tau)(w_t + b_t)L.$$

As should be clear, under this set of assumptions the ratio of the stock of knowledge to that of physical capital, is constant at all points in time. Specifically $x_t = \bar{x} \equiv (\tau/(1-\tau))(2+\kappa+\rho)/(1+\kappa)$.

Consequently, it can be shown (by substituting for $b_t L = \kappa/(1+\kappa)(1+r_{t+1})s_{t-1}L = \kappa/(1+\kappa)(1+r_{t+1})K_t$ and using equilibrium factor prices) that the growth rate of the economy will be given by

$$g_A = g_K = (1-\tau)s\left[\frac{\kappa}{1+\kappa}f(\bar{x}L) + f'(\bar{x}L)\bar{x}L\frac{1}{1+\kappa}\right],$$

where $s \equiv (1+\kappa)/(2+\rho+\kappa)$.[10]

Comparing this equation with Eq. (10) (for $\theta = 1$) reveal that they are identical save for the presence of $\bar{x}$—the constant A/K ratio. Observing that $\bar{x}$ is independent of L it is clear that the corollary carries over to the present model featuring endogenous R&D.

From a practical perspective the extended model shows that the relevant scale variable is efficiency units of labor, $\bar{x}L$, and not just raw labor, L. Still, if we adopt a CES production function, as in the last section, the quantitative significance of increasing $\bar{x}L$ would clearly be identical to the impact recovered from changing L. Adding endogenous R&D therefore does not overturn the fundamental point: the savings motive critically influences the sign (and size) of the impact of scale on growth.

4. Concluding remarks

This paper's analysis shows that the impact of scale on growth, judged from standard endogenous growth models, depends on the savings motive. If the bequest motive is paramount, savings will be funded mainly by capital income. In endogenous growth models the central assumption is that of a constant marginal product of capital in the long run. As labor is complementary to capital in the production function, a larger labor force will increase the real rate of return, savings, and growth. In models where the bequest motive is all-important, a positive scale effect is therefore an inherent feature, unless the model is somehow modified. However, if the life-cycle motive dominates, savings will depend mainly on wages. Due to diminishing returns to labor, an increasing labor force will not necessarily increase savings and growth. Our numerical experiments show that if the bequest motive and the life-cycle motive are about equally important for capital accumulation, the scale effect is likely to be quantitatively small and peters out when the size of the labor force rises. In fact, the scale effect may even turn negative as the labor force becomes sufficiently high. In a setup where growth is driven by capital accumulation and R&D, this result carries over. However, the notion of scale changes from the pure size of the labor force, to the labor force in efficiency units.

The analysis emphasizes the need for a clearer understanding of the income sources of aggregate savings. Previous research has shown that the relative importance of bequest and life-cycle savings is central to a variety of macroeconomic outcomes as it implies different links between wage and capital income on the one hand, and aggregate savings on the other. The present paper adds the impact of scale on growth to the list; understanding the sources of aggregate savings might yield important insights into the growth process.

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[8] In their paper, Antinolfi et al. (2001) immediately normalize the size of the labor force to one, for which reason their analysis is silent about the scale issue. In addition, their analysis does not allow for bequest. Hence, the present model can be seen as a generalization of their framework.

[9] The R&D specification we adopt here is essentially what is labeled "the lap equipment formulation" in the literature. See Rivera-Batiz and Romer (1991).

[10] Observe that s is obtained if we insert $\theta = 1$ into s^w, defined in Section 2.2.

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References

Aiyar, S., Dalgaard, C.-J., 2008. Accounting for productivity: Is it OK to assume that the world is Cobb–Douglas? Journal of Macroeconomics, forthcoming.
Altonji, J.G., Hayashi, F., Kotlikoff, L., 1997. Parental altruism, inter vivos transfers: theory and evidence. Journal of Political Economy 105, 1121–1166.
Antinolfi, G., Keister, T., Shell, K., 2001. Growth dynamics and returns to scale: a bifurcation analysis. Journal of Economic Theory 96, 70–96.
Arestis, P., Demetriades, P., 1997. Financial development and economic growth: assessing the evidence. Economic Journal 107, 783–799.
Baker, M., 2007. A structural model of the neolithic revolution. Journal of Economic Growth 13, 257–292.
Barro, R., Sala-i-Martin, X., 1995. Economic Growth. McGraw-Hill, New York.
Bertola, G., 1993. Factor shares and savings in endogenous growth. American Economic Review 83, 1184–1198.
Bertola, G., 1996. Factor shares in OLG models of growth. European Economic Review 40, 1541–1560.
Blanchard, O., Fisher, S., 1989. Lectures in Macroeconomics. MIT Press, Cambridge, MA.
Caballe, J., 1998. Growth effects of taxation under altruism and low elasticity of intertemporal substitution. Economic Journal 108, 91–104.
Chirinko, R.S., Fazzari, S.M., Meyer, A.P., 2002. That elusive elasticity: a long-panel approach to estimating the price sensitivity of business capital. Working Paper, Federal Reserve Bank of St. Louis.
Dalgaard, C.-J., Kreiner, C.T., 2001. Is declining productivity inevitable? Journal of Economic Growth 6, 187–203.
DeLong, B.J., 2003. Bequests: a historical perspective. Mimeo, University of California at Berkeley.
Dinopoulos, E., Thompson, P., 1998. Schumpeterian growth without scale effects. Journal of Economic Growth 3, 313–335.
Dynan, K., Skinner, J., Zeldes, S.P., 2002. The importance of bequests and life-cycle saving in capital accumulation: a new answer. American Economic Review 92, 274–278.
Galor, O., 1996. Convergence? Inferences from theoretical models. Economic Journal 106, 1056–1069.
Howitt, P., 1999. Steady endogenous growth with population and R&D inputs growing. Journal of Political Economy 107 (4), 715–730.
Jones, C.I., 1995. R&D-based models of economic growth. Journal of Political Economy 103, 759–783.
Jones, L.E., Manuelli, R., 1990. A convex model of equilibrium growth: theory and policy implications. Journal of Political Economy 98, 1008–1038.
Jones, L.E., Manuelli, R., 1992. Finite lifetimes and growth. Journal of Economic Theory 58, 171–197.
Modigliani, F., 1986. Life cycle, individual thrift, and the wealth of nations. American economic review 76, 297–313.
Peretto, P.F., 1998. Technological change and population growth. Journal of Economic Growth 3, 283–311.
Peretto, P.E., Smulders, S., 2002. Technological distance, growth and scale effects. Economic Journal 112, 603–624.
Romer, P., 1987. Growth based on increasing returns due to specialization. American Economic Review Papers and Proceedings 77, 56–62.
Rivera-Batiz, L.A., Romer, P.M., 1991. Economic integration and endogenous growth. Quarterly Journal of Economics 106, 531–555.
Rose, A.K., 2006. Size really doesn't matter: In search of a national scale effect. Journal of the Japanese and International Economies 20, 482–507.
Simon, J., Sullivan, R.J., 1989. Population size, knowledge stock, and other determinants of agricultural publication and pateting: England, 1541–1850. Explorations in Economic History 26, 21–44.
Strulik, H., 2005. The role of human capital and population growth in R&D-based models of economic growth. Review of International Economics 13, 129–145.
Uhlig, H., Yanagawa, N., 1996. Increasing the capital income tax may lead to faster growth. European Economic Review 40, 1521–1540.
Young, A., 1998. Growth without scale effects. Journal of Political Economy 106, 41–63.