The Physiological Foundations of the Wealth of Nations[*]

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Abstract. In the present paper we advance a theory of pre-industrial growth where body size and population size are endogenously determined. Despite the fact that parents invest in both child quantity and productivity enhancing child quality, a take-off does not occur due to a key "physiological check": if human body size rises, subsistence requirements will increase. This mechanism turns out to be instrumental in explaining why income stagnates near an endogenously determined subsistence boundary. Key predictions of the model are examined using data for ethnic groups as well as for sub-national regions.

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JEL: O11, I12, J13.

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1. INTRODUCTION

In large parts of the world, persistent income per capita growth was not observed until a few decades ago, and in some instances - mainly Sub-Saharan Africa - the process still has to begin in earnest. Understanding why this is the case continues to motivate research on economic growth. Yet it is worth bearing in mind that the absence of growth is not unusual in a long-run historical perspective. What is unusual is actually the phenomenon of economic growth, as stagnation seems to have been the norm during most of human history (Galor, 2005). This makes it important to understand why all societies failed to grow until recently; if key mechanisms that prevented growth historically can be exposed, it might leave important clues as to why some societies appear to stagnate at low levels of income to this very day.

In this light it is problematic that important aspects of the pre-industrial growth record remain ill understood. In particular, existing theories ignore the dynamics of human body size and its links to population dynamics and average living standards. This state of affairs is problematic in two related respects.

First, ignoring body size prevents a full understanding of the forces that engendered stagnating income per capita levels during most of human history. Evidence surveyed below strongly suggests that societies throughout history faced a fundamental body size – population size trade-off; a trade-off involving how many children to bear, and how well to feed each of them. This evidence indicates that episodes of rising income should have instigated intensified investments in offspring body size (in addition to child quantity). But in this case the standard Malthusian positive check may not suffice to bring the growth process to a halt as the tendency for diminishing returns to labor input would be counterbalanced by body-size-induced productivity gains. If so, what (other) mechanisms contributed to the observed long-run constancy of income per capita prior to the take-off? Why did episodes of technological change fail to generate step-wise improvements in income per capita in tandem with rising body size? By ignoring the dynamics of body size during the Malthusian epoch the existing literature cannot address these central issues.[1]

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[1] See e.g., Fogel (1994) and Weil (2007) for evidence on the positive link between body size and labor productivity. Note that a virtuous circle of rising nutrition and attendant increases in body size and income could in principle be quantitatively significant. Fogel (1997) calculates that about 30% of UK growth 1780-1979 can be accounted for by improvements in nutrition.

Second, by ignoring body size the existing literature is largely silent about the determination of the level of income per capita during Malthusian times: “the level of subsistence”. Biologically, subsistence requirements are inescapably linked to the size of the human body; bigger bodies are more energy requiring. Without an adequate theory of the long-run determination of human body size the notion of subsistence remains obscure, and predictions about comparative income differences across pre-industrial societies become untenable. As a consequence the existing literature cannot fully describe the economic circumstances prevailing prior to industrialization.

The present paper proposes a theory of long-run growth according to which body size as well as “subsistence requirements” are endogenously determined. Parents face a trade-off between how many children to have and how well to feed each of them. At the same time they are constrained by income and metabolic consumption requirements. A positive shock to income will induce parents to expand fertility and nutritional investments in their children. However, in the following period a “physiological check” sets in: adults of the next generation are larger and as such more energy requiring. This limits fertility and nutritional investments per child, and serves to equilibrate the economy. As a result, the economy remains in a state of stagnation even though parents invest in child quality.

The differential timing of the take-off to sustained growth has helped to shape the current global distribution of per capita income (Galor, 2005, 2010; Lucas, 2000; Cervelatti and Sunde (2013) and Dalgaard and Strulik (2013)). Yet in order to understand the occurrence, timing and speed of the transition it is necessary to fully understand the country-specific economic circumstances that prevailed at the time of the take-off. The present paper proposes a theory of how key preconditions were formed and how they varied at the eve of the “industrial revolution”.

The model features overlapping generations. Individuals live for two periods; childhood and adulthood. Children are passive recipients of nutrition, determined by the parent, who is the economically active agent; we ignore matching in the marriage market and allow agents to reproduce asexually. The economic components of the model are fairly standard. A unique output good is produced using body-size-augmented labor, land, and technology; since we are focusing on pre-industrial times, technology is assumed to be parametrically fixed, and land is also a fixed factor of production. Accordingly, diminishing returns to labor input prevail. Aside from this the theory builds on three novel elements.

First, besides own consumption parents derive utility from the quantity and quality (i.e., nutrition) of their children. Both quality and quantity are normal goods. Second, the metabolic rate of the parent, our conceptualization of "subsistence", is increasing in both body size and fertility. The parent is required in each period to cover these needs. Whereas body size is predetermined at the time of optimization, fertility is a control variable. Theory and empirical evidence from biology provide this important "physiological" constraint, right down to the functional form and its parameters. The substantive implication of the constraint is that if the number of children is expanded the parent faces elevated consumption needs due to pregnancy, leaving less resources for quality investments in the offspring. Taken together these two elements generate a natural size-number trade-off which is strongly supported by empirical evidence reviewed below. The third element is the law of motion for body size, which we derive from first principles, starting at the cellular level. The substantive implication of it is that more nutrition during childhood leads to larger adults.

These elements interact in the following way. A positive productivity shock will induce parents to have more children and invest in more nutrition per child. The better nourished children will grow up to be larger adults than their parents. However, their elevated body size implies that they will be more energy-requiring adults. This prevents them from providing the same amount of nutrition per child that their parents provided, even if their productivity and income is the same as that of their parents. Thus the increase in body size is "checked" by rising metabolic needs, a fact, which puts downward pressure on nutritional investments as well as expenditure on family size. Consequently, growth does not take hold, and the economy remains in a state of stagnation. The new steady-state is characterized by unaltered body size, constant income per capita, and higher population density.

Hence, within a country, episodes of technological change should neither lead to rising income per capita nor to greater average body size; the only lasting impact from technological advancement would be greater population density. These predictions are consistent with what appears to be known about pre-industrial development in Europe. In particular, historians have documented that average height of Europeans exhibited no significant trend over the two millennia preceding the take-off into sustained growth (Kunitz, 1987; Koepke and Baten, 2005), whereas it is well known that population density gradually rose (interrupted, chiefly, by the Plague). This does not mean, of course, that European average body size remained exactly constant. Across centuries average body size did undergo marked changes. But any gains in average stature proved to be temporary in nature; they eventually eroded away, preventing a time trend from emerging.

The model allows for an improved understanding of these dynamics; qualitatively, as explained above, but also quantitatively. To demonstrate the latter, we begin by calibrating our model such that it replicates the evolution of European population from year 0 C.E. to 1800 C.E. Subsequently we ask the model to "forecast" changes in body size over this period, which we then compare to available data. The calibrated model proves to be rather effective in "predicting" the evolution of body size in Europe, during the two millennia that preceded the fertility transition and the take-off to growth.

In order to gauge the relevance of the models' steady state predictions we turn to regression analysis. The key issue is whether productivity has a *differential* impact on population density and body size, as predicted by the model. In examining this claim we study two separate samples, both of which should be describable by a Malthusian model on a priori grounds. Our first sample consists of ethnic-groups from Murdock's (1967) Ethnographic Atlas, which we have augmented by data on population density and body size (weight) drawing on published studies in anthropology and ethnography. In some respects this is an ideal setting for a falsification test of the theory as these societies certainly had not undergone the demographic transition at the time of observation. In fact, many are hunter-gatherers. At the same time one is left wondering if the theory also speaks to more modern societies.

Hence, our second sample involves pixel-level (1×1 degree latitude/longitude) information stemming from contemporary nation states that had not undergone the demographic transition at the end of the 20th century. Needless to say, these countries are not randomly selected; they are all characterized by a non-decreasing total fertility rate until the end of the 20th century. However, it is important to observe that the sample is not selectively constructed at the subnational level, which is where we attempt to elicit information about the validity of the theory's predictions.

In order to overcome concerns about reverse causality, we employ "external" (geographically determined) measures of productivity that capture the richness of the environment of the ethnic-group or land suitability for crop growth within a particular (pixel sized) region. In either setting we find a strong positive impact on population density from the external productivity measures, yet an economically and a statistically insignificant impact on body size. In order to minimize the risk that our OLS results are caused by the influence from confounding factors we include a demanding set of productivity determinants reflecting geography, culture and institutions. Despite this we find, in both settings, that productivity works to elevate population density but not body size, as predicted by the model.

The present paper is related to the literature on growth in the very long run, which models Malthusian stagnation and the transition to modern growth (e.g., Galor and Weil, 2000; Galor and Moav, 2002; Lucas, 2002; see Galor, 2006 for a survey). Particularly related is research which aims to understand the mechanics of pre-industrial development (e.g., Aiyar, Dalgaard and Moav, 2008; Ashraf and Galor, 2011; Sharp, Strulik and Weisdorf, 2012; Voigtländer and Voth, 2010). In contrast to the present paper, however, these studies ignore the evolution of body size, and assume the level of subsistence consumption is exogenously given (if introduced at all).[2]

The paper is also related to the anthropometric history literature, where height and fertility has been discussed in a unifying way. Weir (1993), in particular, provides a perspective on the quality-quantity trade-off that is similar to ours, and uses it to conceptually motivate an empirical analysis of the impact from the French fertility transition on height developments.[3]

The paper proceeds as follows. In Section 2 we survey evidence on the quantity-quality trade-off between fertility and body size. Section 3 develops the model. Section 4 proves existence, stability, and provides comparative statics, whereas Section 5 examines comparative dynamics. Section 6 examines the ability of the model to quantitatively account for the dynamics of the Western European economy prior to the fertility transition, and Section 7 examines key steady state predictions of the model by way of regression analysis. A final section concludes.

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[2] A related contribution by Abdus and Rangazas (2011) does allow subsistence consumption to be endogenously determined by body size and activity level. In contrast to the present paper, however, individuals do not take their metabolic needs into account when optimizing (and metabolism does not increase with fertility), nor does physical stature matter to productivity. Rather than focusing on the causes of stagnation, Abdus and Rangazas use their their model to show a long-run constancy of calorie consumption during roughly the last two centuries; in their model increasing demand for food as a source of utility is off set by a declining activity level at work.

[3] More broadly, the anthropometric literature has long observed a link between changes in body size and movements in fertility, since nutrition during childhood influences not only adult stature but may also have a direct effect on fertility. For example, the age of menarche is reached earlier within well fed populations. A careful discussion is found in Komlos (1989), analyzing the period of take-off in the Habsburg Monarchy. Extending the theory below to include some of these additional mechanisms could be a potentially interesting topic for future work.

2. Size versus Number of Off-Spring: A quantity-quality trade-off

In this section we review evidence from biology, anthropology, and economics which strongly suggest an operative quantity-quality trade-off between body size and family size. The important implication is that families throughout history have made quality investments, in the sense of nutrition. Periods of plenty would therefore imply an expanding number and size of people, which challenges the sufficiency of Malthus' positive check in equilibrating the economy at a constant level of subsistence.

Convincing evidence in favor of a quantity-quality trade-off is found in biology. At the level of the individual mammalian species, the inverse association between body size and rate of reproduction is by now well documented (Charnov and Ernest, 2006; Walker et al., 2008). Indeed, experimental studies manipulating offspring number or environment exist, which document the trade-off (Roff, 2002).

In the context of human populations micro evidence of the size-number trade-off is also available. In subsistence societies the inverse association between family size and size of the offspring has received support by Hagen et al. (2006) who examine the trade-off in the "Shuar" society, which resides in the tropical rainforests of Ecuador. Similarly, Walker et al. (2008) demonstrate a negative link between body size and family size across a sample of 16 subsistence-based societies.[4]

From the perspective of the present study the evidence amassed by biologists and anthropologists is of substantive importance, since it makes plausible that the size-number trade-off is a fundamental one. The systematic link between reproduction and body size in economically primitive societies suggests that the trade-off must have been active for a very long time; certainly before the take-off to sustained growth occurred in Western Europe. As a consequence, temporary income gains should be expected to translate into greater nutritional expenditure on children, implying "quality" investments. The fact that the trade-off transcends the human species implies that the mechanism responsible for it must be generic; the "physiological check", much like Malthus' positive check, fulfills this criteria.

In modern societies evidence of the trade-off is found as well. In an early contribution Douglas and Simpson (1964) examined the results from a national survey of health and development in the UK. Specifically, the survey focused on 1,557 boys and 1,456 girls born in 1946. The physical

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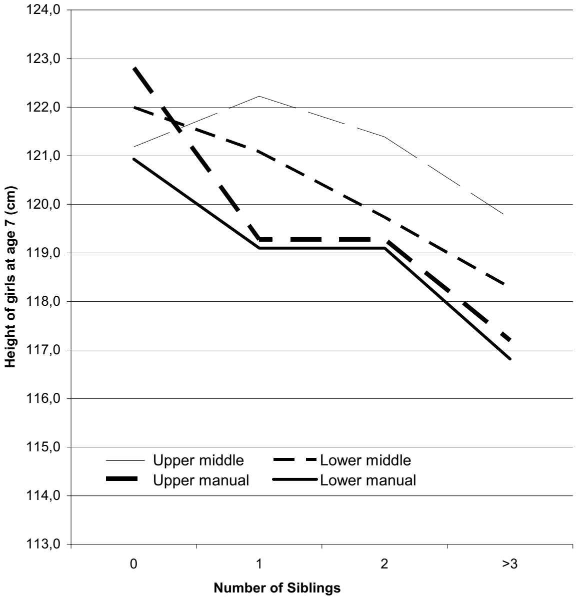
[4] See Cole (2000) and Silventoinen (2003) as well.

Figure 1. Body Size of UK Girls at Age 7 and the Number of Siblings



Notes. The individual lines refer to individual social classes, as defined in the text. Data source: Douglas and Simpson (1964), Table 7.

development of these children was tracked and central health indicators were collected, including height, date of entry into puberty and age of menarche. In addition socio-economic indicators for the households were obtained, including occupation and educational background of the parents. This enabled Douglas and Simpson to categorize the families into social classes, ranging from "lower manual" to "upper middle".[5] Figure 1 shows the association between the mean body size of girls at age of 7 and the number of siblings in the family, as reported by Douglas and Simpson.

The general pattern that emerges is one where a large family size is associated with smaller mean body size of the offspring. In addition, for the size of the family given, the average body size of the offspring, measured by height, generally increases as the socio-economic circumstances improve. Douglas and Simpson document that similar patterns persist to the ages of 11 and 15 for both boys and girls. The 1946 cohort has more recently been analyzed by Kuh and Wadsworth (1989). Their regression-based analysis confirms the general impression conveyed

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[5] In the "upper middle class" we find parents with a secondary education, families where the father has a non-manual occupation, and where at least one of the parents was brought up in a middle class family with similar characteristics. In contrast, the "lower manual" group is characterized by the father being a manual laborer, by both parents only having primary education, and by upbringing; both parents were raised in a working-class family. Between these two extremes we find the "lower middle class" and "upper manual class"; these groups are differentiated from "upper middle class" and "lower manual class" mainly by their educational attainment.

by Figure 1; conditional on a host of environmental factors, each additional sibling implies a reduction in mean height of about 6 mm.

At a higher level of aggregation Weir (1993) demonstrates a strong negative association between height and fertility rates across regions in France during the period 1840-1911. Following up on Weir's findings, Schneider (1996) documents similar patterns across nine European countries, covering the period 1750-1920. Controls for health and income do not overturn these findings.

In sum, the evidence discussed above pertaining to human societies as well as non-human samples quite strongly suggest an operative quantity-quality trade-off between size and number of offspring. In the next section we provide a model which encompasses this fundamental trade-off.

3. The Model

We develop the model in a series of steps. First, Section 3.1 provides a brief introduction to the field of allometry, from which we draw an essential equation in pinning down subsistence consumption. Section 3.2 then explains how subsistence consumption is determined in the model, after which we turn to the intergenerational law of motion for body size in section 3.3. Finally, Section 3.4 and 3.5 discuss the preferences and optimization problem of parents and the production side of the economy, respectively.

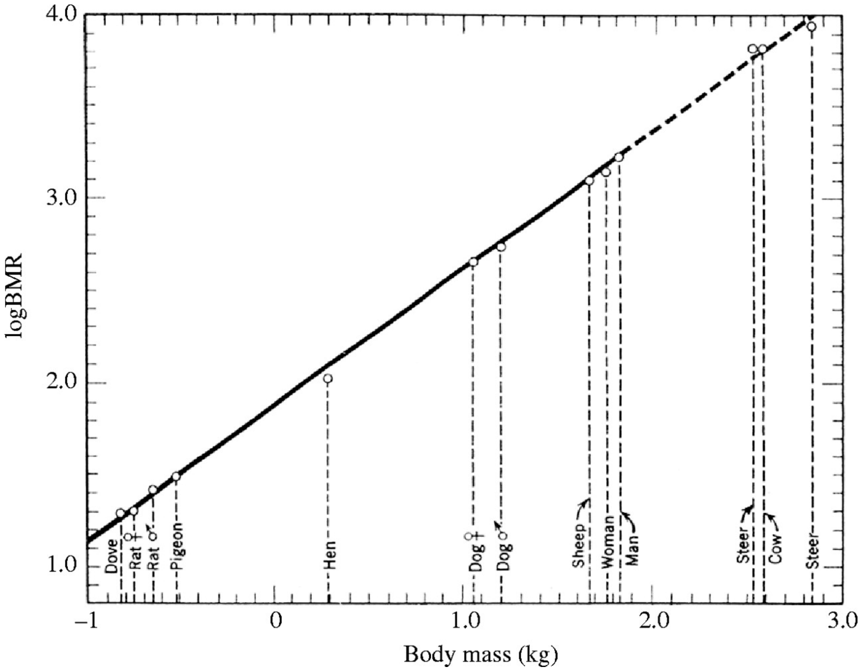
3.1. An Introduction to Allometric Scaling and Energy Consumption.

Allometric scaling is a technique used in biology to study how selected biological variables of an organism correlate with the size of the organism. A fundamental allometry is found between energy consumption B and body mass m of a mammal, and is known as "Kleiber's Law" (Kleiber, 1932):

$$B = B_0 \cdot m^b, \quad \text{with } b = 3/4, \tag{1}$$

where B is the basal metabolic rate and B_0 is a species-dependent constant. Thus, drawn on log-log paper the energy-body mass relationship is linear with slope of 3/4, see Figure 2. A slope of 3/4 has been verified by Brody (1945) for almost all terrestrial animals yielding the famous "mouse-to-elephant curve".

Figure 2: Kleiber's Law



Kleiber's original Figure as reproduced in West and Brown (2005).

Recently biologists and physicists managed to provide a theoretical explanation for Kleiber's law. A living organism needs to feed its cells. For that purpose energy and material is transported through hierarchically branching networks like blood vessels in mammals. The network in use, however, is not of arbitrary structure. Given that organisms have evolved through natural selection, it must be one that minimizes energy used for transport i.e. one that minimizes hydrodynamic resistance. West et al. (1997) have shown that organisms, viewed as energy transporting networks that minimize energy dissipation, fulfil Kleiber's law.

The beauty of this theory lies in its foundations on first principles. This makes it very general, and in fact it has already been applied to a multitude of biological problems from "genomes to ecosystems" (West and Brown, 2005). Some of the applications and extensions, for example those on fertility, are clearly relevant for economic analysis.

3.2. **Subsistence Requirements.** We employ Kleiber's law in order to characterize adult subsistence requirements in an overlapping generations model with endogenous fertility. According to the OLG setup life is separated into two periods. First, childhood (after weaning), defined as the period of body growth and dependence on food provided by the parent. Second, adulthood, defined as the period of constant body size, work effort and reproduction.

The body size of an adult is predetermined by the individual's nutritional intake during childhood. Accordingly, we assume that the body size of an adult remains constant.[6] Subsistence consumption of an adult, however, is not completely predetermined, since it also depends on fertility. From the physiological literature we use the fact that rearing up a child from conception to weaning increases the mother's metabolic needs by a factor ρ (Prentice and Whitehead, 1987; Sadurkis et al., 1988). Denoting by B_t energy used up by the parents own body, and n_t the number of children, total metabolic needs of an adult, E_t, is thus given by:

$$E_t = (1 + \rho \cdot n_t) \cdot B_t = (1 + \rho \cdot n_t) \cdot B_0 m_t^b. \tag{2}$$

In (2) E_t measures metabolic needs in terms of energy, expressed, for example, in kcal. per period. In order to convert energy into goods we introduce the energy exchange rate ϵ, which is measured in kcal. per unit of a unique consumption good. While consumption expenditure is a control variable for parents, the energy extracted from a unit of consumption depends on the food staple available, which we treat (similar to technological progress) as exogenous.

Employing the energy exchange rate we get *subsistence requirements*, i.e. the metabolic needs of an adult in terms of consumption goods, $\bar{c}_t = E_t/\epsilon$. Note that, in contrast to the existing literature in economics, the subsistence requirement is *not* a constant but depends on the individual body size and fertility in adulthood. The variable $\bar{c}_t$ is therefore generation-dependent and thus indexed by t.

3.3. **The Intergenerational Law of Motion for Body Size.** In order to establish the intergenerational link between body sizes we draw on West, Brown, and Enquist's (2001) model of ontogenetic growth.[7] The starting point is an energy conservation equation which reads as follows:

$$E_t^c = b_c N_t + e_c (N'_{t+1} - N_t) \tag{3}$$

where E_t^c is energy consumption during childhood after weaning (prior consumption is covered by adult metabolic needs), N_t denotes the number of human cells after weaning, N'_{t+1} is the number of cells of the child as a grown up, b_c the metabolic energy a cell requires during childhood for maintenance and replacement, and e_c the energy required to create a new cell. Hence the left

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[6] From now on we refer to m_t as body size rather than body mass. This is done for semantic reasons. The term "body size" is closer to the the literature in anthropology and economic history, which focusses on human height. It also avoids confusion with the body mass index.

[7] "Ontogeny" describes the origin and the development of an organism from the fertilized egg to its mature form.

hand side is energy "input", and the right hand side captures energy use. Solving (3) for N'_{t+1} we obtain the number of cells of an adult as a function of the number of cells of a child after weaning and energy intake during childhood:

$$N'_{t+1} = \frac{E_t^c}{e_c} + \left(1 - \frac{b_c}{e_c}\right) N_t. \tag{4}$$

To proceed we insert the fact that body mass consists of the mass of a single cell $\bar{m}$ times the number of cells. This implies for the size of an adult that $m_{t+1} = \bar{m}N'_{t+1}$. Moreover, using the observation that after weaning the size of a child equals μ times the size of the mother (Charnov, 1991, 1993) we have $\bar{m}N_t = \mu m_t$.[8] Substituting N_t and N'_{t+1} in (4) and and solving for m_{t+1}, gives (5):

$$m_{t+1} = \frac{\bar{m}}{e_c} E_t^c + \left(1 - \frac{b_c}{e_c}\right) \mu m_t. \tag{5}$$

This intergenerational law of motion for body size has a simple interpretation: The size of the adult, m_{t+1} is determined by energy consumption during childhood, E_t^c, plus initial size, μm_t, adjusted for energy needs during childhood, $-(b_c/e_c)\mu m_t$. Finally, denote by c_t the consumption of a child, to be determined below from optimization. Then total energy intake during childhood is $c_t \cdot \epsilon = E_t^c$. Inserting this into (5) leaves us with a law of motion for body size across generations:

$$m_{t+1} = a \cdot \epsilon \cdot c_t + (1 - d) \cdot \mu \cdot m_t \tag{6}$$

where $a \equiv \bar{m}/e_c$ and $d \equiv b_c/e_c$. As explained below, in conjunction with Proposition 3, d and a can be viewed as deep physiological parameters, which may exhibit some variation across humans due to natural selection. That is, due to processes whereby large or small bodies were selected depending on e.g. climate related circumstances.

3.4. **Individual's Optimization.** A parent maximizes utility derived from child quality and quantity where quality is in the Beckerian (1960) sense measured by expenditure for consumption (i.e. nutrition) per child, c_t. We allow the parent also to derive utility from own consumption p_t (i.e. consumption of non-food goods) and impose for simplicity a logarithmic form for the utility function:

$$U(c_t, n_t) = \log(c_t) + \beta \log(p_t) + \gamma \log(n_t), \qquad \gamma > 1, \tag{7}$$

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[8] A physiological explanation for this observation is that child development until weaning depends on energy consumption in utero and during the breastfeeding phase. Since larger mothers consume absolutely more energy the offspring should be larger at this point as it receives a fraction thereof.

with β and γ denoting the weight of parental consumption and child quantity in utility, and n_t the number of offspring.

Total child expenditure is constrained by parental income and subsistence consumption of the parent. Furthermore we assume that each parent is endowed with one unit of time and rearing a child requires a certain fraction τ of parental time, $\tau \geq 0$. Let y_t denote potential family income, which is earned when all time is spent working. Combining the budget constraint with energy requirements (2) and applying the energy exchange rate leads to a single constraint:

$$y_t(1 - \tau \cdot n_t) - p_t - c_t n_t - (1 + \rho n_t) B_o m_t^b / \epsilon = 0. \tag{8}$$

Observe that the model captures the full cost profile of children: the metabolic costs which are relevant until weaning ($(1 + \rho n_t) B_o m_t^b$) as well as the economic costs (time τ , nutrition c) that pertain to the period after weaning.

Parents maximize (7) s.t. (8) by choosing fertility n_t, nutritional expenditure per child c_t, and non-food consumption p_t. From the first order conditions we obtain optimal child quality and quantity,

$$c_t = \frac{\rho (B_0/\epsilon) m_t^b + \tau y_t}{\gamma - 1}, \tag{9a}$$

$$n_t = \frac{(\gamma - 1)\left[y_t - (B_0/\epsilon) m_t^b\right]}{(\beta + \gamma)\left[\rho + \tau y_t\right]}, \tag{9b}$$

together with parental consumption $p_t = \beta\left[y_t - (B_0/\epsilon) m_t^b\right]/(\beta + \gamma)$. Observe that $\partial n_t / \partial m_t < 0$ and $\partial c_t / \partial m_t > 0$. Greater metabolic costs of fertility induce larger parents to prefer to have less children and to nourish them better. Note furthermore that $\partial n_t / \partial y_t > 0$ and $\partial c_t / \partial y_t > 0$. With higher income parents prefer to have more children *and* to nourish them better implying that the quantity-quality trade-off is driven by metabolic needs and not by income.

It could be argued that entering child nutrition into the utility of parents is a shortcut representation of a more appropriate quality-quantity trade-off. Strictly speaking, so the argument could go, (nutritional) quality of offspring is better measured by the *size* of grown up children, rather than by the nutritional provisions that make the offspring grow.

Unfortunately, allowing m_{t+1} to enter the utility function instead of c_t destroys the simplicity and elegance of the baseline physiological Malthusian model. But it leads to very similar results.

We provide support for this claim by developing a version of the model with "utility-from-body-size" in Appendix C; the main comparative static results derived below (Propositions 2-3) continue to hold. In Section 4 we furthermore demonstrate that the utility-from-body-size model displays adjustment dynamics that are very similar to those observed for the baseline model. The baseline model is thus a good approximation to the utility-from-body-size version at the steady-state and off the steady-state.

3.5. **Production and Demographics.** We assume that total income is determined at the macro-level by a body size adjusted technology. Specifically, potential aggregate income is given by $Y_t = Am_t^{\phi}\tilde{L}_t^{\alpha}X^{1-\alpha}$, $\alpha \in (0,1)$. As usual A captures technological knowledge, $\tilde{L}_t$ denotes aggregate labor input in period t, and X is land. The parameter ϕ quantifies the return to body size, $\phi > 0$. Larger lean workers are more productive because they exert more muscle force. While there were always some overweight people in society, widespread obesity was not observed before the late 20th century. This means that the representative worker in Malthusian times presumably possessed a lean body. Because muscle force is proportional to muscle cross-section area, measured in meters^2, we observe that it rises with height as h^2 and with weight as $m^{2/3}$ (Astrand and Rodahl, 1970; Markovic and Jaric, 2004, Samaras, 2007). Of course not all tasks of the production processes rely on 'brute force' to the same extent. Theoretical reasoning and empirical estimates in sport physiology suggest that individual performance in different tasks scales with body size as m^{ϕ}, in which $\phi = 2/3$ for exerting force (as for example plowing and digging), $\phi = 0$ for moving and $\phi = -1/3$ for supporting body weight (Markovic and Jaric, 2004). In Dalgaard and Strulik (2011) we investigate in a partial equilibrium context how heterogeneity of tasks and body size explains wage inequality and rationing on the labor market. In the present representative agent context, however, we have to assign an average value to ϕ that approximates the average return of body size on income (see the calibration below).

Following Galor and Weil (2000) we assume that labor is paid according to its average product such that potential income per adult is given by

$$y_t = Am_t^{\phi}(L_t/X)^{\alpha-1}. \tag{10}$$

Since each adult actually supplies $1 - \tau n_t$ units of labor, actual income per worker is given by $Am_t^{\phi}(L_t/X)^{\alpha-1}(1-\tau n_t)^{\alpha} = (1-\tau n_t)^{\alpha}y_t$ and income per capita is given by $\tilde{y} = (1-\tau n_t)^{\alpha}y_t/(1+n_t)$. According to standard Malthusian theory, i.e. ignoring body size, the technology implies stagnation in the long-run unless general productivity A is growing without bound since land is constant. Taking the physiological fact into account that larger bodies exert more muscle force and are thus more productive ($\phi > 0$), Malthusian stagnation follows less immediately since, in principle, living standards could improve continuously in line with increasing body size. As will be shown below, long-run stagnation in this case, requires a further condition – the physiological check – to hold.

Finally, the adult population evolves according to

$$L_{t+1} = n_t L_t, \tag{11}$$

where n_t is given by equation (9b). With equations (1) to (11) the model is complete.

4. Steady-State and Comparative Statics

4.1. **Existence and Stability of the Steady-State.**

The model can be represented by a two-dimensional dynamic system of body size and population size. To obtain it we substitute energy consumption per child (9a) into the law of motion for body size (6) and fertility (9b) into the law of motion for population size (11). This leads to the system (12), with y_t determined by (10).

$$m_{t+1} = \frac{a\epsilon}{\gamma - 1}\left[\rho(B_0/\epsilon)m_t^b + \tau y_t\right] + (1-d)\mu m_t \tag{12a}$$

$$L_{t+1} = \frac{(\gamma - 1)\left[y_t - (B_0/\epsilon)m_t^b\right]}{(\beta+\gamma)[\rho(B_0/\epsilon)m_t^b + \tau y_t]} \cdot L_t. \tag{12b}$$

Proposition 1. **Existence of a steady-state.** *There exists a unique steady-state* (m^*, L^*) *at*

$$m^* = \left\{\frac{aB_0}{(\gamma-1)[1-(1-d)\mu]}\left[\rho + \frac{\tau((\beta+\gamma)\rho + \gamma - 1)}{\gamma - 1 - (\beta+\gamma)\tau}\right]\right\}^{1/(1-b)} \tag{13a}$$

$$\frac{L^*}{X} = \left\{\frac{A\epsilon(\gamma - 1 - (\beta+\gamma)\tau)}{[(\beta+\gamma)\rho + \gamma - 1]\,B_0}\right\}^{1/(1-\alpha)} \cdot (m^*)^{\frac{\phi - b}{1-\alpha}}. \tag{13b}$$

At the steady-state potential income is given by

$$y^* = \frac{[(\beta+\gamma)\rho + \gamma - 1]\,B_0}{[\gamma - 1 - (\beta+\gamma)\tau]\,\epsilon} \cdot (m^*)^b \tag{13c}$$

and income per capita is given by $\tilde{y}^* = (1-\tau)^\alpha y^*/2$.

Proof. At a steady-state equilibrium we observe $n^* = 1$ and thus from (9b) we find that y_t is given by (13c). Inserting (13c) into (12a) evaluated at $m_{t+1} = m_t$ provides body size at the steady-state. Using (10) and (13c) in (12b) we obtain population density at the steady-state. Noting again that $n^* = 1$ such that each family consists of one adult and one child provides income per capita at the steady-state. □

In order to assess stability and other important features of the steady-state we introduce the physiological check.

ASSUMPTION 1. **The Physiological Check.** *The body size elasticity of metabolic needs exceeds the body-size elasticity of productivity:* $b > \phi$.

Applying Kleiber's law, the physiological check requires that $\phi < 3/4$. In the calibration below we argue in favor of a value for ϕ of 0.25, implying that the physiological check is fulfilled. As will become evident below, the case where the physiological check is *not* fulfilled involves dynamics that are inconsistent with the pre-industrial growth record suggesting the check must have been in place throughout history. Intuitively, the physiological check requires that the elevated subsistence needs exceed the productivity gains associated with rising body size. In this regard it is interesting to observe the correspondence to Malthus' positive check, which also involves a restriction on elasticities: the population size elasticity of food demand exceeds the population size elasticity of food supply, $1 > \alpha$.

In order to investigate stability we insert (10) into (12a), which provides the isocline along which body size remains constant, $\Delta m = m_{t+1} - m_t = 0$, implicitly given by

$$G(m_t, L_t) = [1 - (1-d)\mu]\,\frac{\gamma - 1}{a\epsilon} - \rho\frac{B_0}{\epsilon}m_t^{b-1} - \tau A m_t^{\phi-1} L_t^{\alpha-1} = 0. \tag{14}$$

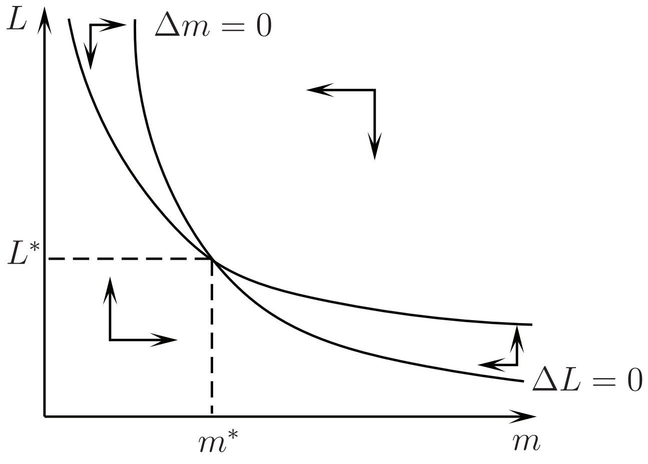
Implicitly differentiating we obtain $\mathrm{d}L_t/\mathrm{d}m_t < 0$; the $\Delta m = 0$–isocline is negatively sloped. Observe that $\Delta m < 0$ when m_t lies to the right of the isocline and $\Delta m > 0$ when m_t lies to the left of the isocline. Moreover, observe that $L_t \to 0$ for $m_t \to \infty$ and $L_t \to \infty$ for $m_t \to \bar{m} > 0$. The resulting curve and arrows of motion are shown in the phase diagram of Figure 3.

The $\Delta L = 0$–isocline in (m_t, L_t)-space is obtained by solving (9b) for $n = 1$ and inserting the result into (12a). This provides (15).

$$\frac{L_t}{X} = \left[\frac{A\epsilon(\gamma - 1 - (\beta+\gamma)\tau)}{((\beta+\gamma)\rho + \gamma - 1)B_0}\right]^{1/(1-\alpha)} \cdot m_t^{\frac{\phi-b}{1-\alpha}}. \tag{15}$$

Given the physiological check, $b > \phi$, the isocline is a hyperbola with $L_t \to 0$ for $m_t \to \infty$ and $L_t \to \infty$ for $m_t \to 0$. Recall that for the $\Delta m = 0$–isocline $L_t \to \infty$ for $m_t \to \bar{m} > 0$. Hence, there is a unique intersection between the $\Delta L = 0$ and $\Delta m = 0$–isoclines. Finally, observe that, when the physiological check is operative, $\Delta L_t < 0$ above the isocline and $\Delta L_t > 0$ below the isocline.

Figure 3: Phase Diagram



The arrows of motion in Figure 3 suggest that the steady-state is a globally stable spiral or node. In Appendix B we evaluate stability numerically and show that the equilibrium is indeed stable when both the positive check ($1 > \alpha$) and the physiological check ($b > \phi$) are operative. This means that the standard assumption of decreasing returns to labor input made in (10), which was necessary and sufficient to ensure stability of stagnation in the standard Malthusian model is necessary but not sufficient to ensure stability in the physiologically founded Malthusian model.[9]

We demonstrate in Appendix B that the positive check is not sufficient to generate stability. For a wide range of parameters, the economy becomes unstable if the physiological check is not operative *although* the positive check is operative, i.e. although $\alpha < 1$. Intuitively, the $\Delta L = 0$ isocline becomes positively sloped for $\phi > b$ and the arrows of motion point toward larger L above the isocline. If ϕ becomes sufficiently large the arrows of motion for m also point towards

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[9] Notice that this claim is based on the observation that instability does exist for some $\alpha < 1$, which can be verified numerically. Of course, the numerical analysis cannot substitute for a theoretical proof of stability of the equilibrium of stagnation.

larger body sizes above the $\Delta m = 0$ isocline. Taken together, this leads to a perpetual expansion path in northeastern direction. Without an operative physiological check it is thus theoretically possible that both population size and body size expand perpetually. The absence of such a trajectory in the historical record suggests that the positive check *and* the physiological check must have been in place.[10] Henceforth we thus assume that the physiological check holds and proceed to examine the steady-state predictions of the model.

4.2. **Comparative statics.** Observe that technology A enters the equation for L^* but does not matter for m^*. In combination with (13c) this leads to the following conclusion:

PROPOSITION 2. **Technological innovations.** *A discrete increase in productivity (A) leads to a more densely populated area but leaves equilibrium body size and income per capita unaffected.*

In the model, a temporary increase in productivity will lead to temporary increases in nutrition and fertility. However, due to the biological feedback loop from subsistence, nutritional expenditures per child will be lower (and so will be fertility) in the following generation. This process will continue until $n = 1$ once again. At this point the temporary gain in body size has fully eroded, and the only result from the innovation is a larger population size. In the long run income per capita does not change since the larger population fully offsets the increase in A. Note also that similar results can be derived for alternative parameter changes associated with technology improvement. Lower dependence on limited land (larger α) and higher efficiency in using brawn (increasing ϕ) both lead to a higher population density without an equilibrium effect on body size and – according to (13c) – on income.

There is convincing evidence testifying to the fact that technological change did increase population density in the aggregate, in pre-industrial times (e.g., Ashraf and Galor, 2011; Baker, 2008). However, the *lack* of a long-run time trend during the last two millennia as a whole, in terms of body size, has also been established (Kunitz, 1987; Koepke and Baten, 2005). The present theory thereby suggests that these potentially conflicting observations are in fact reconcilable.

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[10] Alternatively, it appears to be plausible that there exist a physiologically determined upper limit of body size that humans can attain. In that case, instability would imply that the representative individual is of maximum feasible size, an equally unobserved phenomenon in the history of mankind.

At the same time we reiterate that these steady-state results do not preclude a positive *short-run* impact of productivity gains on income and body size. Comparative dynamics will be discussed below.

Physical anthropologists and biologists argue that some of the observed differences in human body size, across regions and countries, can be attributed to selection (see e.g., Walker and Hamilton, 2008). In particular, it is argued that larger individuals have had a selective advantage in colder climate zones: This idea has its roots in *Bergman's rule*, which states that the body size of mammals tends to rise, as one moves away from the equator. A significant negative correlation between human body size and mean annual temperature was first demonstrated by Roberts (1953) who estimated in a sample of 116 societies that average body mass decreases by 0.3 kilogram per degree Fahrenheit. Bergmann's rule has been confirmed by several follow up studies (e.g. Croignier, 1981, Ruff, 1994, Katzmarzyk and Leonard, 1998, and Gustafson and Lindenfors, 2009). Its most popular explanation is based on the observation that as one moves into colder climate zones it becomes more challenging to keep warm and that as body size goes up the surface to volume ratio declines, which limits the extent of heat loss per unit of body mass. Since limited heat loss is an advantage in cooler climates, selection may have favored larger bodies in cooler areas. Among anthropologists it is now widely accepted that some intercultural variation in body size is the result of genetic, i.e. evolutionary differences (Ruff, 2002). With economic development and the associated secular increase of body size the association of body size with climate became somewhat weaker across populations of the late 20th century, although it is still clearly visible (Katzmarzyk and Leonard, 1998).

The genetic differences that may have emerged in human societies are captured in our model by the physiological parameters a, d, B_0, ρ, and μ. These physiological parameters determine the energy needed to create and maintain cells during ontogenetic growth and in adulthood. In particular the parameter $d \equiv b_c/e_c$ appears to be a likely candidate. To see this observe that equation (3) does not allow for heat loss. Accordingly, the consequences of heat loss are implicit in the parameters of the energy conservation equation. Specifically, lower energy costs of running and maintaining a body cell would capture lower heat loss. In the model this is equivalent to a lower value for b_c and thus d. The evolutionary argument above could thus be interpreted as saying that individuals with lower values for d would have a selective advantage in cold climate since it implies less heat loss, which is useful in keeping body temperature up in cold environments. Observe that lower energetic cell costs in this way, according to the model, would produce adults of larger body size (which according to Kleiber's law need more energy to maintain their metabolism). As a result, in areas where d is smaller (i.e., in colder environments) individuals will be larger.

Proposition 3. **Biological innovations**. *In areas where humans are selected to be larger income per capita is larger, and population density lower.*

This means that if there are physiologically determined differences of body size across populations (reflected by variation in any of the physiological parameters across steady-states), then those populations that have been selected to be of larger body size inhabit less densely populated areas and earn higher income per capita at the Malthusian steady-state.

Proposition 3 implies the following corollary

Corollary 1. *Suppose there are country-specific differences in the genetic determinants of body size. Then across country-specific steady-states there is*
(1) a negative association between population density and body size,
(2) a positive association between income per capita and body size.

Part (1) is known in the biological literature as Damuth's law (Damuth, 1981). According to our theory this empirical regularity can be ascribed to greater metabolic costs of child rearing for larger individuals, which lowers fertility and in the steady state reduces population density. Part (2) is consistent with the empirically observed positive correlation between body size and income per capita (e.g., Steckel, 1983; Brinkman et al., 1988). The corollary thus implies that body size is a sensible indicator of long-run income per capita. According to our model the link emerges due to greater body-size related productivity, and because population density is lower in places with larger individuals. Notice that Proposition 3 compares Malthusian steady-states across populations. Within a population and over time we will observe variation of body size as an off-steady-state phenomenon for given genetic make up. These variations, to which we turn next, reflect the nutritional standard of living in a particular period of time, as commonly assumed by anthropologists and economic historians.

5. COMPARATIVE DYNAMICS

5.1. **Model Calibration.** In order to study the quantitative implications of the model we proceed with a calibration. We start with the biological components. We put $B_0 = 70$ and $b = 0.75$ according to Kleiber's (1932) law. We set $m^* = 54.3$ to match average weight of females in contemporary less developed countries.[11]

In medieval Europe and China life expectancy at age 20 was around 30 years (Clark, 2007). Accordingly we set the length of the adult period to 30 years. In order to get an estimate for ρ we begin by observing that during pregnancy the metabolic rate rises on average by 5%, 10% and 25% in the first, second and third trimester, respectively (Butte and King, 2007). Pregnancy per child takes on average 3/4 of a year. In medieval Europe a women gave birth to about 5 to 6 children (Clark, 2007) which implies for our unisex model that a woman incurs the metabolic cost of about 3 births (of which, at the steady-state, one survives up to adulthood). This means elevated caloric needs for pregnancy per unit length of the adult period by factor $(0.05/3 + 0.1/3 + 0.25/3) \cdot 3/4 \cdot 3/30$. During lactation the basal metabolic rate of a woman who is exclusively breastfeeding her child rises by about 450 calories (Butte and King, 2007). Assuming a basal metabolic rate of 1500 calories for our 54.3 kg average mother, this implies elevated caloric needs by factor 450/1500 during the breastfeeding period. We put the age at weaning to 18 months according to the average value reported for England in the Middle Ages (Mays, 2010). Based on this observation, breastfeeding per child takes on average 1.5 years and we estimate elevated caloric needs for lactation per unit length of the adult period by factor $450/1500 \cdot 1.5 \cdot 3/30$. Taken together, we end up with an estimate for ρ of $[(0.05/3 + 0.1/3 + 0.25/3) \cdot 3/4 + (450/1500) \cdot 1.5] \cdot 3/30 = 0.055$.

To calibrate μ, we observe that the weight of a 18 months old girl (i.e. at the calibrated age of weaning) is 9.4 kg according to World Health Organization Child Growth Standards, evaluated at the 25th percentile.[12] This implies that $\mu = 9.4/54.3 \approx 0.173$. We set the time cost of children $\tau = 0.06$, following Moe (1998) and Lagerloef (2003).

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[11] In a previous draft we examined a cross-section of 38 LDCs that underwent the demographic transition in 1980 or later, according to the "dates" for the fertility decline established by Reher (2004). This is the sample within which the average female weight is 54.3 kg; see Dalgaard and Strulik (2010, Table 2).

[12] This standard is documented in de Onis et al. (2007). It is worth observing that the basis for the weight-for-age curve is solely US children. Since US children may not be an accurate description of less developed economies we chose to use the 25th percentile, rather than the median. The choice of the 25th percentile is admittedly a somewhat arbitrary choice. In any event, the calibrated parameter values are not very sensitive to the exact choice of weight-for-age curve. For instance, μ (and d below), only changes to a very minor degree if we use the 50th percentile instead.

For calibration of d, the amount of energy used for cell maintenance relative to cell creation, we turn to West et al.'s (2001) model on ontogenetic growth in continuous time, and investigate the human growth process. For the calibration we need body size at three different points in time. In addition to weight at age 18 months (end of weaning, 9.4 kg) and asymptotically (i.e., 54.3 kg), we used the weight of a five year-old, which is 16.5 kg (again, at the 25th percentile). This leaves us with $d = 0.37$.[13]

Table 1: Numerical Specification of the Economy

description	notation	value	based on
human metabolism	B_0	70	Kleiber (1932)
Kleiber's law	b	3/4	Kleiber (1932)
equilibrium body-size	m^*	54.3	avg. empirical sample
relative child size after weaning	μ	0.173	WHO, de Onis et al. (2007)
energy for pregnancy and lactation	ρ	0.055	Mays (2010), Butte and King (2007)
energy requirement for body growth	d	0.37	West et al. (2001)
standardized size of body cell	a	0.84	implied
time requirement of child rearing	τ	0.06	Moe (1998)
labor share	α	0.65	Clark (2007)
body size elasticity in production	ϕ	0.25	Strauss (1986)
utility weight for non-food	β	4.5	Deaton and Muellbauer (1986)
utility weight for children	γ	4.0	Brown et al. (2004)
length of a period in years	ψ	30	Clark (2007)
population density	L^*	1.0	normalized
life-time income per adult at subsistence	$\tilde{y}^*$	$12,000$	Maddison (2003)
productivity	A	9415	implied
energy exchange rate	ϵ	0.076	implied

For the economic part of the model we set $\alpha = 0.65$ according to Clark's (2007) estimates. We set $\gamma = 4$ in keeping an evolutionary argument behind the utility function, which is given in Appendix B. Deaton and Muellbauer (1986) estimate that in contemporaneous poor countries children cost their parents about 30-40 percent of what they spend on themselves (ignoring time costs). Assuming that children live with their parents during half of their parents life this implies that a child costs about 35/2=17.5 percent of the parents' own consumption. We adjust the weight of parental non-food consumption in utility such that we match this estimate. This provides $\beta = 4.5$. An implied prediction is that the total food share of expenditure is 85

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[13] The continuous time ontogenetic growth equation is given by $\dot{m} = \delta m^b + dm$, where d is the energy used for cell maintenance relative to cell creation (West et al. , 2001). From that we obtain $d = \log\left([1-(m_0/m^*)^{1/4}]/[1-(m_v/m^*)^{1/4}]\right) \cdot 4/v$. Accordingly, $m_0 = 9.4$, $m_v = 16.5, v = 3.5$ and $m^* = 54.3$.

percent, which squares well with Clark's (2007) observation of the food expenditure share for poor societies in history .

To calibrate the impact of body size on productivity (i.e., ϕ) we proceed in two steps. As a first step, we observe that productivity can be related to nutritional needs. Using the production function (10) and Kleiber's law (1), we obtain:

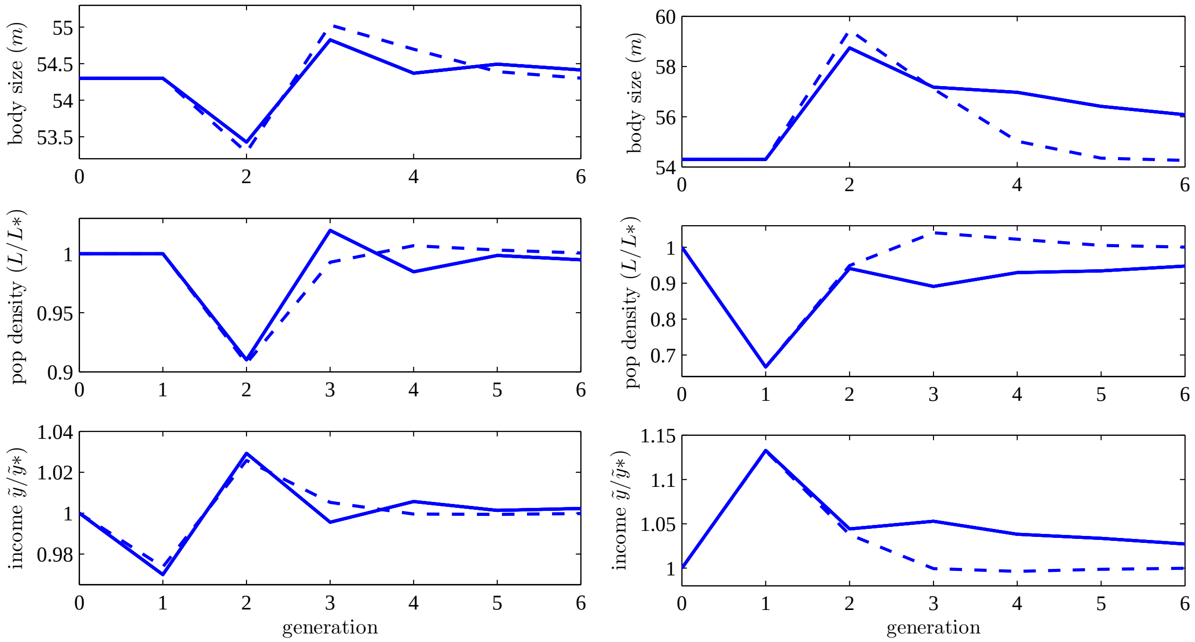
$$\log(y_t) = \log(Z) + \frac{\phi}{b}\log(B_t) + (1-\alpha)\log(X/L)$$

with $\log(Z) \equiv \log(A) + (\phi/b)\log(B_0)$. Assuming that energy needs are a reasonable proxy for energy consumption we can employ Strauss' (1986) estimates for the impact of nutrition on farm productivity to calibrate ϕ. Strauss finds that the elasticity of farm output with respect to nutrition is about 1/3 (with a standard error of 0.11), which in theory reflects ϕ/b. Since b is 3/4, it follows that a reasonable value for ϕ is 1/4, which we will use below.

We normalize the (initial) steady-state size of the population of adults to unity. Suppose that steady-state output per capita is 400 (international dollars) per year as estimated for most European countries during the early and high Middle Ages (Maddison, 2003). Then, during the period of adulthood income is $\$ 12,000 = 30 \cdot 400$. Using the values for m^*, L^*, and y^* we get an estimate of (initial) factor productivity A of 9415. We determine the remaining two parameters, a and ϵ, by solving the numerically specified model at m^* and L^*. This provides the estimates $a = 0.85$ and $\epsilon = 0.076$. Table 1 summarizes the notation, the numerical specification, and the sources it is based on.

5.2. **Dynamic Properties of the Model.** In this section we investigate adjustment dynamics by two numerical experiments: a drought and the black death. In both experiments the economy rests initially (period 0) at the steady-state. The drought experiment temporarily reduces productivity A by three percent in period 1. In the phase diagram the parameter change leads to a temporary upward shift of both the $\Delta m = 0$-locus and the $\Delta L = 0$-locus leaving the intersection at m^* unchanged. As a consequence the economy leaves the steady-state in south western direction. People react on the deteriorated conditions by having less kids and by providing less nutrition per child. Adjustment dynamics shown by solid lines in Figure 4.A set in. The benchmark model is reflected by solid lines.

Figure 4: Impulse Responses



A. Drought

B. Black Death

Left: temporary reduction of productivity A in period 1 by 3%. Right: reduction of population size by 1/3 in period 1. Solid lines: benchmark economy. Dashed lines: alternative economy with body size in the utility function (See Appendix C). All parameters values from Table 1.

Lower productivity leads to a *temporary* decline in body size and population density in the following ("period 2") generation.[14] On less densely populated land the adults of period 2 are relatively prosperous, and since they are also relatively small, income exceeds subsistence needs. Consequently they can afford to increase fertility and nutrition per child, which then increases density and body size in the following period. The economy thus adjusts in damped cycles towards the steady-state.

Because the model allows adjustment in both nutrition and fertility it can produce and explain oscillating adjustment dynamics of income and population size. The standard Malthusian model, in contrast, predicts monotonous adjustment dynamics (Ashraf and Galor, 2011). This distinctive feature of the model thus allows us to capture Malthus' (1888) original ideas on the oscillatory nature of the adjustment of population after a shock (Lee and Loschky, 1987), and provides an endogenous mechanism that explains the cyclical evolution of fertility, population

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[14] Long-lasting effects of droughts and famines during childhood on adult height and weight have been established in previous research; see e.g. Alderman et al. (2006); Meng and Qian (2009); Maccini and Yang (2009).

size, and body size in medieval and early modern times (Duncan and Thomas, 2004; Woitek, 2004)

Impulse responses for the utility-from-body-size model (cf. Appendix C), based on the same calibration as the benchmark model, are shown by dashed lines in Figure 4. The alternative model produces only mildly overshooting behavior but otherwise quantitatively similar impulse responses. The results of the baseline model are thus a fairly accurate approximation to the results stemming from the utility-from-body-size model.

The numerical experiment displayed in Figure 4.B. show responses to an exogenous decline of the population by 1/3 (i.e., the Black Death). In line with the standard Malthusian model the drastic reduction of population leads to an increase of the marginal product of labor and thus income. Higher income induces parents to increase fertility and to nourish their children better implying that the next adult generation is more numerous and heavier. But since body size exceeds its steady-state level this situation is not sustainable. Pressured by high subsistence needs parents reduce fertility and nutrition and the economy adjusts towards the steady-state from above. The utility-from-body-size model produces a mildly overshooting response of population size and thus a faster convergence of body size towards the steady-state but displays otherwise similar adjustment dynamics to the baseline one.

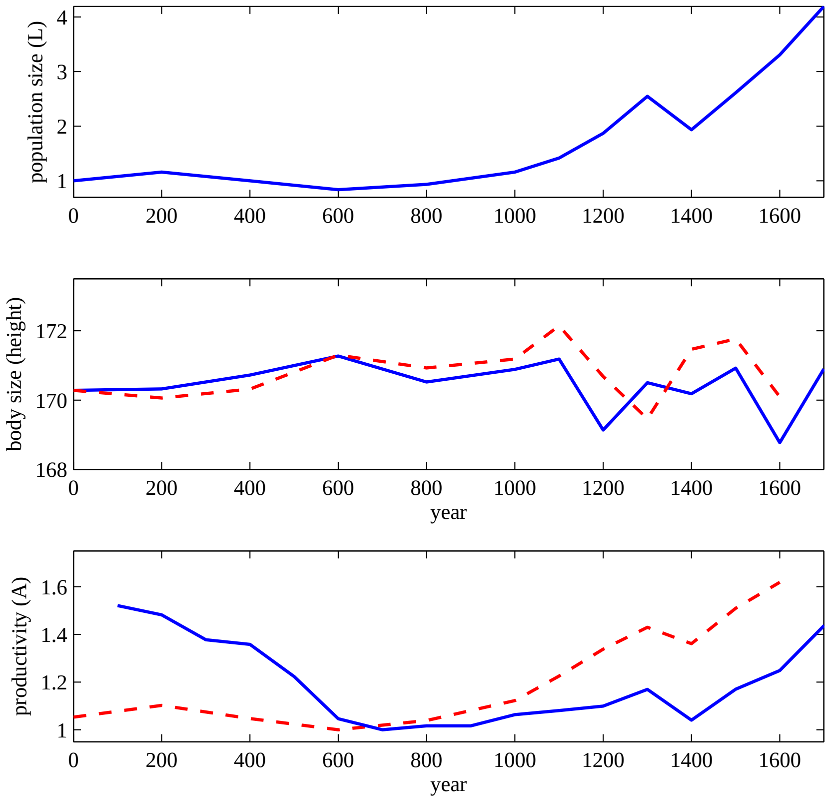
6. Understanding Pre-industrial Dynamics

In this section we gauge the ability of the model to account for the dynamics of Pre-industrial development within Western Europe. For most of the preceding millennia, prior to the onset of the industrial revolution, the growth trajectory of the European population was slightly upward sloping with occasional disruptions, for example, those caused by the Black Death. Overall, the Western European population increased by about a factor of four from the first to the 17th century (Kremer, 1993). Interestingly, for average European body size, i.e. stature, there was no visible trend during the Middle Ages. The study by Koepke and Baten (2005), drawing on height data derived from European skeletal remains, find strong evidence in favor of oscillations in stature around a constant trend level of height (circa 170 cm) during the last two millennia. Statistical tests reveal that the deviations from trend are significant.

Taken together this evidence suggests that prior to the industrial revolution human societies witnessed step-wise increases in population (punctuated by occasional declines), while at the same time height exhibited very little trend in either direction (see also Clark, 2007). Instead, cyclical movements in height, around a constant trend, seem to have been the norm. We know from the comparative statics in Section 4 that our theory is well situated to explain these movements by gradual technological change (productivity growth). After any permanent improvement of technology, population density is predicted to adjust to a higher level while body size is predicted to fall back to the initial steady-state level.

In order to assess the quantitative predictive power of the model we design the following experiment. We take the historical data on European population from Kremer (1993) and on body size (height) from Koepke and Baten (2005). We then calibrate in any century productivity A such that the model values for m_t, L_t, L_{t+1} match the historical data. Subsequently, we use the calibrated model (as specified in Table 1) along with the estimated time series for productivity to predict m_{t+1}.

Figure 5: Predicting Average Stature in Europe



Solid lines: historical time series according to Kremer (1993), Koepke and Baten (2005), and Morris (2010). Productivity shocks are calibrated such that the model economy produces the historical evolution of population size. Dashed lines: predicted body size and productivity.

In order to compare with the historical data we have converted the historical height numbers into weight using a constant body mass index (BMI) of 20. After the simulation, body mass is re-converted by the same BMI into height and the predicted time series is compared with the empirical data.[15] Admittedly, the data on height should be interpreted with care, as they are based on skeleton remains from a variety of archaeological excavations, which may not be equally representative. Still, this is to our knowledge the only source of intertemporally comparable data on body size, which spans the period of European history where Malthusian forces arguably were of first order importance.

Figure 5 shows the result of this exercise. By construction, the imputed productivity series fully explains the empirical population series and thus the secular increase of population size by about a factor of four over the 17 centuries. These productivity changes induce the model to predict mean-reverting fluctuations of body size that match the historical data reasonably well. In particular, for most centuries the model correctly predicts the direction of change for body size. The only period in history where the model fails drastically is around the time of the Black Death episode. The explanation here is very intuitive. The big slump of population growth in the 14th century was to a large extend caused by sharply increasing mortality (instead of productivity changes). Since there is no explicit role for mortality in the model, it misinterprets the slump as solely caused by declining productivity, which leads it to predict too few births and too little nutrition in the century before the population decline. As a result, the model gets the stature cycle wrong. After the Black Death episode the model resumes to predict changes of body size reasonably well.

Dashed lines in the bottom panel in Figure 5 shows the calibrated series for productivity (A). The model predicts a U-shaped path for A, with a minimum around the 7th century. Overall, our imputed series for A suggests a 1.6 fold increase in productivity from year 0 to the end of the period.

How plausible is the imputed productivity series? This is obviously a difficult question to answer since A is unobservable, absent direct measurements of output and production inputs. But a rough of idea of what the answer is likely to be can be obtained from the figure, where the solid line in the lower panel of Figure 5 show the evolution of the "social development

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[15] The body mass index is defined as height (in meters) divided by the square root of weight (in kg). Ideally we would like to compare the models' predictions directly to historical data on body size in the sense of weight; to our knowledge such data does not exist, unfortunately.

index" compiled by Morris (2010). This index is thought to capture (p. 144): "the bundle of technological, subsistence, organizational, and cultural accomplishments through which people feed, clothe, house, and reproduce themselves, explain the world around them, resolve disputes within their communities, extend their power at the expense of other communities, and defend themselves against others attempts to extend power"; it seems reasonable to expect that this index ought to be positively correlated with A in our model. To construct the index, Morris weighs four "traits" together: (i) the extent of energy capture; (ii) number of citizen's in the largest city; (iii) war making capacity and (iv) the state of information technology.[16] The solid line in Figure 5, however, only involves the average of (i), (iii) and (iv) as the size of the largest city undoubtedly is correlated with over-all population density; had (ii) been included in the index a positive correlation with our imputed A would follow almost by construction. To facilitate a comparison between the imputed A and Morris' index the series are normalized by their value in the 7th century (roughly the minimum according to both series).

Bearing in mind that the social development index is best viewed as a proxy for productivity it is unsurprising that there is a marked level difference to our imputed A series. The significant question is whether the two series evolve in sync, which largely is the case. The exception is the period from year 0 to 200 C.E., where our imputed series exhibits a slight increase whereas Morris' index declines. A simple explanation might be that Koepke and Baten's data on body size pertain to observations for Western and Central Europe (locations in contemporary Germany, Benelux, Austria, Northern France, Switzerland and the UK), whereas Morris' series speaks to the Roman Empire in its entirety. It seem likely that there was a significant differences in development between the core parts of the Roman Empire and Northern Europe during the first few centuries C.E. This will cause our imputed series for A to be underestimated until the Roman Empire starts its century long period of decline. As a result, the imputed series for A will underestimate the decline in productivity from the zenith of the Roman Empire and into the Dark Ages. But apart from the period 0-200 C.E. the two series do track each other to a reasonable extent across the centuries.

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[16] Details of the method are found at `http://ianmorris.org/socdev.html`.

7. PRE-INDUSTRIAL COMPARATIVE DEVELOPMENT: REGRESSION ANALYSIS

7.1. **Empirical Specification.** The key steady state prediction of the model is that population density should increase in response to greater productivity, whereas body size (weight) should be unaffected by productivity in the long-run. Hence, productivity holds a differential impact on the size and number of people. Provided a reasonable proxy for productivity can be obtained along with data on population density and body weight this prediction can be tested.

From Propositions 1 and 2 it follows that steady state population density and body weight can be written:

$$\log(x_i) = \alpha_0 + \alpha_1 \log(A_i) + \beta' \mathbf{Z}_i + \varepsilon_i \tag{16}$$

where $\log(x_i)$ is either population density or body size (kg), $\log(A_i)$ is productivity, whereas $\mathbf{Z}_i$ contains additional controls. The unit of analysis i is either ethnic groups or "regions", as described below. We cannot observe the steady state level of either variable so we follow Ashraf and Galor (2011) and proxy it by observed counterparts.

The claim in Proposition 2 is that $\alpha_1 > 0$ when the left hand side variable is population density, but $\alpha_1 = 0$ when body size is the dependent variable.

7.2. Data.

7.2.1. *Ethnic group data.* Our first sample involves societies present in the Ethnographic Atlas (Murdock, 1967). The Ethnographic Atlas contains cultural, geographical, and economic characteristics for roughly 1270 ethnic groups around the world, and has recently been employed in several studies within the economic sciences (e.g., Alesina et al, 2012; Michaelopolous and Papaioannou, 2012).

Unfortunately, the *Atlas* neither contains information about population density nor on body size for which reason we have had to collect the data. The online Appendix lists the detailed sources observation-by-observation. At this point we simply note, in the interest of brevity, that two key sources for population density is Binford et al. (2012) and Marlowe (2005). Together with other published studies referenced in the online Appendix we end up with a data set covering 196 ethnic groups that also are found in the Ethnographic Atlas. The societies for which we have data on population density are largely hunter-gatherers; only five societies with data on population density subside chiefly on agriculture (i.e., 50% dependence or more).

In terms of body size, measured by male body weight, we have been able to find data for 91 societies covered by the *Atlas*.[17] Key sources are Binford et al. (2012) along with the compilation in Eveleth and Tanner (1976), which we have supplemented with data from studies in anthropology and ethnology, as laid out in the online Appendix. This sample comprises both societies that subside on agriculture and via hunting and gathering. Regrettably, the overlap with the "density sample" is minuscule preventing a meaningful "same-sample" check of the productivity link. Fortunately, such a check is possible in our other sample, which we describe below.

In proxying A we are mainly interested in "external" measures of productivity, which therefore cannot be said to be influenced by our dependent variables; density and average body weight, respectively. In the context of societies that subside on agriculture the most obvious measure would be geographically determined "land quality", as produced in e.g. Michaelopolous (2012). This sort of measure combines information about soil conditions along with information about precipitation, temperature and sun hours. However, in light of the fact that we are dealing with both agricultural societies and hunter-gatherers (for which the agro-based land quality measure may not be optimal) we have opted for a more parsimonious approach.

As a proxy for "A" we include a measure of soil conditions ("soil quality") and (log) rainfall (1000 mm), separately. Temperature is also found in the control set (as noted below), but it might pick up other sources of influence on density and weight; e.g., via disease incidence. As a consequence we do not focus on temperature when testing the link between A, density and body size. Soil conditions are captured by the fraction of land area, in a 200 km radius around the center of the ethnic group, where crop growth is feasible judged from factors such as soil depth, fertility, drainage, texture, chemical and terrain slope agriculture (see online Appendix for a global map). Precipitation is a similar (200 km) average, using weather data for the period 1901-2000. When testing the theoretical link between A and population density and body size, respectively, we will examine the individual and joint significance of these two variables; both variables are expected to be positively correlated with productivity. While this is our main approach to the measurement of productivity in the ethnic-group sample, we also provide results

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[17] Ideally, we would want data on female body weight. These data are, however, are a lot scarcer than male weight for which reason we use the latter. Still, it is worth noting that male and female weight appears to be highly correlated across ethnic groups. A regression of female body weight on male body weight (N=33)"returns a slope estimate of 0.99, significant at the 1% level, and a statistically insignificant intercept of -0.15.

from using "land quality"; these results are found in the online appendix along with a map that depicts the global distribution of the variable.

The main concern is that omitted variable bias might be tainting our results; reverse causality can safely be ruled out on a priori grounds. To control as best we can for potential confounders we include an extensive set of additional productivity controls in our regressions. Inspired by the literature on fundamental determinants of prosperity (Acemoglu, 2009, Ch. 4), we include controls that fall in three broad groups: "Geography", "Institutions" and "Culture". The controls are thus motivated by their potential auxiliary impact on productivity (beyond soil quality and rainfall).

As "Geography" controls we include variables that speak to the *location* of the ethnic group: distance to ocean, land area within 100 km of coast and settlement patterns. The latter variable is a 1 to 8 indicator ranging from nomadic (1) to complex settlements (8), and is based on V30 in the *Atlas*. We also include controls of a *climatic* nature: temperature, absence of frost, absolute latitude and continent dummy's. To control for ethnic *institutions* we follow Michaelopolous and Papaioannou (2012) and employ "Jurisdictional Hierarchy Beyond Local Community", which is a 0 - 4 indicator in which a higher number testifies to greater institutional complexity. In order to control for *cultural* sources of influence on productivity we employ language subfamily fixed effects based on V99 in the *Atlas*. While the potential number of cultural fixed effects is large, only 22-23 subfamilies are in practise represented in our samples. Finally, to deal with the fact that our samples comprise both hunter-gatherers and agriculturalists we also control for the *mode of subsistence*. Specifically: the ethnic group's dependence on animal husbandry and agriculture (V4 and V5 in the *Atlas*). The sources of the controls are provided in Appendix D, along with summary statistics for density, weight, the environmental determinants of A as well as latitude. The latter shows that societies in our samples range from the southern part of Latin America to the Arctic North.

7.2.2. *Pixel-level data.* The ethnic-group sample involves societies that a priori should be describable by our model, which then allows us to potentially falsify key predictions of the theory. Yet the question is whether the model is able to speak to more modern societies. To address this question we have had to overcome two challenges. The first is the problem of data availability on body size and the other is to identify contemporary societies where our model should be relevant on a priori grounds.

On the first front we have chosen to rely on Demographic Health Surveys (DHS), which today covers about 90 countries. Of course, only a fraction of these countries have yet to undergo the fertility transition, and our model only speaks to a pre demographic transition setting. Since country coverage therefore inevitably will be small we resort to sub-national data in the analysis. This is possible because DHS has begun to produce geo-referenced surveys, which contains information about the location where the survey data was collected. With this information in hand, we were able to construct estimates for average female body weight at each of the reported DHS enumeration areas, which we subsequently use to calculate "regional" (respondent weighted) averages. In our data set each "region" is a pixel of size one degree latitude by one degree longitude.

In order to deal with the second obstacle, we began by consulting Reher (2004) who provides estimates for the "year" of the fertility transition around the world based on Crude Birth Rates (CBR). The advantage of using CBR is its availability for many countries and years. A drawback is that the estimated transition year often deviates from one based on total fertility rates (or marital fertility). For instance, whereas the English fertility transition is conventionally said to occur in the 1880s (e.g. Hatton and Martin, 2010), Reher's estimate is 1910. Hence, we subsequently scrutinized the evolution of total fertility rates for the countries that Reher's study suggests underwent the fertility transition at the end of the 20th century (or not at all). This allowed us to identify four countries that (also) exhibits non-decreasing total fertility rates until 1990 (or later), and for which geo-referenced data is simultaneously available: Ethiopia, Mali, Niger and Uganda.[18] This leaves us with a total of 233 pixel-level observations for population density, and 159 observations for female weight.[19]

Needless to say, the countries are not randomly selected since they all are characterized by a non-decreasing total fertility rate until the end of the 20th century. However, it is important to observe that the sample is not selectively constructed at the *sub-national level*, which is where we attempt to elicit information about the validity of the theory's predictions.

As our key control for productivity we employ (log) land quality from Michaelopolous (2012). We view this variable as a sensible proxy for A in the present sample, which comprises countries that rely on agriculture to a considerable extent; as late as 1990 the share of GDP stemming from

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[18] The DHS surveys that we employ were conducted around the year 2000, and contain information about the weight of females in the age group 15-44. Accordingly, all the surveyed individuals were born prior to the fertility decline.

[19] See the online Appendix for further details.

Agriculture was 54% in Ethiopia; 46% in Mali; 35% in Niger and 57 % in Uganda, according to World Development Indicators.

As in the ethnic-group setting we include a list of controls for ”geography” (including controls for location), “culture” and “institutions”. However, at a more detailed level some controls inevitably differ from those employed in the ethnic-group analysis. Specifically, “Land area” is a control in the pixel-level analysis, since the size of individual pixel’s (in square kilometers) varies depending on their exact location on the globe. Moreover, since the countries in focus are all from Sub-Sahara Africa, we include controls that previous studies have suggested to be particularly important in this region: malaria ecology (e.g. Sachs and Malaney, 2002) and terrain ruggedness (Nunn and Puga, 2010). To control for culture and political institutions we invoke country fixed effects. Finally, as temperature now is implicit in the land quality index we do not introduce it separately in our baseline regressions, so as to avoid collinearity. Yet, in our robustness checks we introduce it along with additional geographic determinants of productivity, as noted below. Details on sources, and summary statistics, are found in the Appendix.

7.3. **Results.**

7.3.1. *Ethnic-group analysis.* Table 2 reports our result for the ethnic-group sample. The first six columns involve population density as dependent variable, whereas the last six columns concern average weight. To conserve space we only report the estimates for the parameters of interest; i.e., for the environmental proxies for A. We always control for continental fixed effects, and language fixed effects. Moving from left to right in the table we progressively add controls so as to gauge the stability of the estimates for our A-variables; column 6 and 12 thus includes all controls simultaneously. At the bottom of the table we report the p-values associated with the joint exclusion of soil quality and precipitation.

The general finding is that both productivity proxies are statistically significant determinants of population density, regardless of the mix of controls. This is true for the variables individually, and jointly. In our full specification we are able to account for about 70% of the variation in population density across ethnic groups. The partial correlation between either proxy for A and population density is depicted in the online Appendix; their significance do not appear to be driven by outliers.

Table 2: Land Productivity, Population Density and Body Size across Ethnic Groups

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
variables			log pop density						log weight			
Soil quality	1.599***	1.238**	0.666**	1.673***	1.698***	0.369**	0.027	0.038	0.012	0.012	0.063	0.016
	(0.350)	(0.511)	(0.238)	(0.373)	(0.368)	(0.143)	(0.082)	(0.074)	(0.046)	(0.061)	(0.065)	(0.054)
Precipitation (log)	1.020***	0.977***	0.811***	1.058***	1.095***	0.806***	-0.010	-0.021	0.035***	-0.017	-0.016	0.013
	(0.037)	(0.066)	(0.042)	(0.046)	(0.046)	(0.034)	(0.018)	(0.023)	(0.010)	(0.011)	(0.016)	(0.022)
Continental FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Language FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Location	no	yes	no	no	no	yes	no	yes	no	no	no	yes
Climate	no	no	yes	no	no	yes	no	no	yes	no	no	yes
Institutions	no	no	no	yes	no	yes	no	no	no	yes	no	yes
Agricultural dep.	no	no	no	no	yes	yes	no	no	no	no	yes	yes
Soil=0 and Prec. =0	0.000	0.000	0.000	0.000	0.000	0.000	0.80	0.65	0.01	0.06	0.59	0.81
Observations	196	190	196	193	196	190	91	85	91	85	90	84
R^2	0.522	0.563	0.615	0.542	0.548	0.697	0.575	0.624	0.664	0.623	0.585	0.703

Notes: (i) ”Continents” refers to continental fixed effects; ”Language fixed effects” refer to linguistic sub-families for which there are up to 57 (V99 in the Atlas). Our full specification 6 (12) involves 23 (22) such fixed effects; ”location” controls for distance from coast and river, landmass within 100 km of coast and settlement patterns (V30 in the Atlas); ”climate” controls for absolute latitude, temperature and days without frost; ”institutions” control for Jurisdictional Hierarchy Beyond Local Community (V33 in the Atlas); ”agricultural dep” controls for dependence on animal husbandry and agriculture (V4 and V5 in the Atlas). All regressions contain a constant term. (ii) The 3rd row from below tests whether arable land and precipitation are jointly insignificant. (iii) *** , **, * denote significance at 1, 5 and 10%, respectively. (iv) Standard errors are clustered at the linguistic sub-family level. (v) Identical sample regressions were not attempted due to a minimal sample overlap (N=19).

It is obviously impossible to rule out that the significance of our environmental proxies for A could be due to some (set of) omitted variable(s). But following Nunn and Wantchekon (2011) we can provide a sense of how likely it is that unobservables can account for the observed link between our productivity measures and density. The check consists of calculating a measure of how strong selection on unobservables needs to be, relative to observables, in order to account for the significance of either productivity proxy. In the present case we find that selection on unobservables need to be five times greater than selection on observables, in order to explain the rainfall - productivity correlation away. While the soil quality - density link appears more fragile, by this metric, the former result suggests that omitted factors are unlikely to account for the results in Column 1-6 in their entirety.[20]

From Column 7 onwards we examine the productivity-body size nexus. Moving to the right in the table it is apparent that the economic and statistical significance of soil quality and precipitation is not much affected by the introduction of additional controls. In fact, the point estimates are essentially the same in column 12 as in column 7. These results are not driven by outliers as seen from the partial correlations depicted in the online appendix.

In two of the specifications, however, the two variables do appear as jointly significant, and in one of these rainfall is significant, individually (see columns 9 and 10). But generally, and in the setting were we add a full set of controls, soil quality and precipitation are (individually and) jointly insignificant at conventional levels.

Taken together these results support the model. There appears to be a differential impact of productivity on density and body size; whereas productivity supports population density it is not statistically significant in promoting bigger people. Notice also that that the economic significance of the productivity proxies diminishes visibly when moving from density being the dependent variable to body size. This is most easily seen from the parameter estimates associated with rainfall, as they are elasticities. The point estimate in Column 6 is 60 times larger than the counterpart in Column 12. Hence the statistical insignificance of rainfall in Column 12 is clearly not just a matter of an imprecisely estimated parameter.

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[20] The measure discussed in Nunn and Wantchekon (2011) is calculated as the ratio between the point estimate given a full set of controls (here: column 6 of Table 2), β_f, and the difference in estimates between a restricted set of controls, β_r and the full set of controls: $\beta_f/(\beta_r - \beta_f)$. To calculate the measure we ran a stripped down regression of density on rainfall and soil quality, controlling only for language fixed effects and ensuring that the sample is identical to the one from column 6 of Table 2 (N=190). This returns a point estimate for log rainfall of 0.97, and 1.67 for soil quality. The relevant measure for rainfall is thus 0.81/(0.97-0.81) = 4.8.

Turning to the issue of robustness, one might start by observing that some of the ethnic groups in our sample pertains to people from contemporary rich countries (e.g., Icelanders; Dutch). Murdock (1967) notes that their inclusion in the *Atlas* might be debated but proceeds to argue in favor of their inclusion based on (p. 101) "...his conviction that the exclusion of the Western peoples and their cultures from the ethnographic universe is totally unwarranted." Still, in additional robustness checks we have tried omitting all European societies, as well as ethnic groups that arguably identify as currently "rich" countries (e.g. the Japanese). Eliminating such ethnic groups only reduces the sample to a minor extent, and has virtually no impact on the results reported in Table 6 (see online Appendix).[21] Finally, as noted above, we also examined the impact from an alternative productivity proxy (land quality) on density and body size. The results are qualitatively similar, and discussed in the online Appendix.

7.3.2. *Pixel-level analysis.* Table 3 contains the main results for the pixel-level sample. The structure of the table basically mimics that of Table 2; the first six columns concern population density, whereas the following six relates to the productivity-body size nexus. In the present context it is possible to check whether the differential impact obtains in the same sample, which is examined in a final column 13.

The overall results are similar to those from the ethnic groups sample. Our external productivity measure is positively (and significantly) correlated with population density, but insignificantly correlated with body size. Moreover, the parameter estimates seem fairly stable despite the progressive inclusion of plausible confounders. The partial correlations are depicted in the online Appendix; it is clear that the results do not appear to be driven by outliers. The models accounts for 50-70% of the variation in population density and body weight. Moreover, we can confirm a positive link between population density and land quality in the same sample where we obtain an insignificant correlation with body size (cf. column 13).

It is worth noting that there is considerable difference in the economic significance of the two sets of estimates; whereas improvements in land quality by 1% increases density by about 0.3-0.4%, the similar figure for body weight is essentially zero. Hence, the differential impact

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[21] One new result, though, is that the joint significance of our productivity proxies in Table 2 (column 10), evaporates in the more limited sample, demonstrating its non-robustness.

Table 3: Land Productivity, Population Density and Body size across regions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
variables	log pop density						log weight						log pop density
Land quality (log)	0.476***	0.520**	0.459*	0.447*	0.441***	0.326***	-0.017	-0.001	0.001	0.001	-0.004	0.001	0.325**
	(0.033)	(0.093)	(0.167)	(0.186)	(0.067)	(0.035)	(0.010)	(0.003)	(0.006)	(0.002)	(0.005)	(0.005)	(0.084)
Country FE	no	yes	yes	yes	yes	yes	no	yes	yes	yes	yes	yes	yes
Climate	no	no	yes	no	no	yes	no	no	yes	no	no	yes	yes
Geography	no	no	no	yes	no	yes	no	no	no	yes	no	yes	yes
Location	no	no	no	no	yes	yes	no	no	no	no	yes	yes	yes
Observations	233	233	233	229	229	229	159	159	159	157	157	157	157
R^2	0.253	0.285	0.345	0.344	0.530	0.587	0.196	0.648	0.673	0.659	0.672	0.699	0.501

Notes: "climate" controls for malaria ecology and absolute latitude; "Geography" controls for log area of the pixel and terrain ruggedness; "Location" controls for distance to major river, distance to ocean and distance to capital, respectively. (ii) *** , **, * denote significance at 1, 5 and 10%, respectively. (iii) Standard errors are clustered at the country level. (iv) Column 13 re-estimates the full specification for the sample in column 12. (v) The (1 × 1 degree lat/log) pixels derive from four countries as discussed in the text: Ethiopia, Mali, Niger and Uganda. (vi) The dependent variable in columns 1-6 and 13 is population density in 2000; in 7-12 it is average (female) body size.

from land quality is not simply a matter of imprecisely estimated parameters in the body weight setting.

If we examine the measure of how strong selection on unobservable variables need to be to account for the land quality/ population density link we obtain a factor of 2.2. While this is a tall order it is a more modest requirement than what we attained above in the ethnic-group sample, and our ability to motivate the variation in density and weight is slightly lower in the pixel-level setting.

Hence, in an effort to further examine the robustness of our baseline results, we started by including additional confounders. Specifically: temperature, topography (elevation) as well as distance to the nearest country border. As expected, the inclusion of temperature reduces the point estimate for land quality, but it retains statistical significance albeit at the 10 % level of significance. In the full model with all controls added simultaneously the point estimate in the population density model is about 0.2. Land quality remains uncorrelated with body weight (see the online Appendix).

In another check we examined whether the results appear to be influenced greatly by one of the individual countries; if they are, this too could be a sign of important omitted confounders. The results do not appear to be fragile. Omitting countries one by one, with a full set of controls, provides very similar results to those reported in Table 3 (see the online Appendix).

Finally, we also examined the correlation between body weight and economic activity at the pixel level (reported in the online Appendix). Part of the motivation is that previous research has established a positive link between body size and average income at the aggregate level, for which reason it is of interest to see if a similar finding obtains regionally (see e.g., Steckel, 1983; Weil, 2007). Moreover, the proposed model predicts the existence of a positive correlation between income and body size (cf. the corollary to Proposition 3).

In order to measure economic activity we employ the luminescence of individual pixel's seen from space at night, as proposed by Henderson et al (2012) and employed in an African context in Michaelopolous and Papaioannou (2012). Conditional on our baseline controls from Table 3 as well as population density (to ensure we focus on *per capita* income; see Michaelopolous and Papaioannou, 2012) we indeed uncover a positive correlation between weight and earth lights; per capita income tends to be higher in regions where individuals are bigger.[22]

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[22] Michaelopolous and Papaioannou (2012) document that the "lights data" predicts livings standards measured by a wealth index available from DHS.

Obviously, we can make no claims about causality in these regressions. Theory suggests a positive bi-directional casual link between size and income, and the regression analysis cannot identify (the magnitude of) each channel separately. Either way, however, the fact that a positive correlation is detectable in the data is consistent with the predictions of the model. Taken together with the results pertaining to the steady state determination of population density and body size we believe the empirical analysis provides fairly strong support of the main predictions of the proposed model.

8. Conclusion

The theory developed in the present paper proposes that child quality investments have been made throughout history. These quality investments in child nutrition were productive and served to elevate individual productivity. Nevertheless, in the long run income stagnated.

We hypothesize that stagnation persisted due to a "physiological check": when body size increases, subsistence requirements grow, which limits further nutritional advances and stabilizes the economic system around an endogenously determined subsistence boundary. We argue that this is a key mechanism in explaining the absence of per capita income growth, which characterized most of human history, and represents a fundamental reason why episodes of technological change did not instigate rising body size and productivity. In fact, as shown above, with quality investments taking place the Malthusian positive check is not sufficient to equilibrate the economy.

The two millennia that preceded the fertility transition in Europe witnessed gradually rising population density, punctuated by occasional declines for disease related reasons. At the same time, however, body size did not exhibit any upward or downward trend.

Our theory elucidates the mechanics behind this fact and the developed model is able to quantitatively account fairly well for the periodic changes in body size over the period in question. Similarly, key steady state predictions of the theory find support across ethnic groups as well as across regions within countries at a pre-fertility-transition state of development. Specifically, we document a differential impact from environmental richness on density and body size; higher productivity spurs population density but does not lead to physically lager individuals.

We believe that the present study can form the basis for further research in a number of different directions. One avenue of research would be to employ our framework in order to obtain a further understanding of the long-run historical growth record. A key stumbling block for research in pre-industrial development is the paucity of reliable estimates for income per capita over time and across countries.[23] Our model can be helpful in this regard, looking forward. While it is well known that skeletal remains can be used to provide height estimates (e.g., Koepke and Baten, 2005), it is equally possible to produce weight estimates (e.g., Auerbach and Ruff, 2004). With time series data in hand for body weight, it becomes possible to gauge changes in income across time and space, with reasonable quantitative accuracy. This is feasible as per capita income near subsistence is given by $y \propto m^b$, with $b = 3/4$. Hence, a one percent increase in body weight is associated with an increase in income by 3/4 percent. Similarly, if cross country estimates for historical body weight are calibrated using skeletal remains it becomes possible to examine the extent of comparative income differences, prior to the take-off, and key determinants thereof.

Another line of inquiry could be to further our understanding of the long-run movements of body size, before, during and after the demographic transition. In recent work we have taken a first step in terms of studying the transition from stagnation into sustained growth. Specifically, in Dalgaard and Strulik (2014) we augment the model above by endogenous human capital accumulation. In addition, technological change depends on human capital augmented labor, and we allow for the feature that greater levels of technological sophistication increases the return on skill accumulation (Galor and Weil, 2000). A key prediction of the theory is that societies which historically were inhabited by bigger individuals are likely to undergo the demographic transition, and thus the take-off to sustained growth, relatively early. The reason is that, due to the higher physiological costs of child quantity, bigger parents needs less of an encouragement to reduce family size and intensify quality investments; both in terms of nutrition and education. Accordingly, the theory predicts that places where individuals were physiologically bigger in pre-industrial times would experience the take-off to growth sooner. Accordingly, factors that historically worked to increase (steady state) body size should therefore hold significant explanatory power vis-a-vis contemporary cross-country income inequality; differences in human body size that have emerged due to positive selection, for example. The wealth of nations likely have physiological foundations.

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[23] The standard reference on pre-industrial development is of course Maddison (2003). Yet the accuracy of these data are often questioned by economic historians. See e.g. Persson (2010) and references cited therein.

References

Abdus S. and P. Rangazas, 2011. Adult Nutrition and Growth, *Review of Economic Dynamics* 14, 636-649.

Acemoglu, D., 2009. *Introduction to Modern Economic Growth.* Princeton University Press

Alderman, H., Hoddinott, J., and Kinsey, B., 2006, Long term consequences of early childhood malnutrition, *Oxford Economic Papers* 58, 450-474.

Alesina, A., P. Giuliano and N. Nunn, 2012. On the Origins of Gender Roles: Women and the Plough. Working paper (Harvard U).

Aiyar, S., C-J. Dalgaard and O. Moav, 2008. Technological progress and regress in pre-industrial times, *Journal of Economic Growth* 13, 125-44

Ashraf, Q., and O. Galor. 2011. Dynamics and Stagnation in the Malthusian Epoch. *American Economic Review* 101, 2003-41.

Astrand, P.O. and Rodahl, K., 1970, Body dimensions and muscular work, in P.O. Astrand and K. Rodahl (eds.) *Textbook of Work Physiology*, McGraw-Hill, New York, 321-39.

Auerbach, B.M. and C.B Ruff, 2004. Human Body Mass Estimation: A Comparison of Morphometric and Mechanical Methods. *American Journal of Physical Anthropology* 125, 331-342

Baker, M., 2008. A structural model of the transition to agriculture. *Journal of Economic Growth*, 13, 257–292

Becker G., 1960. An economic analysis of fertility. In: *Demographic and economic change in developed countries.* Conference of the Universities-National Bureau Committee for Economic Research, a Report of the National Bureau of Economic Research. Princeton University Press, Princeton, NJ, pp 209-240

Binford, L.R., Johnson, A.L., White, D.R., and Eff, E.A., 2012, *Coded Rdata and Codebook on Height and Population Density.*

Brody, S., 1945, *Bioenergetics and Growth*, Van Nostrand-Reinhold, New York.

Brown, J., J. Gillooly, A. Allen, V.M. Savage and G. West, 2004, Towards a metabolic theory of ecology, *Ecology* 85, 1771-89.

Brinkman, H-J., J.W. Drukker, and B. Slot, 1988, Height and income: A new method for the estimation of historical national income series, *Explorations in Economic History* 25, 227-64.

Butte, N.F. and J.C. King, 2005, Energy requirements during pregnancy and lactation, *Public Health Nutrition* 8, 1010-1027.

Charnov, E.L., 1991, Evolution of life history variation among female mammals, *Proceedings of the National Academy of Sciences of the United States of America* 88, 1134-1137.

Charnov, E.L., 1993, *Life History Invariants*, Oxford University Press, Oxford.

Charnov, E.L. and S.K.M. Ernest, 2006. The offspring - size / clutch - size trade-off in mammals. *The American Naturalist* 167, 578-82.

Clark, G., 2007, *Farewell to Alms: A Brief Economic History of the World*, Princeton University Press.

Cole, T.J., 2000, Secular trends in growth, *Proceedings of the Nutrition Society* 59, 317-324.

Croginer, E., 1981, Climatic and anthropometric variations in Europe and the Mediterranean area, *Annals of Human Biology* 8, 99-107.

Dalgaard, C-J. and H. Strulik, 2010. The Physiological Foundations of the Wealth of Nations. Discussion Paper No. 10–05 (University of Copenhagen).

Dalgaard, C-J. and H. Strulik, 2011, A physiological foundation for the nutrition-based efficiency wage model, *Oxford Economic Papers* 63, 232-253.

Dalgaard, C-J. and H. Strulik, 2014. Physiological Constraints and Comparative Development. Department of Economics, University of Copenhagen, Discussion paper 14-21.

Damuth, J., 1981, Population density and body size in mammals, *Nature* 290, 699-700.

Damuth, J., 1993. Cope's rule, the island rule and the scaling of mammalian population density, *Nature* 365, p. 748-50.

Deaton, A.S. and Muellbauer, J., 1986, On measuring child costs: With applications to poor countries, *The Journal of Political Economy* 94, 720–744.

de Onis, M., Onyango, A.W., Borghi, E., Siyam, A.,Nishida, C. and Siekmann, J., 2007, Development of a WHO growth reference for school-aged children and adolescents, *Bulletin of the World Health Organization* 85, 660-667.

Douglas J.W.B. and H.R. Simpson, 1964, Height in relation to puberty family size and social class: A longitudinal study, *The Milbank Memorial Fund Quarterly* 42, 20-34.

Duncan, C.J. and Scott, S., 2004, The key role of nutrition in controlling human population dynamics, *Nutrition Research Reviews* 17, 163-175.

Eveleth and Tanner, 1976. Worldwide variation in human growth. Cambridge University Press.

Fogel, R.W., 1994, Economic growth, population theory, and physiology: The bearing of long-term processes on the making of economic policy, *American Economic Review* 84, 369-395.

Fogel, R.W., 1997, New findings on secular trends in nutrition and mortality: Some implications for population theory, in: Mark Rosenzweig and Oded Stark (eds.), *Handbook of Population and Family Economics*, Volume 1A, New York, Elsevier.

Galor, O., 2005, From stagnation to growth: Unified growth theory, in P. Aghion and S. Durlauf (eds.) *Handbook of Economic Growth*, North Holland, Amsterdam.

Galor, O., 2010, The 2008 Lawrence R. Klein Lecture - Comparative Development: Insights from Unified Growth Theory, *International Economic Review* 51, 1-44.

Galor, O. and D. Weil, 2000, Population, technology and growth: From Malthusian stagnation to the demographic transition and neyond, *American Economic Review* 90, 806-828.

Galor, O., and O. Moav, 2002. Natural selection and the origin of economic growth. *Quarterly Journal of Economics*, 117, 1133-1191

Gustafsson, A. and Lindenfors, P., 2009, Latitudinal patterns in human stature and sexual stature dimorphism, *Annals of Human Biology* 36, 74-87.

Hagen, E.H., H.C. Barrett, and M.E. Price, 2006, Do human parents face a quality-quantity tradeoff? Evidence from a Shuar Community, *American Journal of Physical Anthropology* 130, 405-418.

Hansen, G. and E. Prescott, 2002, Malthus to Solow, *American Economic Review* 92, 1205-1217.

Hatton, T.J. and Martin, R.M., 2010. Fertility decline and the heights of children in Britain, 1886–1938. *Explorations in Economic History* 47, 505–519

Henderson, J.V. and Storeygard, A. and Weil, D.N., 2012. Measuring Economic Growth from Outer Space. *American Economic Review* 102, 994–1028

Katzmarzyk, P.T. and Leonard, W.R., 1998, Climatic influences on human body size and proportions: ecological adaptations and secular trends, *American Journal of Physical Anthropology* 106, 483-503.

Kleiber, M., 1932, Body size and metabolism, *Hilgardia* 6, 315-353.

Koepke, N. and J. Baten, 2005a, The biological standard of living in Europe during the last two millennia, *European Review of Economic History* 9, 61-95.

Komlos, J., 1989, *Nutrition and Economic Development in the Eighteenth-Century Habsburg Monarchy*, Princeton University Press.

Komlos, J., 1993. The secular trend in the biological standard of living in the United Kingdom 1730-1860. *Economic History Review*, 46, 115-44.

Komlos, J., 1998. Shrinking in a growing economy? The mystery of physical stature during the Industrial Revolution. *Journal of Economic History* 58, 779-802.
social science history, *Social Science History* 28, 191-210.

Kremer, M., 1993, Population growth and technological change: One million B.C. to 1900, *Quarterly Journal of Economics* 108, 681-716.

Kuh, D.L. and M. Wadsworth M, 1989, Parental height: childhood environment and subsequent adult height in a national birth cohort, *International Journal of Epidemiology* 18, 663-668.

Kunitz, S., 1987, Making a long story short: A note on mens height and mortality in England from the first through the nineteenth centuries, *Medical History* 31, 269-80.

Lee, M.L. and D. Loschky, 1987, Malthusian population oscillations, *Economic Journal* 97, 727-739.

Lagerof, N.-P., 2003, Gender equality and long run growth, *Journal of Economic Growth* 8, 403-426.

Lucas, R.E., Jr., 2000, Some Macroeconomics for the 21st Century. *Journal of Economic Perspectives* 14, pp. 159-168.

Lucas, R.E. Jr., 2002, The industrial revolution: Past and future, in: Lucas, R.E. Jr., *Lectures on Economic Growth*, Cambridge, Massachusetts: Harvard University Press.

Maccini, S. and Yang, D., 2009, Under the weather: health, schooling, and economic consequences of early-life rainfall, *American Economic Review* 99, 1006-1026.

Maddison, A., 2003, *The World Economy – A Millennial Perspective*, OECD.

Malthus, T. R., 1888, *An Essay On The Principles of Population*, 9th ed. London: Reeves & Turner.

Markovic, G., and Jaric, S., 2004, Movement performance and body size: the relationship for different group of tests, *European Journal of Applied Physiology*, 92, 139-49.

Marlowe, F.W., 2005. Hunter-gatherers and human evolution, *Evolutionary Anthropology: Issues, News, and Reviews*, 14, 54–67

Mays, S., 2010, The effects of infant feeding practices on infant and maternal health in a medieval community, *Childhood in the Past: An International Journal* 3, 63-78.

Moe, K.S., 1998, Fertility, time use, and economic development, *Review of Economic Dynamics* 1, 699-718.

Morris, I., 2010, *Why the West Rules – For Now*, Profile Books, London.

Michaelopolous, S., 2012. The Origins of Ethnolinguistic Diversity *American Economic Review*, 2012, 102(4): 15081539

Michaelopolous, S and E. Papaioannou, 2012. Pre-colonial Ethnic Institutions and Contemporary African Development. *Econometrica* (forthcoming)

Murdock, G.P., 1967. Ethnographic Atlas. University of Pittsburgh Press, Pittsburgh, PA.

Nunn, N. and D. Puga, 2010. Ruggedness: The Blessing of Bad Geography in Africa, Forthcoming: *Review of Economics and Statistics.*

Nunn, N and L. Wantchekon, 2011. The Slave Trade and the Origins of Mistrust in Africa. *American Economic Review* 101, 3221-3252

Persson, K.G., 2010. The End of the Malthusian Stagnation Thesis. Mimeo (University of Copenhagen).

Prentice, A.M. and R.G. Whitehead, 1987, The energetics of human reproduction, *Symposia of the Zoological Society of London* 75, 275-304.

Meng, X. and Qian, N., 2009, The long term consequences of famine on survivors: Evidence from a unique natural experiment using China's Great Famine, NBER Working Paper No. w14917.

Reher, D. S., 2004. The demographic transition revisited as a global process. *Population Space and Place* 10, 19–42.

Roberts, D.,F., 1953, Body weight, race, and climate, *American Journal of Physical Anthropology* 11, 533-558.

Roff, D.A., 2002. *Life History Evolution*, Sunderland, MA, Sinauer Associates.

Ruff, C., 1994, Morphological adaptation to climate in modern and fossil hominids, *Yearbook of Physical Anthropology* 37, 65-107.

Ruff, C., 2002, Variation in human body size and shape. *Annual Review of Anthropology* 31, 211-232.

Sachs, J. and P. Malaney, 2002. The economic and social burden of malaria. *Nature* 415, 680-85.

Sadurkis, A., N. Kabir, J. Wagner, and E. Forsum, 1988, Energy metabolism, body composition, and milk production in healthy Swedish woman during lactation, *American Journal of Clinical Nutrition* 48, 44-49.

Samaras, T.T., 2007, *Human Body Size and the Laws of Scaling: Physiological, Performance, Growth, Longevity and Ecological Ramifications*, Nova Science Publishers, New York.

Schneider, R., 1996, Historical note on height and parental consumption decisions, *Economics Letters* 50, 279-83.

Silventoinen, K., 2003, Determinants of variation in adult body height, *Journal of Biosocial Science* 35, 263-285.

Steckel, R., 1983, Height and per capita income, *Historical Methods* 16, 1-7.

Strauss, J., 1986. Does better nutrition raise farm productivity? *Journal of Political Economy* 94, 297–320.

Sharp, P. and Strulik, H. and Weisdorf, J., 2012. The determinants of income in a Malthusian equilibrium, *Journal of Development Economics* 97, 112-117.

Voigtländer, N. and Voth, H.J., 2009. The Three Horsemen of Growth: Plague, War and Urbanization in Early Modern Europe. CEPR Discussion Paper No. DP7275

Walker, R.S. and M.J. Hamilton, 2008, Life-history consequences of density dependence and the evolution of human body size, *Current Anthropology* 49, 115–12.2

Walker R., M. Gurven, O. Burger and M.J. Hamilton, 2008, The trade-off between number and size of offspring in humans and other primates, *Proceedings of the Royal Society* 275, 827-33.

Weil, D., 2007, Accounting for the effect of health on economic growth, *Quarterly Journal of Economics* 122, 1265-1306.

Weir, D.R., 1993, Parental consumption decisions and child health during the early French fertility decline: 1790-1914, *Journal of Economic History*, 259-74.

West, G.B and J. H. Brown, 2005, The origin of allometric scaling laws in biology from genomes to ecosystems: Towards a quantitative unifying theory of biological structure and organization, *Journal of Experimental Biology* 208, 1575-92.

West G.B., J. H. Brown and B.J. Enquist, 1997, A general model of the origin of allometric scaling laws in biology, *Science* 276, 122-26.

West G.B., J. H. Brown and B.J. Enquist, 2001, A general model of ontogenetic growth, *Nature* 413, 628-31.

Woitek, U., 2003, Height cycles in the 18th and 19th century, *Economics and Human Biology* 1, 243-257.

Appendix A: Proof of Proposition 3

The proof inspects the derivatives of (13) with respect to the physiological parameters a, d, μ, B_0, and ρ. For body size we observe immediately $\partial m^*/\partial a > 0$, $\partial m^*/\partial B_0 > 0$, $\partial m^*/\partial \mu > 0$, $\partial m^*/\partial \rho > 0$, and $\partial m^*/\partial d < 0$. For income we have $\partial y^*/\partial a = \omega b m^{b-1} \partial m^*/\partial a > 0$, $\partial y^*/\partial \mu = \omega b m^{b-1} \partial m^*/\partial \mu > 0$, $\partial y^*/\partial d = \omega b m^{b-1} \partial m^*/\partial d < 0$, $\partial y^*/\partial B_0 = \omega/B_0 m^b + \omega b m^{b-1} \partial m^*/\partial B_0 > 0$, and $\partial y^*/\partial \rho = (\beta+\gamma)\tau B_0 / \left[\gamma - 1 - (\beta+\gamma)\tau\right] / \epsilon m^b + \omega b m^{b-1} \partial m^*/\partial \rho > 0$, in which we have defined $\omega \equiv \left[(\beta+\gamma)\rho + \gamma - 1\right] B_0 / \{\left[\gamma - 1 - (\beta+\gamma)\tau\right] \epsilon\}$. Notice the positive relationship between potential income y^* and income per capita $\tilde{y}^*$.

For population density we have $\partial(L^*/X)/\partial a = -\theta\sigma m^{-\sigma-1} \partial m^*/\partial a < 0$, $\partial(L^*/X)/\partial \mu = -\theta\sigma m^{-\sigma-1} \partial m^*/\partial \mu < 0$, $\partial(L^*/X)/\partial d = -\theta\sigma m^{-\sigma-1} \partial m^*/\partial d > 0$, $\partial(L^*/X)/\partial B_0 = -\theta/[(1-\alpha)B_0] m^{-\sigma} - \theta\sigma m^{-\sigma-1} \partial m^*/\partial B_0 < 0$, and $\partial(L^*/X)/\partial \rho = -(A/\omega)^{\frac{1}{1-\alpha}-1}/(1-\alpha)(\beta+\gamma) \cdot \left[\gamma - 1 - (\beta+\gamma)\tau\right] / \left[\gamma - 1 + (\beta+\gamma)\rho\right] m^{-\sigma} - \theta\sigma m^{-\sigma-1} \partial m^*/\partial \rho < 0$, with $\theta \equiv (A/\omega)^{1/(1-\alpha)} > 0$ and $\sigma \equiv (b-\phi)/(1-\alpha) > 0$.

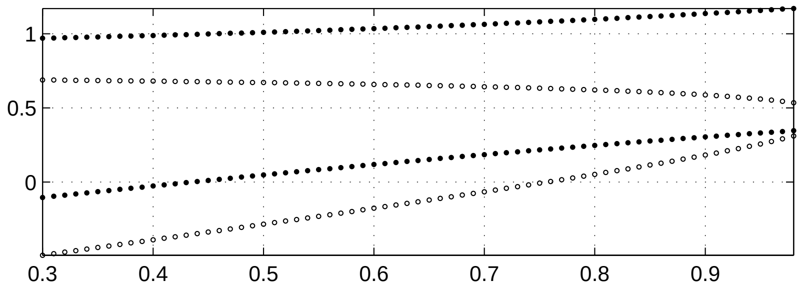
Appendix B: On the Numerical Analysis of the Model

Stability. We evaluate the Jacobian determinant of system (12) numerically. Given the value of b fixed by nature, we have identified the elasticities α and ϕ whose change induces the strongest reaction of dynamic behavior. Figure A shows the eigenvalues of the Jacobian and alternative α. White dots on the left hand side demonstrate stability of our benchmark calibration (parameter values from Table 1). Black dots reflect results for the case of $\phi = 1.5$ indicating a strong violation of the physiological check (all other parameter values from Table 1). The Malthusian equilibrium is now unstable for $\alpha > 0.40$, i.e. for any reasonable labor share.

An Evolutionary Argument for the size of γ. In light of the models' sensitivity to the value of γ it is of interest to try to gauge a likely size of this parameter. As it turns out, the weight of child quantity in the utility function can be given an evolutionary motivation, from which a magnitude is implied.

Suppose nature maximizes genetic fitness, given by the total number of descendants produced. Denote by π_t the fraction of surviving children. Then genetic fitness of the current generation is given by $\prod_{t=0}^{\infty} \pi_t n_t$. As the solution of the maximization problem is invariant to a monotonous transformation of the objective function, we let nature maximize the logarithm of genetic

Figure A: Eigenvalues



Parameters from Table 1 and alternative values of α. White dots: benchmark case from Table 1. Black dots: violation of the physiological check: $\phi = 1.5$.

fitness so that the objective becomes $\max \sum_{t=0}^{\infty} \log \pi_t + \sum_{t=0}^{\infty} \log n_t$. From the allometric literature we know that longevity (survival probability) of free living animals scales with body mass at factor 1/4 (e.g., Brown et al., 2004), i.e. $\pi_t \propto m_t^{\psi}, \psi = 1/4$. Evaluating (6) at the steady-state, we find that $m_t \propto c_t$ and thus $\log \pi_t \propto \psi \cdot \log c_t$. Inserting this information into "nature's objective function" and dividing through by ψ, we find that maximizing genetic fitness is tantamount to maximizing $\sum_{t=0}^{\infty} [\log(c_t) + (1/\psi) \cdot \log(n_t)]$. This is fulfilled when parents's of each generation maximize (7) for $\beta = 0$ and $\gamma = 1/\psi$. Note that evolutionary considerations thus predict $\gamma \approx 4$. This is the value we will use in our calibrations. Note that this relatively high value for γ serves to stabilize the economic systems (cf above).

Observe, finally, that this argument precludes that parents derive utility from own consumption, beyond metabolic needs. In a previous version of this research (Dalgaard and Strulik, 2010) we show that all our key results go through if $\beta = 0$, i.e. if parents only care about child nutrition and family size, but not about own consumption beyond metabolic needs.

Appendix C: Body Size of the Grown Up Child in the Utility Function

This section develops a version of the model where parents derive utility from child body size (instead of food expenditure for children). Parents now maximize (17) subject to (6) and (8).

$$U(c_t, n_t) = \beta \log(p_t) + \log(m_{t+1}) + \gamma \log(n_t) \tag{17}$$

By contrast to the baseline model, sophisticated parents take the law of motion for body size into account in their maximization of utility i.e. they understand how child nutrition affects ontogenetic growth. Solving the first order conditions for the variables of interest leads to the solution (18) .

$$p_t = \frac{\beta[y_t - (B_0/\epsilon)m_t^b]}{\beta + \gamma} \tag{18a}$$

$$c_t = \frac{a\epsilon[\tau y_t + \rho(B_0/\epsilon)m_t^b] - \gamma(1-d)\mu m_t}{(\gamma - 1)a\epsilon} \tag{18b}$$

$$n_t = \frac{(\gamma - 1)\left[y_t - (B_0/\epsilon)m_t^b\right]}{(\beta+\gamma)[a\epsilon(\tau y_t + \rho(B_0/\epsilon)m_t^b] - (1-d)\mu m_t} \tag{18c}$$

Inserting nutrition expenditure (18b) into (6) provides the law of motion for body size (19a) and inserting fertility (18c) into (11) provides the law of motion for population size (19b).

$$m_{t+1} = \frac{a(\tau\epsilon y + \rho B_0 m_t^b) - (1-d)\mu m_t}{\gamma - 1} \tag{19a}$$

$$L_{t+1} = \frac{(\gamma-1)a(\epsilon y - B_0 m_t^b)}{(\gamma+\beta)\left[a(\rho B + \tau\epsilon y) - (1-d)\mu m_t\right]} \cdot L_t, \tag{19b}$$

with y determined by (10). At the steady-state, $n_t = n^* = 1$. Using this information in (18c) and solving for y provides (20).

$$y^* = \frac{\gamma+\beta}{a\epsilon} m^* + \frac{B_0}{\epsilon}(m^*)^b. \tag{20}$$

The utility-from-child-size model thus preserves the positive association of income and body-size. Since y^* does not directly depend on a, d, or ρ, the model preserves the feature of a positive association across steady-states when countries differ (genetically) in stature of their inhabitants. Substituting (20) into (19a) provides body size at the steady-state.

$$m^* = \left[\frac{(\tau+\rho)aB_0}{\gamma - 1 - (\gamma+\beta)\tau + (1-d)\mu}\right]^{1/(1-b)}. \tag{21}$$

This expression looks very similar to the original m^* from the baseline model. Inserting y^* into (19b) provides population density at the steady-state.

$$\left(\frac{L}{X}\right)^* = \left(\frac{a\epsilon A}{(\gamma+\beta)(m^*)^{1-\phi} + aB_0(m^*)^{b-\phi}}\right)^{\frac{1}{1-\alpha}}. \tag{22}$$

Note that population density does not directly depend on a, d, or ρ. This means that the utility-from-child-size model preserves the feature of a positive association between body size and income across country-specific steady-states when countries differ (genetically) in stature of their inhabitants. The expression (22) is less compact than in the baseline model. But visual inspection reveals that the utility-from-child-size model preserves the features of the baseline model stated in Proposition 2, 3 and in Corollary 1.

APPENDIX D: DATA SOURCES

TABLE C.1: SUMMARY STATISTICS: MAIN VARIABLES

variable	obs	mean	stdev	min	max
ethnic group sample					
pop density	196	0.32	0.51	0.00	5.05
male weight	91	59.78	8.03	43.40	77.30
soil quality	196	0.72	0.32	0.00	1.00
land quality	196	0.35	0.34	0.00	1.00
precipitation	196	0.08	0.06	0.01	0.26
latitude	196	35.10	26.60	-55.00	78.00
latitude	91	16.50	25.90	-39.00	78.00
pixel-level sample					
pop density	233	45.46	66.21	0.15	347.04
female weight	159	52.98	4.43	44.33	71.2
land quality	233	0.32	0.33	0.001	0.99
absolute latitude	233	10.73	4.65	0	19

ETHNIC-GROUPS SAMPLE

- **Population density** Average population density. See online appendix for detailed sources
- **Body size**. Average male weight (kg). See online appendix for details
- **Rainfall** Average precipitation for the period 1900-2000 in a 200 km radius around the center of the ethnic group defined by lat/lon as reported in the Ethnographic Atlas (EA). Source: `http://www.cru.uea.ac.uk/cru/data/hrg/`
- **Soil quality**. Fraction of land area (200 km radius around center of ethnic society) which that is not "unsuitable" for crop growth based on soil conditions. Plate 27 of FAO's 2002 GAEZ database `http://webarchive.iiasa.ac.at/Research/LUC/SAEZ/`
- **Land quality**. Share of total area, which is arable according to the impact class measure (impact classes 1+2+3+4+5). Source: Plate 47 of FAO's 2002 GAEZ database: `www.iiasa.ac.at/Research/LUC/SAEZ/`
- **Temperature** Average precipitation for the period 1900-2000 in a 200 km radius around the center of the ethnic group. Source: `http://www.cru.uea.ac.uk/cru/data/hrg/`

- **Continent dummies**. Include Europe, Asia, North America, South America, Africa, and Oceania.
- **Distance to coast or river**. Distance to nearest coast or ocean from center of society.
- **Percent near coast** Percent of total area which lies within 100 km of icefree coast or navigable river, defined by Center for International Development, Harvard. Shapefile: `www.hks.harvard.edu/centers/cid`
- **Days without Frost**. Based on climate information during the period 1901-1930. Source: `http://www.ipcc-data.org/obs/cru_ts2_1.html`.
- **Absolute latitude**. V104 in the EA
- **Settlement patterns**. 1-8 indicator, with 1 being nomadic and 8 being complex settlement. V30 in the EA.
- **Jurisdictional Hierarchy Beyond Local Community** Following Michaelopolous and Papaioannou (2012), this is an ordered variable ranging from 0 to 4 indicating the number of jurisdictional levels (political complexity) in each society above the local level. Based on V33 in the EA.
- **Language fixed effects**. Based on V99 in the EA.
- **Agricultural dependence**. Dependence on animal husbandry and agriculture. V4 and V5 in EA.

Pixel-level sample

- **Body size**. Average female weight (kg). The basic data is collected by Demographic Health Surveys `http://www.measuredhs.com/data/available-datasets.cfm`. Some of the surveys are labelled "GPS datasets". Access can be obtained for the purpose of research upon request. In these data sets the place of data collection is linked to coordinates. With this data in hand we aggregated across enumeration areas to obtain the (weighted) average female body weight by (1 degree latitude, 1 degree longitude) pixel in the countries under consideration.
- **Population density, Land quality, Malaria ecology, distance to capital, distance to border, "earthlights"** Source: Michaelopolous and Papaioannou (2012) (MP). MPs data is found at a resolution of 0.125*0.125 decimal degrees. For present

purposes the data was “aggregated” to a 1*1 resolution by taking an area weighted average.

- **Latitude (degrees), elevation (m above sea level), temperature (average annual level 1980-2008, C degrees), precipitation (average annual level 1980-2008, ’000 mm), area (sq km), distance to ice-free ocean or major navigable river (km).** Source: Yale UniversityGeographically based Economic (G-Econ) data version 3.4. Data are available at `http://gecon.yale.edu.`.