Dual labour markets and nominal rigidity

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Abstract

The conventional menu cost framework performs poorly with realistic labour supply elasticities: the menu costs required for price rigidity are very high and the welfare consequences of monetary disturbances are negligible. We show that the presence of dual labour markets greatly improves the performance of the framework both by reducing the minimum effective menu costs and by boosting the welfare consequences of monetary disturbances are negligible. We show that the presence of dual labour markets provides an explanation of procyclical productivity and the shrinking of wage differentials during booms, in line with stylized facts on business cycles.

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1 Introduction

The insight that nominal inertia of prices and wages can be explained in an imperfectly competitive setting by the presence of lump-sum costs associated with price or wage changes was developed in the original New Keynesian literature (e.g. Mankiw, 1985; Blanchard and Kiyotaki, 1987; Ball and Romer, 1989, 1990, 1991) and has since been developed in a number of directions.[1] With price or wage setters, nominal prices are expected to be at or near to the optimal levels, with no first order effects of price changes on payoff. Furthermore, with imperfect competition – implying a socially suboptimal equilibrium – nominal inertia gives rise to first order welfare effects as output and employment vary. Thus, nominal disturbances may generate large welfare effects due to the presence of small menu costs.

Whilst the argument is simple and intuitive, the traditional menu-cost models faced a number of problems:

- the menu costs required for rigidity are too large for realistic values of the labour supply elasticity.[2] The weight of empirical evidence is that the labour supply elasticity is rather small, so that an increase in employment will result in a significant upward pressure on wages and hence on prices. With a perfectly inelastic labour supply, menu costs of any finite size cannot prevent price and wage changes.

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[1] Recent contributions to the menu-cost literature include Danziger (1999), Dixon and Hansen (1999), Dotsey *et al.* (1999), Bennett and LaManna (2001), Kleven and Kreiner (2001), and Kreiner (2002).

[2] Pencavel (1986) concludes from a survey of cross-section studies that the compensated elasticity for men is around 0.1. Higher estimates have though been found in life-cycle frameworks, e.g. MacCurdy (1981) reports elasticities in the range 0.1 to 0.45. And, even higher estimates have been found for women, see Killingsworth and Heckman (1986). These values are significantly below the elasticities considered by Blanchard and Kiyotaki (1987), who used the range of 2 to 5.

- Even if menu costs do result in a nominal shock increasing output, traditional models predict that productivity is either acyclical or countercyclical, which conflicts with the evidence that demand disturbances yield procyclical productivity (e.g., Rotemberg and Summers, 1990; Basu, 1996).

- Since labour markets are assumed to be symmetric, all wages are identical. Therefore, the models cannot explain the cyclicality of wage differentials, i.e. the compression of union-nonunion wage differences during booms found in empirical studies (see Melow, 1981; Freeman, 1984; Wunnava and Honney, 1991).

The aim of this paper is to solve these three problems by developing the labour market side, combining the traditional menu-cost model with dual labour markets.

The dual labour market approach has become an established part of labour market theory since Harris and Todaro (1970) and is confirmed by a substantial amount of empirical evidence; see Dickens and Lang (1985) and the survey in Saint-Paul (1996, pp. 62-68). In our framework, there is a *primary* labour market with monopolistic wage setting and a *secondary* labour market characterized by perfect competition. Thus, in equilibrium there is a wage differential in favour of primary labour. The output market has a representative monopolistic sector as in the traditional approach. The special case of no secondary labour market corresponds to the Blanchard and Kiyotaki (1987) model (BK hereafter), which we take as our benchmark for the traditional approach.

In general, we find that labour market duality improves the performance of the menu-cost approach. As in BK, we look at the response to a monetary expansion when

there are menu costs in both output and labour markets. The overall level of minimum effective menu costs as a percentage of GDP is smaller with dual labour markets, and the welfare consequences of nominal disturbances are larger. Moreover, in accordance with empirical evidence wage differentials are countercyclical and productivity may be procyclical. Furthermore, we focus on the problematic case of a low labour supply elasticity, where BK performs particularly badly. With a labour supply elasticity as low as 0.2 it turns out that the minimum effective level of menu costs is still within reasonable limits with our dual labour market approach.

Why does labour market dualism enhance the role of menu-costs? Note that the dual labour market equilibrium is characterized not only by a low *level* of aggregate employment (as in the traditional model) but also a *misallocation* of employment away from the primary to the secondary labour market. Following a positive demand shock, wages are pushed up in the competitive secondary labour market. In the primary labour market, menu costs lead unions to keep primary wages constant and expand employment. Hence, the wage differential in favour of primary labour shrinks, and employment in the primary market increases relative to employment in the secondary market. Since productivity is higher in the primary market, such a change in the allocation of employment increases overall productivity, thereby boosting both GDP and welfare.

The menu costs needed for wage rigidity in the labour market are smaller with the dual labour market. To grasp the intuition for this result, note that in the standard framework an increase in production can come about only by an increase in aggregate employment, whereas in our framework there is the additional possibility of *labour real-*

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location. As it is less costly for the trade unions to reallocate labour than it is to expand the aggregate number of working hours, the costs of keeping wages fixed are lower in the dual labour market case, implying that the menu costs required for wage rigidity go down. Although the minimum effective menu costs of firms tend to increase because of higher wages in the secondary market, overall the minimum effective menu costs of firms and workers are reduced by the introduction of a dual labour market.

The remainder of the paper is organized in the following way. In Section 2 we outline the basic structure of the model and the properties of the equilibrium with fully flexible prices. In Section 3 we analyze the occurrence of nominal rigidity caused by price and wage adjustment costs, and the implications for welfare, productivity, and wage differentials. Finally, Section 4 concludes and discusses the related literature.

2   The model without menu costs

In this section we outline the basic general equilibrium model with fully flexible wages and prices. A simple dual labour market is incorporated following Calvo (1978): we assume that each individual may work in either a primary labour market, where wages are determined by trade unions, or in a secondary labour market characterized by perfectly competitive wage setting. Firms produce differentiated goods using labour input hired in both the primary and the secondary labour markets. Thus, the model is one of *intra*sectoral dualism where high-wage jobs coexist with low-wage jobs within each firm. However, the model can easily be reinterpreted as one of *inter*sectoral dualism where wages differ across sectors.

Our framework is constructed in such a way that it has the simple, single labour market model of BK as a special case.[3] Furthermore, to facilitate comparison with the BK paper we use the same notation.

2.1 Households

Each individual household is identified by (j, h) where $j \in [0, 1]$ denotes the skill of the individual and $h \in [0, 1]$ is an index for individuals with this particular skill. The utility level of individual (j, h) is given by

$$U(j, h) = \left(\int_{i=0}^{1} C(i, j, h)^{(\theta-1)/\theta} \, di \right)^{\theta/(\theta-1)} - N(j, h)^{\beta}, \quad \beta > 1, \quad \theta > 1 \qquad (1)$$

where $C(i, j, h)$ denotes consumption of good $i \in [0, 1]$, θ is the elasticity of substitution between any two goods, $N(j, h)$ denotes the number of working hours, and β determines the steepness of the individual labour supply curve. The budget constraint of the household is given by

$$\int_{i=0}^{1} P(i) C(i, j, h) \, di \leq W(j, h) N(j, h) + \int_{i=0}^{1} V(i, j, h) \, di \equiv I(j, h) \qquad (2)$$

where $P(i)$ is the price of good i, $W(j, h)$ is the wage rate of the individual, and $V(i, j, h)$ is the household's (lump sum) share of profits from firm i. Maximizing (1) subject to (2) with respect to $C(i, j, h)$ gives the demand for good i of the household

$$C(i, j, h) = \left(\frac{P(i)}{P} \right)^{-\theta} \frac{I(j, h)}{P} \qquad (3)$$

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[3]Apart from a few trivial simplifications, that is. We exclude money from the utility function and assume instead like Ball and Romer (1989, 1990, 1991) that a simple transactions technology determines the relation between aggregate spending and real money balances. We also assume that goods and workers are distributed on the unit interval. This removes a lot of constants from the Blanchard and Kiyotaki framework and avoids the critique of the Dixit-Stiglitz framework put forward by d'Aspremont *et al.* (1996).

where P is the consumer price index given by

$$P \equiv \left(\int_{i=0}^{1} P(i)^{1-\theta} \, di \right)^{1/(1-\theta)} \tag{4}$$

By using eqs (1) through (4) we obtain the indirect utility function

$$U(j,h) = \frac{W(j,h) \, N(j,h) + \int_{i=0}^{1} V(i,j,h) \, di}{P} - N(j,h)^{\beta} \tag{5}$$

The labour market is segmented in two, a primary labour market where the wage set-ting is imperfectly competitive and a secondary labour market characterized by perfect competition. Before choosing consumption each individual is assigned to one of these two labour markets.[4] If an individual with skill j finds a job in the primary sector he will perform a task suitable for his particular skill and earn the hourly wage rate $W^{P}(j)$. If unsuccessful in finding a primary sector post, he can find employment in the secondary labour market where productivity is independent of skill and each worker earns the common hourly wage rate W^{S}. All workers with skill j are organized in a trade union, which determines the wage rate and the individual working hours for those members who work in the primary market but is unable to control the conditions in the secondary market. If assigned to the secondary labour market, the individual supply of labour is found by maximizing (5) with respect to $N(j,h)$, which gives

$$N(j,h) = N^{S} \equiv \left(\frac{1}{\beta} \frac{W^{S}}{P} \right)^{\frac{1}{\beta-1}} \tag{6}$$

where $\frac{1}{\beta-1}$ is the wage elasticity of the labour supply.

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[4]Clearly, since primary wages are higher, all workers prefer the primary sector. We assume that all workers are *ex ante* identical and that a random draw selects the lucky workers who get a job in the primary sector.

Finally, we assume that some transactions technology (e.g. a cash-in advance constraint) determines the relation between aggregate spending of the individual households and money balances

$$\int_{h=0}^{1} \int_{j=0}^{1} I(j,h) \, dj \, dh = M \tag{7}$$

2.2 Firms

The technology of firm i is described by

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\begin{aligned}
Y(i) &= \left(L^P(i)^\eta L^S(i)^{1-\eta} \right)^{1/\alpha}, & \alpha \geq 1, \quad 0 \leq \eta \leq 1 \tag{8} \\
L^P(i) &= \left(\int_{j=0}^{1} L^P(i,j)^{(\sigma-1)/\sigma} \, dj \right)^{\sigma/(\sigma-1)}, & \sigma > 1
\end{aligned}
$$

where $L^P(i,j)$ is employment of hours of skill j hired in the primary labour market, $L^S(i)$ is number of hours hired in the secondary labour market, σ measures the degree of substitution between any two types of skill in the primary labour market, and α is a returns to scale parameter. The parameter η determines the relative importance of primary and secondary employment, where the limits correspond to having only one labour market characterized either by perfect competition ($\eta = 0$) or monopolistic competition ($\eta = 1$) as in BK.

Cost minimization implies that the firm demands labour according to

$$L^P(i,j) = \left(\frac{W^P(j)}{W^P} \right)^{-\sigma} \eta \frac{W Y(i)^\alpha}{W^P} \quad , \quad L^S(i) = (1-\eta) \frac{W Y(i)^\alpha}{W^S} \tag{9}$$

where

$$W \equiv \left(\frac{W^P}{\eta} \right)^\eta \left(\frac{W^S}{1-\eta} \right)^{1-\eta} \quad , \quad W^P \equiv \left(\int_{j=0}^{1} W^P(j)^{1-\sigma} \, dj \right)^{\frac{1}{1-\sigma}} \tag{10}$$

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By using eqs (8) and (9) we can write profits as

$$V(i) = P(i)Y(i) - W^P L^P(i) - W^S L^S(i) \tag{11}$$

where the price and output of firm i are constrained by the aggregate demand of the households, derived from (3) and (7), i.e.

$$Y(i) = \left(\frac{P(i)}{P}\right)^{-\theta} \frac{M}{P} \tag{12}$$

Maximizing profits subject to (8) and (12) yields the following price-setting rule of the firm

$$\frac{P(i)}{P} = \left[\frac{\theta\alpha}{\theta-1}\frac{W}{P}\left(\frac{M}{P}\right)^{\alpha-1}\right]^{\frac{1}{1+(\alpha-1)\theta}} \tag{13}$$

2.3 Trade unions

All individuals with skill j are organized in a craft union which determines the hourly wage rate, $W^P(j)$, and the number of working hours, $N^P(j)$, of those members working in the primary labour market.[5] The union maximizes the aggregate utility of its members or, equivalently, the expected utility of a representative member. Accordingly, the choice of the union may be found by maximization of

$$S(j) = \int_0^{H^P(j)} \left(\frac{W^P(j)N^P(j)}{P} - N^P(j)^\beta\right)dh + \int_{H^P(j)}^1 \left(\frac{W^S N^S}{P} - \left(N^S\right)^\beta\right)dh \tag{14}$$

where $H^P(j)$ is the number/fraction of members working in the primary labour market. The first term denotes the utility of those employed in the primary labour market while

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[5] An alternative to this formulation is to let the unions set wages while letting the individual workers choose their own working hours. This would perhaps be more appealing from a descriptive point of view and, indeed, it is possible to construct such a model version albeit the derivations get somewhat more messy. The main advantage of the present formulation, however, is that it has the BK model as the special case of no secondary labour market, corresponding to $\eta = 1$. This enables us to make a clean comparison between the dual labour market model and the previous menu cost literature.

the second term is the utility of those who are unable to find a job in the primary market and therefore opt for secondary employment. The union is constrained by

$$H^P(j) N^P(j) = \int_{i=0}^{1} L^P(i,j)\, di$$

stating that the aggregate number of working hours in the primary market has to equal the aggregate demand for this particular skill. Thus, the union takes into account that higher wage claims or longer hours in the primary labour market will push members into competitively priced secondary jobs. Maximizing (14) subject to the above constraint and the labour demand schedule (9) yields

$$W^P(j) = \frac{\sigma}{\sigma - 1} W^S \quad \forall j \tag{15}$$

$$N^P(j) = N^S \quad \forall j \tag{16}$$

The primary wage rate is set as a mark-up over the opportunity cost, i.e. the wage rate in the secondary market (taken as given by each union). The magnitude of this mark-up is determined by the elasticity of substitution between the different skills. The choice of working hours in the primary labour market determines how the aggregate demand for primary labour is distributed over number of persons and number of hours per person, respectively. As the individual labour supply functions are strictly convex, cf. (6), it is optimal to smooth the number of working hours between the two possible states of an individual. Therefore, the union sets the number of working hours in the primary market equal to the going number of hours in the secondary market.

2.4 General equilibrium

Due to the symmetry of the model we have $W^P(j) = W^P \; \forall j$ and $P(i) = P \; \forall i$. Using these eqs as well as (6), (7), (8), (9), (13), (15), (16), it is possible to derive the equilibrium aggregate income/consumption level (see Appendix 1)

$$Y \equiv \int_{i=0}^{1} \frac{P_i Y_i}{P} di = \left(\frac{\theta - 1}{\theta} \left(\frac{\sigma - 1}{\sigma} \right)^{\eta \beta} \left(\frac{\sigma - \eta}{\sigma} \right)^{1-\beta} \right)^{\frac{1}{\alpha \beta - 1}} \hat{Y} \leq \hat{Y} \qquad (17)$$

where $\hat{Y} \equiv \left(\alpha \beta \eta^{-\eta \beta} (1 - \eta)^{\beta(\eta - 1)} \right)^{\frac{1}{1 - \alpha \beta}}$ is the efficient (Walrasian) income level. It is evident from this eq. that income is below its efficient level and that the inefficiency is larger when goods are less substitutable (smaller θ), when the skills of the workers are less substitutable (smaller σ), and when the labour supply is more elastic (smaller β). A higher η also reduces income, since this expands the relative size of the labour market characterized by monopolistic wage setting. The inefficiency of aggregate income stems from distortions in both the level and the allocation of employment. Firstly, as in BK, the presence of imperfectly competitive behaviour in goods and labour markets causes the individual number of working hours to be below its socially optimal level. Secondly, the dual structure of the labour market generates a misallocation of employment as the high primary wages leads to low primary employment relative to secondary employment.

When there is a perfectly inelastic labour supply, the aggregate employment is at its first best level. Hence, only the misallocation effect is operative. This special case isolates the effect of labour market dualism on aggregate income:

$$\lim_{\beta \to \infty} Y = \left(\frac{\sigma - 1}{\sigma} \right)^{\eta/\alpha} \left(\frac{\sigma}{\sigma - \eta} \right)^{1/\alpha} \hat{Y} \leq \hat{Y} \qquad (18)$$

This eq. highlights the difference between a standard menu cost framework, obtained by setting η equal to 0 or 1, and the generalized model with $0 < \eta < 1$. In the standard case, the equilibrium level of income is at its efficient level when labour supply is inelastic, whereas in the dual labour market case income is below its efficient level despite aggregate employment being at its first best level. Labour market dualism implies that productivity in the primary labour market is higher than productivity in the secondary market ($W^P > W^S$). Consequently, by reallocating workers from secondary employment to primary employment the economy would reap a productivity gain, thereby increasing GDP and consumption without inflicting more disutility of work on the households. Clearly, such a reallocation increases welfare, and therefore the market equilibrium is not a social optimum even with inelastic labour supply.

3   Menu costs, fluctuations and welfare

The previous section analyzed how dual labour markets influence the equilibrium of an economy without any nominal frictions. As with a simple, single labour market, money is neutral since all objective functions are defined on real variables. However, if price/wage setters face small adjustment costs (menu costs) of changing prices/wages, money may be non-neutral and changes in aggregate demand may give rise to fluctuations in real variables. In this section, we ask how the presence of a dual labour market affects the level of menu costs required for price and wage rigidity and the consequences of nominal disturbances. The first subsection deals with the size of menu costs needed to generate price stickiness (minimum effective menu costs), while the second subsection deals with

fluctuations in welfare, productivity, and wage differentials due to nominal disturbances.

It is well-known that imperfect competition is a precondition for the minimum effective menu costs to be small. Accordingly, the difference in competitiveness between the primary and secondary labour markets leads to different reactions to nominal disturbances: this contrast is crucial for understanding the results.

- The wage in the competitive secondary market rises after an increase in labour demand due to a monetary expansion.

- If menu costs are sufficiently high to prevent wage changes in the primary market then a monetary expansion always increases primary employment relative to secondary employment (cf. eq. (9)).

This reallocation of workers from low-paying, low productivity secondary jobs to high-paying, high productivity jobs in the primary labour market alleviates the initial distortion in the allocation of employment, thereby boosting GDP and welfare. However, the wage increase in the secondary market may also increase the minimum effective menu costs of firms by increasing their marginal costs of satisfying the extra demand. Thus, the magnitude of the wage rise in the secondary market following a monetary expansion is crucial both for the menu costs required for rigidity and for the welfare effects.

We start by finding the response of the secondary wage following a monetary expansion given that firms and wage setters in the primary labour market keep their respective prices and wages fixed. This is derived from (6), (7), (9), and (15), and is

given in elasticity form by (see Appendix 2)

$$\zeta \equiv \frac{dW^S/W^S}{dM/M} = \frac{\alpha\left(\sigma-\eta\right)\left(\beta-1\right)}{\eta\left(1-\eta\right)\left(\beta-1\right)+\sigma-\eta} > 0 \tag{19}$$

This elasticity indicates why standard New Keynesian results break down when labour supply becomes perfectly inelastic ($\beta \to \infty$). In this case ζ equals $\frac{\alpha(\sigma-\eta)}{\eta(1-\eta)}$ which goes to infinity when the dual labour market is excluded from the analysis ($\eta \to 0$ or $\eta \to 1$). In the absence of a dual labour market, a marginal increase in production can come about only by an increase in the number of work hours. With $\beta = \infty$ more work hours imply an infinite increase in the marginal disutility of work and hence the competitive wage. These shortcomings of the standard theory do not arise in the presence of a dual labour market as the above elasticity is finite even for a perfectly inelastic labour supply. This is because an increase in production is achieved (with a fixed level of aggregate employment) by reallocating workers away from low-productivity secondary jobs towards high productivity primary jobs. The following subsections analyze this issue in greater detail.

3.1 Minimum effective menu costs

This subsection derives the levels of menu costs of firms and workers that are sufficient to make price and wage rigidity a Nash-equilibrium. For the firms to keep prices fixed, menu costs must be greater than or equal to the increase in profits resulting from adjustment of prices. The profit gain resulting from adjustment or, equivalently, the minimum effective menu costs of firms is derived by making a second order Taylor approximation on (11) around the initial equilibrium, assuming that other firms do not change their prices

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and that workers keep primary wages fixed (which is indeed consistent with rational expectations provided that menu costs are sufficiently large). The aggregate level of minimum effective menu costs of firms in proportion of GDP equals (see Appendix 3)

$$\frac{dV}{Y} \approx F\left(\theta, \alpha, \eta, \zeta, m\right) \equiv \frac{1}{2}\left(\frac{\theta - 1}{1 - \theta + \theta\alpha}\right)\left[(1 - \eta)\zeta + \alpha - 1\right]^2 m^2 \qquad (20)$$

where $V \equiv \int_{i=0}^{1} V_i di$ and $m \equiv dM/M$. Equations (19) and (20) reveal that the minimum effective menu costs of firms are greater in the presence of a dual labour market compared to the case with a single monopolistic labour market. Since monopolistic wages are rigid, whereas competitive wages increase following a monetary expansion, marginal costs of firms increase more when a positive share of the labour market is characterized by competitive wage setting. Higher marginal costs imply that it is more costly for the firm to satisfy the extra demand, thereby driving up the level of menu costs required for price rigidity. However, the above derivation presumes that workers do not adjust the wages in the primary labour market and it is exactly this presumption which is so critical in the conventional models.

Turning to the labour market, we derive the private gain of union j from adjusting the wage in the primary labour market or, equivalently, the minimum effective menu costs of unions. This calculation is done by making a second order Taylor approximation on (5) around the initial equilibrium, assuming that other workers do not change wages in the primary labour market and that output prices are fixed, again in consistence with rational expectations. The aggregate level of minimum effective menu costs for workers

in proportion of GDP is given by (see Appendix 4)

$$\frac{dS}{Y} \approx G\left(\sigma, \eta, \theta, \alpha, \zeta, m\right) \equiv \frac{1}{2}\left(\sigma - 1\right) \frac{\eta\left(\theta - 1\right)}{\alpha\theta} \zeta^2 m^2 \tag{21}$$

where $S \equiv \int_{j=0}^{1} S_j dj$. The menu costs needed for wage rigidity in the primary market are always increasing in the wage response of the secondary market, ζ. According to eq. (19), the wage response, ζ, is always lower with a dual labour market and therefore it follows from the above eq. that the minimum effective menu costs of workers are also lower. Thus, we can state the following proposition on the minimum effective menu costs of firms and workers:

Proposition 1   Comparing the dual labour market setting $(0 < \eta < 1)$ with the single, monopolistic labour market setting $(\eta = 1)$ we have: (i) The minimum effective menu costs of firms, $F\left(\cdot\right)$, is greater with a dual labour market, whereas (ii) the minimum effective menu costs of workers, $G\left(\cdot\right)$, is smaller with a dual labour market.

Proof (i) Follows from (19) and (20) while (ii) follows from (19) and (21).   □

To elaborate further on the implications of having a dual labour market, we present some numerical examples in Table 1 illustrating the minimum effective menu costs following a 5% monetary expansion. In the first three rows we consider a labour supply elasticity equal to $1\frac{2}{3}$ (i.e. $\beta = 1.6$). The first row considers the single, monopolistic labour market $(\eta = 1)$ analyzed by BK which yields very low minimum effective menu costs.[6] In the second row, we introduce a dual labour market by reducing η to 0.5. As explained

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[6] In fact, the menu cost requirements of workers are much lower than in BK due to an error in the original article, see Hansen (1998).

above, this increases the minimum effective menu costs of firms but reduces the minimum effective menu costs of workers.

Table 1.   Minimum effective menu costs as percentage of GDP following a monetary expansion of 5%.

				Firms	**Workers**
β	σ	η	ξ	$F\left(\cdot\right)$	$G\left(\cdot\right)$
1.6	5.0	1.0	0.20	0.00	0.16
	5.0	0.5	0.10	0.06	0.07
	2.5	0.5	0.20	0.06	0.03
6.0	5.0	1.0	0.20	**0.00**	**11.00**
	5.0	0.5	0.10	1.69	3.37
	2.5	0.5	0.20	**1.07**	**0.78**
	1.4	0.3	0.20	1.55	0.12
	3.3	0.7	0.20	0.56	2.55
	2	0.5	0.25	0.85	0.41
∞	5.0	1.0	0.20	0.00	∞
	2.5	0.5	0.20	6.65	5.20

Note: In all examples we use $\alpha = 1.1$ and $\theta = 5$.

It is important to see that this change of η involves two effects. On the one hand the macro degree of imperfect competition is reduced implying that the distortion in the level of employment is alleviated. On the other hand the introduction of asymmetry in the labour market give rise to a deterioration in the allocation of employment. To isolate the effect of asymmetric wage setting we want to introduce a dual labour market while keeping constant the average degree of imperfect competition. Indeed, from a macro perspective the most clean comparison is one between the conventional, symmetric framework and the dual labour market model for a given macro degree of imperfect competition.[7] Employing eq. (15), the average mark-up in the primary and secondary

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[7]The standard New Keynesian results are derived in models with a single monopolistic labour market, which is trying to capture the average of all the different labour markets in real world economies. Accordingly, the mark-up of wages in these models should be thought of as the average degree of imperfect competition in the different labour markets. The hope is then that this procedure provides

labour markets, i.e. the macro Lerner index, is given by

$$\xi \equiv \eta \cdot \frac{W^P - W^S}{W^P} + (1 - \eta) \cdot 0 = \frac{\eta}{\sigma}$$

In the third row, we isolate the effect of a dual labour market by adjusting σ so as to keep constant the macro Lerner index, ξ. In this case there is a much larger reduction of the minimum effective menu costs of workers compared to the single market case.

A labour supply elasticity of $1\frac{2}{3}$ is clearly unrealistic as mentioned in the Introduction. Hence, the next six rows consider a much more realistic elasticity of 0.2 (i.e. $\beta = 6$). In the conventional symmetric model, the minimum effective menu costs of firms are still very low. In fact, with a single monopolistic labour market the labour supply elasticity is irrelevant for the minimum effective menu costs of firms; what matters is the returns to scale parameter, α, and when returns to scale are close to constant, effective menu costs are always low. However, the derivation of the menu costs of firms is based on the assumption that wages are rigid in the primary market and this assumption is not reasonable in the case of an inelastic labour supply. In fact, the minimum effective menu costs of workers are equal to 11% of GDP, which is clearly unrealistic.

In the next row we introduce labour market dualism by reducing η from 1 to 0.5. In this case the minimum effective menu costs of firms are higher, since marginal costs are pushed up by higher secondary wages. However, the minimum effective menu costs of workers are significantly reduced. With a symmetric labour market workers can only accommodate the higher demand by increasing the number of work hours, and this is

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reliable estimates of minimum effective menu costs, welfare effects, etc. The aim of this paper is to investigate whether this is indeed the case given that the macro degree of imperfect competition is really generated by several labour markets (in our case, two) with varying degrees of imperfect competition.

costly when labour supply is inelastic. By contrast, in the asymmetric setting the same effect may be achieved in a much less costly way through a labour market reallocation. Of course, this example does not isolate the effect of a dual labour market, since reducing η also implies less imperfect competition on a macro level. The pure effect of a dual labour market is apparent by comparing the fourth and sixth rows (in bold), where η is 1 and 0.5 respectively, and where the macro degree of imperfect competition, ξ, is held constant at 0.2. It is evident that the minimum effective menu costs of workers are 14 times lower relative to the symmetric setting. In the dual labour market case, prices and wages are sticky when menu costs of firms (workers) are around 1% (0.8%) of GDP. The level of minimum effective menu costs of firms is in line with the empirical evidence in Levy *et al.* (1997) and Dutta *et al.* (1999) but to our knowledge no evidence exists on wage adjustment costs. However, there seems no reason to believe that the costs of wage adjustments are lower than those of price adjustments.

In the next rows, we analyze the sensitivity of menu costs with respect to η and ξ. According to the table, the lower η and the higher ξ, the lower the minimum effective menu costs of workers. Finally, in the last two rows we look at the interesting special case where the elasticity of labour supply is equal to zero. In the standard framework, Keynesian results break down; menu costs of any size cannot generate monetary non-neutrality (the minimum effective menu costs of firms is still low, but it is derived on the assumption that primary wages are rigid, which they won't be). By contrast, in the dual labour market model minimum effective menu costs are finite; if menu costs are sufficiently high, trade unions are willing to keep primary wages fixed and accommodate

the higher demand for primary labour through a labour market reallocation away from the secondary sector. However, it should be noted that with a perfectly inelastic labour supply minimum effective menu costs are not within realistic limits.

3.2 Fluctuations in welfare, productivity, and wage differentials

This section is concerned with the consequences of monetary expansions, assuming that menu costs are indeed able to generate nominal rigidity. We show that labour market dualism boosts the welfare consequences of monetary expansions, and in addition offers a new explanation of the cyclicality of productivity and wage differentials.

Starting with the implications for wage differentials, note that a monetary expansion pushes up wages in the secondary labour market, whereas menu costs prevent wage adjustment in the primary labour market. This implies that the wage differential, $W^P - W^S$, narrows during booms and widens during recessions. This is in line with empirical investigations on the cyclicality of union-nonunion wage differentials by Melow (1981), Freeman (1984), Wunnava and Honney (1991) and others. In addition, this feature of the model is crucial for the consequences for welfare and productivity. The welfare effect of an increase in the money stock is derived by making a second order Taylor approximation on the utility function (1) around the initial equilibrium. This yields (see Appendix 5)

$$\frac{dU}{Y} \simeq W\left(\theta, \alpha, \xi, \beta, \zeta, m\right) = m - \frac{\theta-1}{\theta\alpha}\frac{1-\xi}{\beta-1}\left[\zeta m + \frac{1}{2}\zeta^2 m^2\right] \tag{22}$$

The welfare effect depends on η both through the macro Lerner index, $\xi = \eta/\sigma$, and

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through the wage response of the secondary labour market, ζ. The pure effect of a dual labour market is measured by changing η and at the same time adjusting σ so as to hold ξ constant. Since ζ is always lower in the case of a dual labour market than in the case of a single monopolistic labour market, eq. (22) shows that the welfare effect of a monetary expansion is larger in the presence of labour market dualism. The intuition for this result is not difficult to grasp. Following a monetary expansion, the secondary wages relative to primary wages go up, which implies an expansion of primary employment relative to secondary employment. Since the allocation of labour is distorted away from primary employment in the initial equilibrium, such a reallocation creates a positive welfare effect in addition to the positive welfare effect resulting from a higher employment level also present in the standard framework.

Turning to the change in productivity following monetary disturbances, we define aggregate productivity $Q \equiv Y/N$. As a measure of the cyclicality of productivity, we calculate the percentage change in productivity, Q, relative to the percentage change in GDP (see Appendix 6)

$$X(\beta, \zeta) \equiv \frac{dQ/Q}{dY/Y} = 1 - \frac{\zeta}{\beta - 1} \tag{23}$$

In other words, X is the share of the increase in GDP accounted for by higher productivity. It is a well established empirical regularity that productivity is procyclical (see e.g. Rotemberg and Summers, 1987), and any satisfactory theory of business cycles should be able to capture this stylized fact. However, this is not the case for the BK setup. By setting η equal to 1, it follows from eqs (19) and (23) that productivity is countercyclical with decreasing returns to scale ($\alpha > 1$) and acyclical in the special case with constant

returns to scale ($\alpha = 1$). By contrast, with a dual labour market ($0 < \eta < 1$) productivity may be procyclical with decreasing returns and is always procyclical with constant returns. Furthermore, comparing the standard framework with the dual labour market model, the productivity effect, X, is always greater in the latter case. Intuitively, during a boom employment of primary labour goes up relative to that of secondary labour, and since productivity is higher for primary labour, this reallocation contributes positively to productivity. We are now able to state the following proposition on the consequences of monetary shocks for wage differentials, welfare, and productivity:

Proposition 2   Holding constant the macro Lerner index ξ we have: (i) the wage differential, $W^P - W^S$, is countercyclical with a dual labour market ($0 < \eta < 1$). (ii) The welfare effect of a monetary expansion, $W(\cdot)$, is always greater with a dual labour market. (iii) With a single, monopolistic labour market ($\eta = 1$) productivity is countercyclical, i.e. $X(\cdot) \leq 0$. Productivity is less countercyclical or procyclical with a dual labour market.

Proof   (i) Follows from the fact that primary wages are fixed due to the menu costs, whereas secondary wages increase according to (19). (ii) Follows from (19) and (22). (iii) Follows from (19) and (23). □

Table 2 displays the total minimum effective menu costs of firms and workers as well as the change in welfare and productivity following a monetary expansion of 5%. In the first three rows we consider β equal to 1.6. In this situation, the BK setup performs well; small menu costs (0.16% of GDP) generate large welfare cycles (1.75% of GDP).

Yet, the model conflicts with empirical evidence by generating countercyclical productivity. Introducing labour market dualism, holding constant the macro Lerner index, ξ, the model performs better. Menu costs are smaller, the welfare effect is larger, and productivity is less countercyclical.

In the more realistic case with an inelastic labour supply, i.e. $\beta = 6.0$ corresponding to an elasticity of 0.2, the BK model performs very badly. It takes very large menu costs (11% of GDP) to generate cycles of a much smaller magnitude (1.36% of GDP), and in addition productivity is countercyclical. Reducing η from 1 to 0.5 significantly improves the results. The minimum effective menu costs are more than halved, the welfare effect is boosted, and productivity is now procyclical. Adjusting σ so as to capture the pure effect of a dual labour market, the picture looks even better. The total minimum effective menu costs are only 1.85% of GDP, and the welfare effect is now larger than the menu costs causing it (2.86% of GDP). Furthermore, 32% of the expansion in GDP is accounted for by increased productivity. Thus, the introduction of dual labour markets significantly improves the ability of menu cost models to explain economic fluctuations.

Table 2. The consequences for welfare and productivity
following a monetary expansion of 5%

β	σ	η	ξ	**Menu Costs** $F(\cdot)+G(\cdot)$	**Welfare** $W(\cdot)$	**Productivity** $X(\cdot)$
1.6	5.0	1.0	0.20	0.16	1.75	−10
	5.0	0.5	0.10	0.13	1.46	−6
	2.5	0.5	0.20	0.08	1.98	−2
6.0	5.0	1.0	0.20	**11.00**	**1.36**	**−10**
	5.0	0.5	0.10	5.06	1.88	14
	2.5	0.5	0.20	**1.85**	**2.86**	**32**
	1.4	0.3	0.20	1.66	3.17	41
	3.3	0.7	0.20	3.11	2.44	20
	2	0.5	0.25	1.26	3.24	40
∞	5.0	1.0	0.20	∞	−	−
	2.5	0.5	0.20	12.03	5.00	100

Note: In all examples we use $\alpha = 1.1$ and $\theta = 5$. Figures for Menu
Costs, Welfare, and Productivity are expressed as percentages.

In the next rows, we present a sensitivity analysis with respect to the wage share for primary labour, η, and the macro Lerner index, ξ. The lower η and the higher ξ, the better the performance of the model. Finally, in the last two rows we look at the case of a perfectly inelastic labour supply. In the standard framework, menu costs of any size will not be able to generate price and wage rigidity, and accordingly there is monetary neutrality. By contrast, in the dual labour market model menu costs can, in principle, explain business cycles. Note that since the boom is now solely generated by a reallocation from low-productive, secondary employment towards high-productive, primary employment, all of the income increase is accounted for by procyclical productivity.

4   Concluding remarks

In this paper, we have shown that the introduction of dual labour markets greatly improves the performance of the conventional menu cost framework: aggregate effective

menu costs are lower, the welfare consequences of nominal disturbances are larger, and in addition labour market dualism provides an explanation of procyclical productivity and countercyclical wage differentials. The numerical examples suggest that the presence of dual labour markets does lead to minimum effective menu costs that are within realistic limits even when labour supply is very inelastic. However, it must be said that the welfare effect is still small relative to the menu costs required for price rigidity, implying that the efficiency loss of fluctuations is modest.

While our model is one of *intra*sectoral dualism (high-paying jobs coexist with low-paying jobs within each firm), other writers have focused on the issue of *inter*sectoral dualism, where wages differ across sectors (e.g., Harris and Todaro, 1970; Calvo, 1978; Bulow and Summers, 1986). It can be shown that our model may be reinterpreted as a model of *inter*sectoral dualism. In this reinterpretation we consider an economy with two sectors producing different goods; one sector hires only monopolistic labour, whereas the other hires only competitive labour. This two sector model is formally equivalent to the one sector model of this paper when aggregate consumption is modelled as a Cobb-Douglas aggregate of the two types of goods in the same way as the two types of labour are aggregated in the one sector model. Thus, our results are not restricted to a specific type of labour market dualism.

The mechanism for boosting productivity in this paper is the reallocation of labour from the secondary to the primary labour market in response to a positive demand shock. This is analogous to the industrial policy advocated by Bulow and Summers (1986), namely the subsidising of the primary sector to increase its share of aggregate

employment. This would also have the effect of compressing wage differentials. However, there is also an important difference. An on-going industrial subsidy programme will have a permanent effect, implying a permanent fiscal cost with associated distortionary taxation. By contrast, the productivity effect here is transitory and has no fiscal implications.

Clearly, the importance of menu costs is a matter of some debate. We can take a narrow literal view of menu costs, in which case they are perhaps not so important. However, if we take a broader view (including costs of decision and elements of bounded rationality), then they are more significant. Either way, they are essentially the only theory of nominal rigidities that we have. The question remains as to whether the basic simple intuition underlying the menu cost idea is compatible with plausible magnitudes of parameters. In general, different type of imperfections may potentially contribute to price stickiness by increasing real rigidity as shown by Ball and Romer (1990). Alternatively, two recent papers have shown that asymmetries may have the same effect. Basu (1995) shows that an asymmetric production structure where some firms produce intermediate goods and other firms final goods may amplify the consequences of menu costs. Dixon and Hansen (1999) show that if the degree of imperfect competition is not the same across firms then this may reduce the menu costs required for rigidity and also boost the welfare consequences. Also, these two papers can explain the observed cyclicality of productivity, contrary to the previous literature.

What this paper shows along with Basu (1995) and Dixon and Hansen (1999) is that the pessimism of early papers was due to the representative sector framework and is not

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generic to menu costs outside that framework. The specific contribution of this paper is to show that labour market dualism, often emphasized as an important stylized view of the economy, significantly improves the standard model in terms of effective menu costs, welfare effects, and the cyclicality of productivity. In addition, this approach explains the compression of wage differentials during booms.

1 Derivation of eq. (17)

We start by deriving the number of individuals working in the primary and secondary labour market, respectively. The symmetry of the model implies that $L^P(i,j) = L^P \forall i, j$ and $L^S(i) = L^S \forall i$. Using this and eq. (9) give

$$\frac{L^P}{L^S} = \frac{\eta}{1-\eta}\left(\frac{W^P}{W^S}\right)^{-1} \tag{24}$$

which after insertion of (15) yields

$$L^P = \frac{\eta}{1-\eta}\frac{\sigma-1}{\sigma}L^S \tag{25}$$

Symmetry and equilibrium in the two types of labour markets imply

$$L^P = H^P N^P$$

$$L^S = H^S N^S = \left(1 - H^P\right) N^S$$

where H^S is the number of workers in the secondary market, H^P is the number of workers in the primary labour market, and $H^P(j) = H^P \ \forall j$. Since, $N^P = N^S$, cf. eq. (16), it follows from the three eqs above that

$$H^P = \frac{L^P/L^S}{1 + L^P/L^S} = \frac{(\sigma - 1)\eta}{\sigma - \eta} \tag{26}$$

$$H^S = \frac{1}{1 + L^P/L^S} = \frac{\sigma(1 - \eta)}{\sigma - \eta} \tag{27}$$

From the production function (8), we get

$$Y = \int_0^1 Y(i)\, di = \left(\left(L^P\right)^\eta \left(L^S\right)^{1-\eta}\right)^{1/\alpha}$$

which after insertion of (25) gives

$$Y = \left(\left(\frac{\eta}{1-\eta}\frac{\sigma - 1}{\sigma}\right)^\eta L^S\right)^{1/\alpha}$$

From eqs (6) and (27), we obtain

$$L^S = H^S N^S = \frac{\sigma(1-\eta)}{\sigma - \eta}\left(\frac{1}{\beta}\frac{W^S}{P}\right)^{\frac{1}{\beta-1}}$$

implying that

$$Y = \left(\left(\frac{\eta}{1-\eta}\frac{\sigma - 1}{\sigma}\right)^\eta \frac{\sigma(1-\eta)}{\sigma - \eta}\left(\frac{1}{\beta}\frac{W^S}{P}\right)^{\frac{1}{\beta-1}}\right)^{1/\alpha} \tag{28}$$

This yields a relationship between Y and W^S/P. In order to solve for Y, we now derive a similar relationship from eq. (13), which together with symmetry condition

$P(j) = P \ \forall j$ give

$$Y = \frac{M}{P} = \left(\frac{\theta \alpha}{\theta - 1}\frac{W}{P}\right)^{\frac{1}{1-\alpha}}$$

Insertion of eqs (10) and (15) yields

$$Y = \left(\frac{\theta \alpha}{\theta - 1}\eta^{-\eta}(1-\eta)^{-(1-\eta)}\left(\frac{\sigma}{\sigma - 1}\right)^{\eta}\frac{W^S}{P}\right)^{\frac{1}{1-\alpha}}$$

Combining this with eq. (28), and isolating for Y give eq. (17).

2 Derivation of eq. (19)

Note, that the first order condition (15) does not hold after a monetary shock if menu costs are sufficiently high to keep W_j^P fixed. We start by finding the aggregate labour demand in the secondary labour market from eqs (9) and (12). This yields

$$L^S = (1-\eta)\frac{W}{W^S}\left(\frac{M}{P}\right)^{\alpha} \tag{29}$$

The aggregate labour supply in the secondary labour market is defined as

$$H^S N^S = \left(1 - H^P\right) N^S = \left(1 - \frac{L^P}{N^P}\right) N^S$$

By inserting eqs (6) and (16), we obtain

$$H^S N^S = \left(\frac{1}{\beta}\frac{W^S}{P}\right)^{1/(\beta-1)} - L^P$$

Rewriting this eq. by inserting L^P derived from eq. (9), (12), $P_i = P \ \forall i$, and $W_j^P = W^P \ \forall j$, we have

$$H^S N^S = \left(\frac{1}{\beta}\frac{W^S}{P}\right)^{1/(\beta-1)} - \eta\frac{W}{W^P}\left(\frac{M}{P}\right)^{\alpha} \tag{30}$$

By equilibrating supply, eq. (30), and demand, eq. (29), in the secondary labour market, we get the following eq. determining the secondary wage level

$$
\left(\frac{1}{\beta} \frac{W^S}{P} \right)^{1/(\beta-1)} - \eta \frac{W}{W^P} \left(\frac{M}{P} \right)^{\alpha} = (1-\eta) \frac{W}{W^S} \left(\frac{M}{P} \right)^{\alpha}
$$

and after inserting (10) this yields

$$
\left(\frac{1}{\beta} \frac{W^S}{P} \right)^{1/(\beta-1)} = \left(\frac{\eta}{1-\eta} \frac{W^S}{W^P} \right)^{-\eta} \left(\frac{M}{P} \right)^{\alpha} + \left(\frac{\eta}{1-\eta} \frac{W^S}{W^P} \right)^{1-\eta} \left(\frac{M}{P} \right)^{\alpha} \tag{31}
$$

Now, take the total derivative w.r.t. M and W^S

$$
\begin{aligned}
\left(\frac{1}{\beta} \frac{W^S}{P} \right)^{1/(\beta-1)} \frac{1}{\beta-1} \frac{dW^S}{W^S} &= \left(\frac{\eta}{1-\eta} \frac{W^S}{W^P} \right)^{-\eta} \left(\frac{M}{P} \right)^{\alpha} \left(-\eta \frac{dW^S}{W^S} + \alpha \frac{dM}{M} \right) \\
&+ \left(\frac{\eta}{1-\eta} \frac{W^S}{W^P} \right)^{1-\eta} \left(\frac{M}{P} \right)^{\alpha} \left((1-\eta) \frac{dW^S}{W^S} + \alpha \frac{dM}{M} \right)
\end{aligned}
$$

Dividing on both sides with (31) gives

$$
\begin{aligned}
\frac{1}{\beta-1} \frac{dW^S}{W^S} &= \frac{1}{1 + \frac{\eta}{1-\eta} \frac{W^S}{W^P}} \left(-\eta \frac{dW^S}{W^S} + \alpha \frac{dM}{M} \right) \\
&+ \frac{\frac{\eta}{1-\eta} \frac{W^S}{W^P}}{1 + \frac{\eta}{1-\eta} \frac{W^S}{W^P}} \left((1-\eta) \frac{dW^S}{W^S} + \alpha \frac{dM}{M} \right)
\end{aligned}
$$

To evaluate the change in W^S around the initial equilibrium, we insert (15) in the above eq. and isolate dW^S/W^S. This gives

$$
\frac{dW^S}{W^S} = \frac{\alpha (1 - \eta/\sigma) (\beta - 1)}{\eta/\sigma (1-\eta) (\beta-1) + 1 - \eta/\sigma} \frac{dM}{M}
$$

or

$$
\zeta \equiv \frac{dW^S/W^S}{dM/M} = \frac{\alpha (1 - \xi) (\beta - 1)}{\xi (1-\eta) (\beta-1) + 1 - \xi}
$$

where $\xi \equiv \eta/\sigma$. QED.

3   Derivation of eq. (20)

The profit gain resulting from adjustment of prices is derived by making a second order Taylor approximation on (11) around the initial equilibrium, assuming that other output prices as well as primary wages are rigid. Letting V^1 denote the approximation of profits when firm i chooses not to adjust the price, and letting V^2 denote the approximation of profits when firm i chooses to adjust the price, we have

$$dV\left(i\right) \equiv V^2 - V^1 \approx \frac{1}{2} \frac{\partial^2 V\left(i\right)}{\left(\partial P\left(i\right)\right)^2} \left(dP\left(i\right)\right)^2 + \frac{\partial^2 V\left(i\right)}{\partial P\left(i\right) \partial M} dP\left(i\right) dM \qquad (32)$$

In (32) we have used the envelope theorem, i.e. $\partial V\left(i\right)/\partial P\left(i\right) = 0$. The profit function equals

$$V\left(i\right) = P\left(i\right) \left(\frac{P\left(i\right)}{P}\right)^{-\theta} \frac{M}{P} - W \left(\frac{P\left(i\right)}{P}\right)^{-\theta\alpha} \left(\frac{M}{P}\right)^{\alpha}$$

The first order derivative is given by

$$\frac{\partial V\left(i\right)}{\partial P\left(i\right)} = \left(\frac{P\left(i\right)}{P}\right)^{-\theta} \frac{M}{P} \left(1 - \theta\right) + \alpha\theta \frac{W}{P} \left(\frac{P\left(i\right)}{P}\right)^{-\alpha\theta - 1} \left(\frac{M}{P}\right)^{\alpha}$$

The second order derivatives evaluated in the initial equilibrium are

$$\frac{\partial^2 V\left(i\right)}{\left(\partial P\left(i\right)\right)^2} = \frac{M}{P^2} \left(1 - \theta\right) \left(1 - \theta + \theta\alpha\right) \qquad (33)$$

$$\frac{\partial^2 V\left(i\right)}{\partial P\left(i\right) \partial M} = \frac{1}{P} \left(1 - \theta\right) \left(1 - \alpha - \left(1 - \eta\right) \zeta\right) \qquad (34)$$

where we have used eq. (10) and the definition $\zeta \equiv \frac{dW^S/W^S}{dM/M}$. From (13) we have

$$dP\left(i\right) = P \frac{1}{1 - \theta + \theta\alpha} \left[\left(1 - \eta\right) \zeta + \alpha - 1\right] m, \qquad m \equiv \frac{dM}{M} \qquad (35)$$

By inserting (33), (34), and (35) in (32), we obtain

$$
\begin{aligned}
dV\left(i\right) \quad \approx \quad & \frac{1}{2}\left(1-\theta\right)\left(1-\theta+\theta\alpha\right)\left(\frac{1}{1-\theta+\theta\alpha}\right)^{2}\left[\left(1-\eta\right)\zeta+\alpha-1\right]^{2}m^{2}M \\
& +\left(1-\theta\right)\left(1-\alpha-\left(1-\eta\right)\zeta\right)\left(\frac{1}{1-\theta+\theta\alpha}\right)\left[\left(1-\eta\right)\zeta+\alpha-1\right]m^{2}M
\end{aligned}
$$

or equivalently

$$
dV\left(i\right)\approx\frac{1}{2}\left(\frac{\theta-1}{1-\theta+\theta\alpha}\right)\left[\left(1-\eta\right)\zeta+\alpha-1\right]^{2}m^{2}M
$$

Now, the aggregate minimum effective menu costs relative to GDP can be written in the following way

$$
\frac{dV}{Y}\approx F\left(\theta,\alpha,\eta,\zeta,m\right)\equiv\frac{1}{2}\left(\frac{\theta-1}{1-\theta+\theta\alpha}\right)\left[\left(1-\eta\right)\zeta+\alpha-1\right]^{2}m^{2}
$$

where $V\equiv\int_{i=0}^{1}V\left(i\right)di$. This eq. corresponds to (20). QED.

4 Derivation of eq. (21)

The utility gain of union j resulting from adjusting the wage in the primary labour market is derived by making a second order Taylor approximation on (5) around the initial equilibrium, assuming that other primary wages as well as output prices are rigid, i.e.

$$
dS\left(j\right)\equiv S^{2}-S^{1}\approx\frac{1}{2}\frac{\partial^{2}S\left(j\right)}{\left(\partial W^{P}\left(j\right)\right)^{2}}\left(dW^{P}\left(j\right)\right)^{2}+\frac{\partial^{2}S\left(j\right)}{\partial W^{P}\left(j\right)\partial M}dW^{P}\left(j\right)dM \tag{36}
$$

where S^{1} denotes the utility when worker i chooses not to adjust the wage and S^{2} denotes the utility when worker i chooses to adjust the wage. Note, that we have used

the envelope theorem, i.e. $\partial S(j)/\partial W^P(j) = 0$. The indirect utility is given by

$$
\begin{aligned}
S(j) &= \left(\frac{W^P(j)N^P(j)}{P} - N^P(j)^\beta\right)H^P(j) + \left(\frac{W^S N^S}{P} - (N^S)^\beta\right)(1 - H^P(j)) \\
&= \frac{W^P(j) - W^S}{P}L^P(j) + \frac{W^S}{P}N^S + (N^S)^\beta
\end{aligned}
$$

where we have used eq. (16) and the equilibrium condition $L^P(j) = H^P(j)N^S$.

After inserting (6), we get

$$
S(j) = \frac{W^P(j) - W^S}{P}L^P(j) + \left(\beta^{\frac{1}{1-\beta}} + \beta^{\frac{\beta}{1-\beta}}\right)\left(\frac{W^S}{P}\right)^{\frac{\beta}{\beta-1}}
$$

From (9), we have

$$
L^P(j) = \left(\frac{W^P(j)}{W^P}\right)^{-\sigma}\eta\frac{W}{W^P}\left(\frac{M}{P}\right)^\alpha \tag{37}
$$

which inserted in the above eq. gives

$$
S(j) = \frac{W^P(j) - W^S}{P}\left(\frac{W^P(j)}{W^P}\right)^{-\sigma}\eta\frac{W}{W^P}\left(\frac{M}{P}\right)^\alpha + \left(\beta^{\frac{1}{1-\beta}} + \beta^{\frac{\beta}{1-\beta}}\right)\left(\frac{W^S}{P}\right)^{\frac{\beta}{\beta-1}}
$$

The first order derivative equals

$$
\begin{aligned}
\frac{\partial S(j)}{\partial W^P(j)} &= \frac{1}{P}\left(\frac{W^P(j)}{W^P}\right)^{-\sigma}\frac{W}{W^P}\eta\left(\frac{M}{P}\right)^\alpha(1-\sigma) \\
&\quad + \sigma\frac{W^S}{P}\left(\frac{W^P(j)}{W^P}\right)^{-\sigma-1}\frac{W}{(W^P)^2}\eta\left(\frac{M}{P}\right)^\alpha
\end{aligned}
$$

The second order derivatives evaluated in the initial equilibrium become

$$
\frac{\partial^2 S(j)}{(\partial W^P(j))^2} = (1-\sigma)\frac{L^P(j)}{PW^P} \tag{38}
$$

and

$$
\frac{\partial^2 S(j)}{\partial W^P(j)\,\partial M} = (\sigma - 1)\zeta\frac{L^P(j)}{P}\frac{1}{M} \tag{39}
$$

33

where we have used (15) and (37).

From eqs (6) and (15) we obtain

$$\frac{W^P(j)}{W^S} = \frac{\sigma - 1}{\sigma} \Rightarrow \frac{dW^P(j)}{W^P(j)} = \frac{dW^S}{W^S} \Leftrightarrow dW^P(j) = W^P(j)\,\zeta m \tag{40}$$

Inserting the eqs (38), (39), and (40) into (36) implies

$$dS(j) \approx \frac{1}{2}(1-\sigma)\frac{N^P(j)}{PW^P}\left(W^P(j)\,\zeta m\right)^2 + (\sigma - 1)\,\zeta\frac{L^P(j)}{P}W^P(j)\,\zeta m^2$$

which can be rewritten as

$$dS(j) \approx \frac{1}{2}(\sigma - 1)\,\zeta^2 m^2\frac{W^P(j)\,L^P(j)}{P}$$

The aggregate minimum effective menu costs in proportion of GDP is given by

$$\frac{dS}{Y} \approx \frac{1}{2}(\sigma - 1)\,\zeta^2 m^2\frac{W^P(j)\,L^P(j)\,/P}{Y}$$

where $S = \int_{j=0}^{1} S(j)\,dj$. From eqs (9), (13), (15), and (17), we get

$$\frac{W^P(j)\,L^P(j)\,/P}{Y} = \frac{(\theta - 1)\,\eta}{\theta\alpha}$$

Thus, we have

$$\frac{dS}{Y} \approx G(\sigma, \eta, \theta, \alpha, \zeta, m) \equiv \frac{1}{2}(\sigma - 1)\,\zeta^2 m^2\frac{(\theta - 1)\,\eta}{\theta\alpha}$$

which corresponds to (21). QED.

5 Derivation of eq. (22)

The aggregate utility of the households can be written in the following way

$$U \equiv \int_0^1 \int_0^1 U(j, h)\,dj dh = Y - \left(N^S\right)^\beta$$

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Letting U^1 denote the utility level before the monetary expansion and U^2 denote the utility level after the expansion, a second order Taylor expansion gives

$$dU = U^2 - U^1 \simeq \frac{\partial U}{\partial Y}dY + \frac{\partial U}{\partial N^S}dN^S + \frac{1}{2}\left[\frac{\partial^2 U}{\partial Y^2}\left(dY\right)^2 + \frac{\partial^2 U}{\partial\left(N^S\right)^2}\left(dN^S\right)^2 + 2\frac{\partial^2 U}{\partial Y\partial N^S}dY\,dN^S\right]$$

Deriving the different derivatives of U with respect to Y yields

$$dU \simeq dY + \frac{\partial U}{\partial N^S}dN^S + \frac{1}{2}\frac{\partial^2 U}{\partial\left(N^S\right)^2}\left(dN^S\right)^2$$

Measuring relative to initial income gives

$$\frac{dU}{Y} \simeq \frac{dY}{Y} + \frac{1}{Y}\frac{\partial U}{\partial N^S}dN^S + \frac{1}{Y}\frac{1}{2}\frac{\partial^2 U}{\partial\left(N^S\right)^2}\left(dN^S\right)^2$$

$\Leftrightarrow$

$$\frac{dU}{Y} \simeq \frac{dY}{Y} + \frac{N^S}{Y}\frac{\partial U}{\partial N^S}\frac{dN^S}{N^S} + \frac{1}{2}\frac{\left(N^S\right)^2}{Y}\frac{\partial^2 U}{\partial\left(N^S\right)^2}\left(\frac{dN^S}{N^S}\right)^2$$

Deriving the different derivatives of U with respect to N^S yields

$$\frac{dU}{Y} \simeq \frac{dY}{Y} - \frac{\beta\left(N^S\right)^\beta}{Y}\frac{dN^S}{N^S} - \frac{1}{2}\beta\left(\beta - 1\right)\frac{\left(N^S\right)^\beta}{Y}\left(\frac{dN^S}{N^S}\right)^2$$

where

$$Y = \frac{M}{P} \qquad \Rightarrow \qquad \frac{dY}{Y} = \frac{dM}{M} = m$$

$$N^S = \left(\frac{1}{\beta}\frac{W^S}{P}\right)^{\frac{1}{\beta-1}} \qquad \Rightarrow \qquad \frac{dN^S}{N^S} = \frac{1}{\beta-1}\frac{dW^S}{W^S} = \frac{1}{\beta-1}\zeta m$$

By combining (6), (28), (17) we get

$$\frac{\left(N^S\right)^\beta}{Y} = \frac{1}{\alpha\beta}\frac{\theta-1}{\theta}\frac{\sigma-\eta}{\sigma}$$

Thus, we have

$$\frac{dU}{Y} \simeq W\left(\theta,\alpha,\xi,\beta,\zeta,m\right) = m - \frac{\theta-1}{\theta\alpha}\frac{1-\xi}{\beta-1}\left(\zeta m + \frac{1}{2}\zeta^2 m^2\right)$$

which corresponds to (10). QED.

6 Derivation of eq. (23)

Aggregate productivity is defined as $Q \equiv Y/L$ where $L = L^P + L^S = H^P N^P + \left(1 - H^P\right) N^S$ is aggregate employment measured in hours. This implies that the percentage change in productivity relative to the percentage change in GDP is given by

$$X\left(\beta, \zeta\right) \equiv \frac{dQ/Q}{dY/Y} = 1 - \frac{dL/L}{dY/Y}$$

The relative change in aggregate consumption/production, $\frac{dY/Y}{dM/M}$, is equal to one according to (12). The relative change in aggregate employment can be determined from (6), (16), and (19)

$$\frac{dL/L}{dM/M} = \frac{1}{\beta - 1} \frac{dW^S/W^S}{dM/M} = \frac{\zeta}{\beta - 1}$$

It then follows that

$$X\left(\beta, \zeta\right) = 1 - \frac{\zeta}{\beta - 1}$$

QED.

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